FROZEN EARTH IN SPITSBERGEN

by

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(Manuscript received 13th January, 1922)

1. In regions with a sufficiently low mean annual temperature, combined with slight snowfall, the subsoil is constantly frozen. A layer of snow acts as a good insulator, and the ground is thus protected against excessive cooling in countries with heavy snowfall.

Spitsbergen affords the natural condition for a formation of permanent frozen subsoil. The mean annual temperatur is about — 8° (centigrades), and the snow that comes during the winter, is blown together in heaps, mostly in cirques and other hollows. The snow is also to a great extent blown out of the valleys and deposited on the fjord-ice.

If the temperature is supposed to rise uniformly with one centigrade per 40 metres, the layer of frozen subsoil — earth and rock — should attain the enormous thickness of about three hundred metres, or one thousand feet. In the Swedish Coal Mine (Sveagruvan) in Lowe Sound a temperature of 0° C. was actually found in a depth of about 320 metres below the surface, and 430 metres from the mouth of the level adit. The rock is quite dry.

During the summer the soil thaws to a depth of some two feet, and the ground is constantly water-soaked. The frozen soil is a very important factor in all sort of ground work, diggings and foundations. Frost-safe cellars do not exist in this country; and if the stability of a stone wall or pillar must be absolutely certain, it must be founded on solid rock. The chief difficulty seems to be, that a concrete wall, for instance, acts as a better heat conductor than the water-soaked soil, and may therefore cause thawing at the bottom. If the foundation is sufficiently deep in the frozen soil — five feet or so — there is generally no danger.

The frozen soil continues for some distance under the seabottom; but the upper limit slopes down at a quicker rate than the beach, and disappears farther out under the sea.

The conditions are somewhat different at different places, chiefly owing to the variable temperature of the sea-water, and the profile of the beach. At the pier of the Swedish Mine, the ice was found to stretch under the seabottom to a distance of about one hundred metres from the shore, according to information gathered on the place.

I have made some theoretical speculations as to the explanation of this peculiar distribution of the frozen soil. To obtain a result, we must start with some simplifying assumptions.

2. As a matter of fact, the temperature rises downwards in the ground. The measurements are rather different, but we will here assume that the temperature gradient

is constant, 0°025 per metre, when the conditions are not disturbed by other causes. — The effect of the annual variation of temperature does not reach far down, and may be disregarded. During the summer the most part of the heat is used to melt the frozen earth, and in the autumn the sod will be frozen again. By this process the amplitude of the annual variation is greatly diminished in the layers immediately below the surface. We will therefore assume that the temperature of the surface of the land is constant, T' degrees. This quantity is negative. The temperature of the sea is about zero, or a little higher. As a first approximation, we put this temperature = zero. Moreover, we will assume that the coast is low and flat, and the sea very shallow, so that a section across the coast does not deviate much from a level straight line. This is approximately the case along the west coast of Spitsbergen. The coast line is assumed to be straight. For further considerations, a system of coordinates is introduced. The XY plane is perpendicular to the coast line; the X axis is horizontal, and the Y axis points vertically downwards. The Z axis coincides with the coast line. Now the temperature is determined in every point of the XZ plane thus: for positive values of x, the temperature $T = 0$; for negative x, the temperature is: $T = T'$. With increasing distance from the origin, the temperature shall more and more approach to a simple proportionality with the ordinate y; the temperature is supposed to be independent of the coordinate z. If the soil is supposed to be homogenous in respect to temperature and heat conduction, and the conditions stationary, the following differential equation holds good:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad \text{(I)}$$

We will now divide the temperature function in two parts:

$$T = T_1 + T_2$$

The equation (I) must be fulfilled by both parts of the function. Now we put:

$$T_1 = \frac{T'}{\pi} \cdot u$$

where u denotes the angle between radius vector and the positive X axis. This function fulfils the condition (I) and gives for:

$$u = 0 \ , \ T_1 = 0 \ , \text{ but for } u = \pi \ , \ T_1 = T' \ .$$

But $u = 0$ corresponds to any point on the positive X axis, and $u = \pi$, to any point on the negative X axis. We have:

$$u = \operatorname{arctg} \frac{y}{x}$$

and by introducing this in the expression for T_1, we optain (Fig. 1):

$$T_1 = \frac{T'}{\pi} \cdot \operatorname{arctg} \frac{y}{x}$$

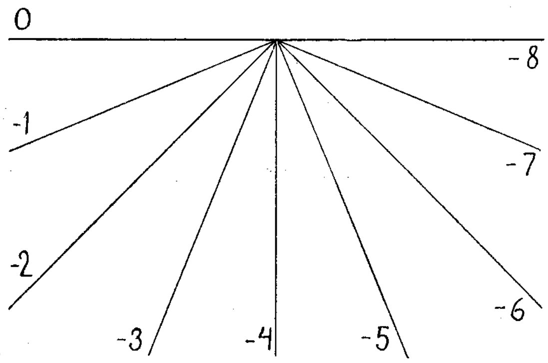

Fig. 1. Field of temperature represented by the function: $T = -\frac{8}{\pi} \cdot u$

Further we will put: $T_2 = qy$

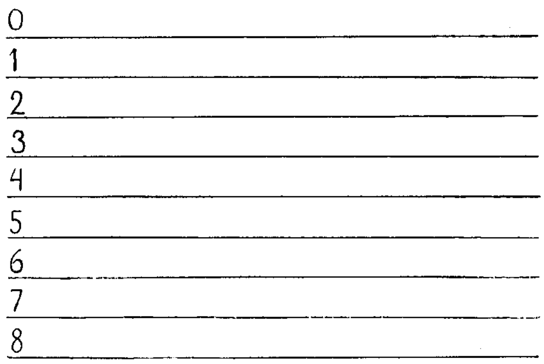

Fig. 2. Field of temperature represented by the function: $T = qy$.

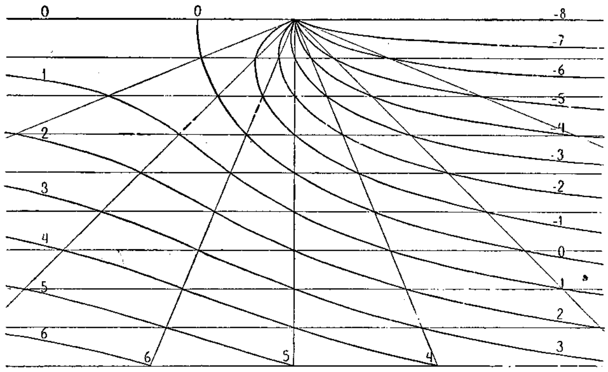
Fig. 3. Field of temperature obtained by superposition of the fields represented by Figs. 1 and 2, that is, the function: $T = qy - \frac{8}{\pi} \cdot u$

which function too fulfils the condition (I), and accounts for the downward increase in temperature (Fig. 2). By a simple superposition of the fields we obtain (Fig. 3):

$$T = qy + \frac{T'}{\pi} \cdot \operatorname{arctg} \frac{y}{x} \qquad \text{(II)}$$

This function fulfils the conditions. By putting:

$$-\frac{T'}{\pi q} = h$$

the equation (II) can be written:

$$T = q\left(y - h \operatorname{arctg} \frac{y}{x}\right)$$

By derivation is obtained:

$$\frac{\partial T}{\partial x} = \frac{qhy}{r^2}, \quad \frac{\partial T}{\partial y} = q - \frac{qhx}{r^2}$$

where:

$$r^2 = x^2 + y^2$$

The magnitude of the gradient of temperature is found:

$$G = q(1 + (h^2 - 2hy)/r^2)^{\frac{1}{2}}$$

and the direction angle of the gradient, w, is determined by:

$$\operatorname{tg} w = \frac{r^2 - hx}{hy}$$

It is seen, that the gradient G approaches to the value $G = q$, with increasing distance r from the origin, and simultaneously, the direction becomes more and more parallel to the Y axis. The second condition is thus also fulfilled.

3. The equation (II) represents the field of temperature. By putting $T =$ a constant an isothermal curve is obtained. All these isotherms pass through the origin. The direction of the tangents in this point is found by writing the equation thus:

$$\frac{y}{x} = \operatorname{tg} \frac{qy - T}{qh}$$

and then:

$$\lim_{\substack{x=0 \\ y=0}} \left(\frac{y}{x} \right) = \operatorname{tg} \frac{-T}{qh} = \operatorname{tg} u$$

The singular point in the origin is a »knot«. Another singular point is situated in: $x = h$, $y = 0$. Here the curve $T = 0$ passes with two branches, one parallel to the X axis, the other parallel to the Y axis. This singular point is a »hyperbolic« point in the field. — We do only consider the part of the XY plane with positive ordinates y, that is, the part situated below the surface. In this way, we need not care as to the multiple values of the periodic function arcustangens. But geometrically the field is symmetrical in regard to the X axis.

After some reckoning all the inflexion points of the curves $T =$ constant are found to lie on the straight line $x = h$.

It may be noted too, that the vertical temperature gradient

$$\frac{\partial T}{\partial y} = q \left(1 - \frac{hx}{r^2} \right)$$

is equal to 0 in all points of the circle:

$$(x - \tfrac{1}{2}h)^2 + y^2 = (\tfrac{1}{2}h)^2$$

This circle passes through both singular points. In points inside the circle, the vertical temperature gradient is negative, in the field outside the circle, it is positive.

The field can be drawn graphically as a superposition of the two fields for T_1 and T_2 , which are easily constructed.

But in this way it is not possible to determine the exact position of the singular point $(h, 0)$.

By introducing the most probable values:

$$T' = -8^\circ$$

$$q = 0.025^\circ \ pr.\ m.,$$

there is obtained:

$$h = \text{about } 100 \text{ metres.}$$

The frozen soil should stretch below the sea-bottom to a distance of 100 metres from the shore, and should here drop off vertically. Now the temperature of the sea-water generally lies a little above zero, and the actual upper limit of the frozen soil will drop off in a more gradual slope. The singular point, where all isotherms should pass, does not of course exist in reality; here the temperature gradient should be infinitely great, which is impossible. But there must be a rapid transition in the temperature of the soil in a belt along the sea margin, and we may expect that the distribution of temperature is approximatively correct, as represented by equation (II).[1]

4. We shall now consider the theoretical distribution of temperature in a more complex case. Suppose a shallow fjord lies between low flat land on both sides; the temperature of the sea-water is zero, and that of the surface of the land is equal to T' degrees centigrade, for instance, — 8°. The fjord may be straight, with constant breadth. On a section, perpendicular to the trend of the fjord, the sea-bottom and the surface of the adjoining land are presented by an approximately level line. This line is chosen as the X axis, with the origin in the middle of the fjord; the Y axis points downwards. The breadth of the fjord is put $= 2b$. By a simple superposition (Figs. 4 and 2) the temperature function is found:

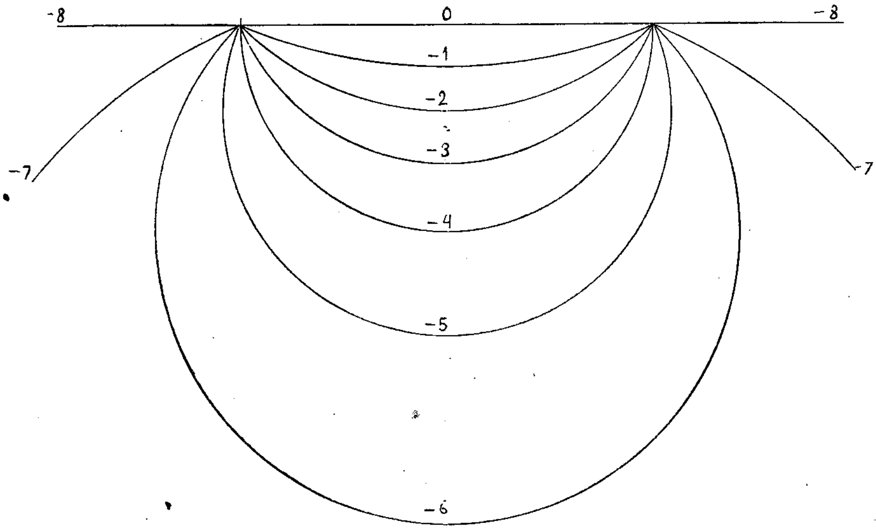

Fig. 4. The field represented by the function: $T = \frac{8}{\pi} \cdot \text{arctg} \frac{2by}{r^2 - b^2}$

$$T = qy - \frac{T'}{\pi} \cdot \text{arctg} \frac{2by}{r^2 - b^2}$$

and by putting:
$$-\frac{T'}{\pi} = qh$$

the equation is obtained in the following form:

$$T = q \left(y + h \ \text{arctg} \frac{2by}{r^2 - b^2} \right) \qquad \text{(III)}$$

By variation of the parameter T this equation represents a series of curves, which all pass through the points $y = 0$, $x = \pm b$. These points are singular, of the kind named »knots«. Moreover there are two other singular points in the XY plane. By differentiation is obtained:

[1] According to *Dr. Bertil Högbom* (Über die geologische Bedeutung des Frostes, Bull, of the Geol. Inst. of Uppsala, Vol. XII, p. 257) the temperature of the rock is — 5° in the Swedish Mine, at a distance of 25 metres from the opening. This would correspond to a mean temperatur of about — 6° of the soil of the surface. The distance h then becomes 76.5 m.

$$\frac{\partial T}{\partial x} = \frac{4qhbxy}{(r^2 - b^2)^2 + 4b^2y^2}$$

$$\frac{\partial T}{\partial y} = q + 2qhb \frac{x^2 - y^2 - b^2}{(r^2 - b^2)^2 + 4b^2y^2}$$

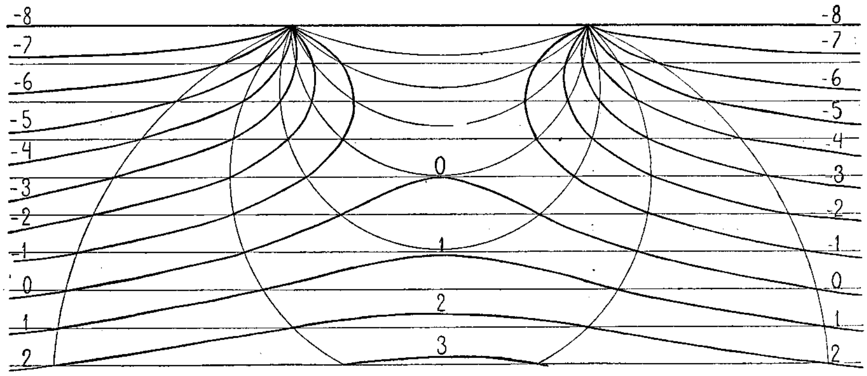

Fig. 5. The field represented by the function: $T = qy + \frac{8}{\pi} \cdot \operatorname{arctg} \frac{2by}{r^2 - b^2}$
where: $\pi qb < 16$, that is, $b < 2h$

Singular points must be situated either in:

$$x = 0\ ,\ y = \pm b \cdot \sqrt{2h/b - 1}$$

or in:

$$y = 0\ ,\ x = \pm b \sqrt{1 - 2h/b}$$

The two singular points are situated on the Y axis or on the X axis, according as b is less or bigger than $2h$. In character, these singular points are »hyperbolic« points or

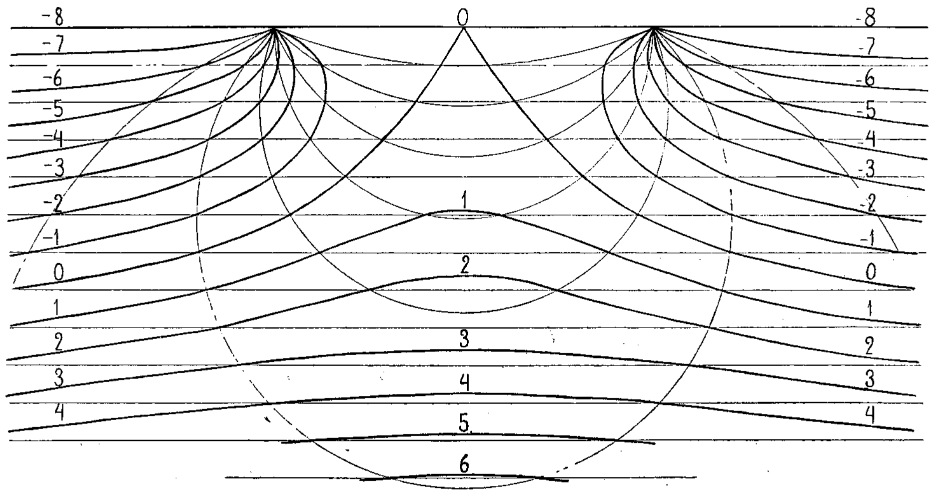

Fig. 6. The field represents the same function as Fig. 5, but here:
$\pi qb = 16$, that is, $b = 2h$

»cols«. In case they are situated on the X axis, the curve $T = 0$ passes through both, one branch coinciding with the X axis, the other intersecting this one at right angles.

If the term b should happen to be exactly like $2h$, a complex singular point is situated in the origin, a sort of triple hyperbolic point, with three branches of the curve

$T = 0$, intersecting each other at angles of 60°. (The prototype of such points may be given by the equation $r^3 \sin 3\varphi = a^3$ in polar coordinates. Here a is an arbitrary parameter). This is verified by writing the equation $T = 0$ thus:

$$2by + (r^2 - b^2) \operatorname{tg} \frac{y}{h} = 0$$

The tangent is developped in a series, and the quotient y/x is found for points approaching to $0,0$.

The curve $T = 0$ marks the limits of the area of temperatures below zero. This area is continuous below the fjord, if b is less than $2h$ (Fig. 5). In the opposite case, (Fig. 7), the area of frost is divided in two distinct parts, by an intervening area of temperatures above zero, along the central parts of the fjord; in the case $b = 2h$, these two parts just meet in the middle of the fjord (Fig. 6).

The conclusion is that a fjord must be at least 400 metres broad, if the soil below the bottom shall not be frozen all over, from one beach to the other. — If the temperature of the sea-water is above zero, the conditions are modified.

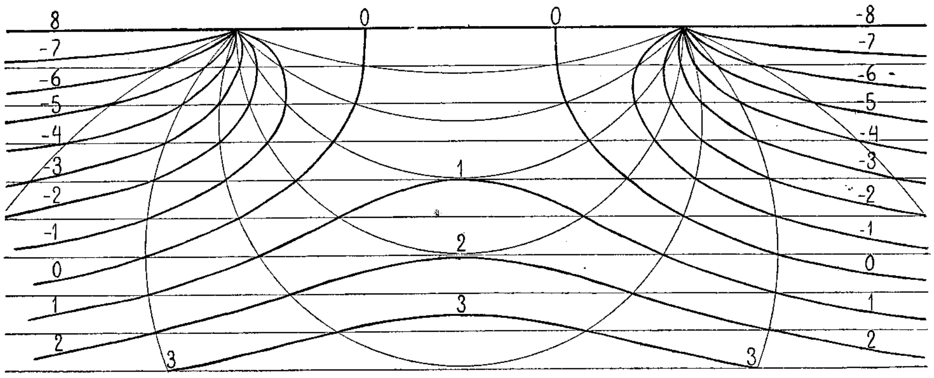

Fig. 7. Same function as Figs. 5 and 6, but now: $\pi qb > 16, b > 2h$

5. Below an active glacier the temperature is practically at the melting point of ice. Water is streaming forth from the glaciers at winter times too. If now a glacier is more than 400 metres broad, the frozen soil cannot reach below the ice, from one margin to the other; along the central parts, there will be a zone of unfrozen ground. The mean temperature of the environs is supposed to be — 8° C. — Below the central parts of large glaciers water may soak down into the ground, and pass below the water-tight layer of frozen ground. In this way it is possible to account for the ground water, that gives rise to rather numerous springs in the floor of some of the broad flat valleys in the central parts of West Spitsbergen. The temperature of the water of these springs may be only a little above zero, as contrasted to the springs of (late) volcanic origin, the water-temperatures of which even rise so high as 28° C. The water of all springs, however, seems to be slightly saline. The cold springs are surrounded by accumulations of ice and sod, and look like mud volcanoes. It is clear that the water that runs out of these springs, must come from somewhere, and if it is not all to be considered as »juvenile« — coming from the unknown interior — it must come from the surface water, and in some way or other it must be able to pass through the thousand feet thick crust of frozen ground.

The explanation set forth in this paper may with some reason be considered to be too purely speculative; but this is mainly owing to the sparseness of investigations into the actual conditions near the seashore, as to the value of the true mean annual temperature of the sea-water, and the exact position of the upper limit of the frozen ground. In Advent Bay, for instance, the frozen ground drops off towards the sea with a considerable slope; this fact indicates, that the sea water has a mean temperature somewhat above zero, say, 1.5 to 2°. It is seen from the diagrams, that a comparatively slight increase in the temperature of the water will cause a rather rapid descent of the curve $T = 0$. In Braganza Bay, at the Swedish Mine, the mean temperature of the sea water seems to be practically at the melting point of ice.

As to the condition of the frozen ground below the glaciers, actual investigations would be extremely difficult and expensive, though not necessarily impossible. A point that ought to be fixed, is the question of the temperature of the ice, in some deeper bore-holes.

As to the thermal conductivity of the soil, there is no doubt some difference, whether the ground is dry, water-soaked, or frozen; but this fact had to be neglected in the theoretical discussions, as an introduction of different co-efficients for this property would make the calculations very complex, and, moreover, had to be founded on quite uncertain assumptions.