INTRODUCTION

The determination of tidal motions of the earth's crust has been made either by means of horizontal pendulums or, as being the case by A. A. Michelson's method,[1]) by means of different instruments for very exact measurements of the variations of the height of the water surface at the ends of long pipes half-filled with water. The latter method has been used in a modified form at the Danish Meteorological Institute.

From thorough experiments it was concluded that the instruments used at the Danish Meteorological Institute for the above-mentioned measurements of the water levels, might be well qualified for measurements of the tidal motions of the earth's crust.

On the basis of this result, the Geophysical Institute, Bergen, and the Danish Meteorological Institute, Copenhagen, made an agreement, according to which, the two Institutions jointly, as a *preliminary* work, should make observations of the motion of the earth's crust. These measurements were to be carried out in a 125 m long tunnel belonging to the Geophysical Institute, Bergen, by means of the instruments made and adjusted at the Danish Meteorological Institute. The object of the measurement should be to determine not only the tides of the solid earth, but also the variations in the position of the earth's crust produced by the varying loading originating from the tides of the sea.

During the days from the 4th to the 9th June 1934 the instruments were installed in the tunnel, and from the 9th to the 25th June 1934 readings were made every hour. The setting up of the instruments and the observations were made by Dr. J. E. FJELDSTAD, assisted by Messrs. AABREK, OLSEN and JAKHELLN, all on the staff of the Geophysical Institute, Bergen, and by J. EGEDAL from the Danish Meteorological Institute.

The authors.

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[1]) A. A. MICHELSON: Preliminary results of measurement of the rigidity of the Earth. — *The Astrophysical Journal,* March 1914, vol. XXXIX. No. 2.

OBSERVATIONS

BY J. EGEDAL

The Niveauvariometer.

For the measurements of the motion of the earth's crust made in the tunnel at Bergen, the method of A. A. MICHELSON has been used. This method requiring very exact measurements of the variations of the height of the watersurface in the long pipe, the observations have been made by means of a very sensitive variometer called *Niveauvariometer.*[1])

The Niveauvariometer consists of a cylindric glass vessel, 7 cm high and 14 cm in diameter. The vessel is partly filled with water communicating with the water in the pipe. In order to avoid evaporation, the water in the variometer is covered with a layer of Nujol, 1—2 cm thick. A cylindric glass float, 1.7 cm high and 5 cm in diameter, is floating on the Nujol. The vertical movements of the float caused by vertical movements of the surface of the Nujol are transmitted by a vertical glass rod to a mirror arrangement.

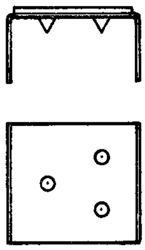

Fig. 1. Mirror arrangement. Seen sidewards (a) and from below (b).

The latter (see Fig. 1 a) consist of a 1.6 cm broad and 0.03 cm thick piece of brass, formed as indicated in Fig. 1. A stainless metal pivot attached to the under side of the brass piece rests in a hole in a small glass plate on the top of the glass rod, while two other pivots (see Fig. 1 b) rest in holes on a fixed glass plate; the latter is attached to a brass beam fastened near its ends to the edges of the glass vessel. The mirror is fixed to the top of the brass piece. The pivots form an isosceles triangle with the base at the two pivots on the fixed glass plate. The heights of the triangles in the Niveauvariometers used at Bergen were about 0.68 and 0.70 cm, respectively.

Above the mirror is placed a total reflecting prism, and by this arrangement the variations of the mirror's position can be observed in the horizontal plane by means of a telescope.

Thus, the variations in the height of the Nujol will be transmitted by means of the float to the mirror arrangement acting as a sort of lever, and further the resulting variations in the position of the mirror can be observed in a telescope furnished with a scale (1 division = 1 mm). The position of the telescope can be controlled by means of a mirror fastened to the Niveauvariometer.

The manner of action of the Niveauvariometer: In Fig. 2 the mirror arrangement is shown. The letter P_1 designates the point of the pivot resting on the rod of the float, P_2 designates the projection of the line connecting the points of the pivots

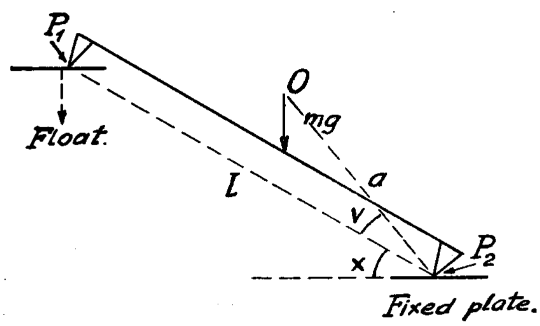

Fig. 2. Mirror arrangement.

[1]) Cfr. J. EGEDAL: On an apparatus for registration of variations in the position of the earth's crust with respect to the plumb-line. *Rep. 18. Scand. Naturalist Congress in Copenhagen,* 26.—31. Aug. 1929.

resting on the fixed glass plate, and l is the distance from P_1 to the just mentioned line. Thus the quantity l is the lenght of the arm of the lever. The angle which the line l forms with the horizontal plane is called x. Calling the height of the float h_f, this quantity can be expressed by the following equation:

$$h_f = h_0 + l \sin x, \qquad (1)$$

h_0 corresponding to the height of the surface of the fixed plate. Therefore, for a small variation Δh_f, we have

$$\Delta h_f = l \cos x \, \Delta x \qquad (2)$$

Now we will examine how a variation of the height of the Nujol, Δh_w, is related to the resulting variation in the height of the float, Δh_f. From tests it is concluded that effects of friction at the pivots and of surface tensions in the Nujol immediately near the float can be disregarded,[1]) but other forces may alter the difference between the height of the float and the height of the Nujol. Calling the variation in this difference, corresponding to a variation of the height of the Nujol Δh_w, Δh_{fw}, we have the following equation:

$$\Delta h_f = \Delta h_w + \Delta h_{fw} \qquad (3)$$

and, by (2)

$$l \cos x \, \Delta x = \Delta h_w + \Delta h_{fw} \qquad (4)$$

The quantity Δh_{fw} depends on the variations in the pressure of the mirror arrangement on the float, and we will therefore examine the equation of equilibrium of the float-mirror system. When

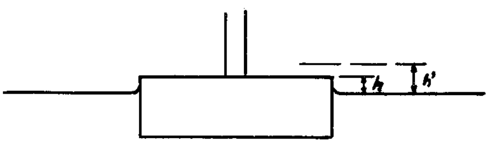

Fig. 3. The float.

friction at the pivots and surface tensions are disregarded, the equation of equilibrium may be written (Fig. 2 and Fig. 3):

$$mga \cos(x + v) - A\varrho\,(h' - h)\, gl \cos x = 0 \qquad (5)$$

where m is the mass of the mirror arrangement, g the accelleration of gravity, a the distance from the centre of gravity of the mirror arrangement, O, to the point P_2, v the angle between two planes both containing the line connecting the points of the two pivots on the fixed glass plate, the one containing the centre of gravity, O, the other the point of the pivot P_1, A the area of the base of the float, ϱ the specific gravity of the Nujol, and h' and h the height of the float in relation to the surface of the Nujol, respectively before and after the placing of the mirror arrangement.

From (5) we have

$$h' - h = \frac{ma \cos(x + v)}{A\varrho l \cos x}$$

which can be written

$$h' - h = \frac{ma}{A\varrho l}(\cos v - \operatorname{tg} x \sin v) \qquad (6)$$

On differentiating we get

$$\frac{\partial x}{\partial h} = \frac{A\varrho l \cos^2 x}{ma \sin v} \qquad (7)$$

and for small variations of h we have

$$\Delta x = \frac{\partial x}{\partial h} \Delta h_{fw}$$

or

$$\Delta x = \frac{A\varrho l \cos^2 x}{ma \sin v} \Delta h_{fw} \qquad (8)$$

From (4) and (8) we get

$$l \cos x \, \Delta x = \Delta h_w + \frac{ma \sin v}{A\varrho l \cos^2 x} \Delta x$$

or

$$\left(l \cos x - \frac{ma \sin v}{A\varrho l \cos^2 x}\right) \Delta x = \Delta h_w \qquad (9)$$

In order to get the relation between the variation of the height of the Nujol (Δh_w) and the readings from the millimetre scale, the quantity Δx (9) should be replaced by the quantity Δn giving, in millimetres, the variations observed on the scale. The relation between Δx and Δn is

$$\Delta x = \frac{\Delta n}{2L} \qquad (10)$$

where L is the length in millimetres of a beam of light from the reflecting part of the mirror to the centre of the scale after a reduction equal to one third of the way which the beam passes through glass.

From (9) and (10) we get

$$\left(l \cos x - \frac{ma \sin v}{A\varrho l \cos^2 x}\right) \frac{\Delta n}{2L} = \Delta h_w \qquad (11)$$

[1]) In a series of tests made earlier the observed height of the Nujol after a variation of 200 μ ($1\,\mu = 0.001$ mm) was changed about 0.2 μ. In another series, the pivots were placed on a polished glass plate and not in holes, and in this case the change corresponded to 0.03 μ.

Thus the Niveauvariometer has the following scale value:

$$\frac{l}{\frac{\Delta n}{\Delta h_w}} = \frac{1}{2\,L}\left(l\cos x - \frac{ma\sin v}{A\varrho l\cos^2 x}\right) \qquad (12)$$

For $l = 0.7$ cm, $a = 0.35$ cm, $v = 10^\circ$, $m = 2$ g, $A = 30$ cm², $\varrho = 0.8$ and x small, the second member of the parenthesis is only 1 per cent of the first one, and while the construction of the mirror arrangement has been made in such a manner that the angle v became as small as possible, the scale value may be considered as approximately equal to $\frac{l}{2L}$. It should be added that both of the members in the parenthesis (12) only vary when x varies, and that, if the quantities l, a, v, m, A and ϱ are those given above, a variation of x from 0° to 5° will change the value of the difference in the parenthesis less than a half per cent. Thus, the scale value can be considered as sufficiently constant in the interval in which the measurements are to be taken.

The height H of the surface of the liquid of the Niveauvariometer over a certain fixed point can be expressed thus:

$$H = H_0 + an + f(t) \qquad (13)$$

where H_0 is the zero-point value, a the scale value, n the reading on the scale and $f(t)$ a function of the temperature.

Of course, variations of the temperature will change the dimensions of the material of which the Niveauvariometer is made. Let us consider the movements caused by temperature variations of the «fixed points» on which the two pivots of the mirror arrangement rest. Between the glass plate for the «fixed points» and the pillar on which the Niveauvariometer is placed, there is first a few millimetres thick brass beam, and then the 7 cm high glass vessel. If the temperature is raised by 1 degree (centigrade), the height of the «fixed points», and consequently also of the two pivots, will, in relation to the pillar, be changed by about 0.6 μ (1 μ = 0.001 mm), a quantity which is not at all neglectable in the present case. The change in the height of a 1 m high pillar for a rise of 1 degree in its temperature amounts to about 10 μ. The effect of the temperature variations on the Niveauvariometer as a whole will not exceed the above found change of 0.6 μ per degree, and consequently be rather small compared with the effect on the pillar, and therefore it will be of no real importance to examine this effect on the Niveauvariometer more closely, or to determine a proper temperature-correction of its readings.

The Hydrostatic Levelling Instrument.

The Hydrostatic Levelling Instrument constructed by D. la Cour for very precise levelling, has been used in the present case only for the determination of the scale values of the Niveauvariometers.

A full description of the details of the instrument is not to be given here, it shall only be mentioned that it consists of a cylindric glass vessel, 3 cm high, with a diameter of 4.5 cm, and that all supporting parts of the instrument consists of invar-rods. The undermost part of the instrument consists of some invar-rods placed one above the other in a 75 cm long gun-barrel. The bottom rod is at its lower end furnished with a thin steel plate, polished at its under side. Thus, the whole instrument is resting on this steel mirror. At the top, just over the centre of the vessel is found a micrometer screw, the axis of which is placed vertically. The lower part of the micrometer screw is furnished with a glass point.

The instrument is used in the following manner: When the water of which the height is to be measured has been placed in the vessel, the micrometer screw is placed at such a height that the glass-point is just over the water. Now the micrometer screw is cautiously screwed downwards until the glass-point just touches the water surface, then the water immediately will rush on the glass point, and at this very moment the observer has to stop the screwing and thereafter to read the micrometer. With great care, the experienced observed will be able to make the observations so well that the standard deviation of a single observation will be about 1—2 μ. The application of the instrument for the determination of the scale values of the Niveauvariometers will be mentioned below.

The installation of the instruments.

The instruments were installed in the tunnel belonging to the Geophysical Institute, Bergen. Through a shaft, 18 m deep, a staircase leads from the basement of the Institute direct to the tunnel. The tunnel which had previously been used for railway traffic is 125 m long and is closed at both ends. The floor inclines nearly 1 m from one end to the other. The direction of the tunnel is approximately from SSW to NNE. The temperature variations in the tunnel are very small, and, as will be seen from the tables below, they are taking place very slowly. At one end of the tunnel there is a spring, the water of which is flowing down the rocky walls, and on account of this the degree of moisture in the tunnel is very high. Disregarding this latter point, the tunnel is the ideal one for measurements of earth-crust motions.

For the Niveauvariometers and the telescopes four pillars were set up, those for the Niveauvariometers were placed 103.13 m from each other. Owing to the distance mentioned a relative height-variation of the pillars of 1 μ corresponds to an inclination equal to 2 thousandths of a second of arc. The distance between the pillar for the Niveauvariometer and the corresponding pillar for the telescope is 3.5 m. The lowest pillar for Niveauvariometer had a height of 93 cm and the highest one of 177 cm. For the constructions of the pillars, cement drain-pipes were used, and the pillars were covered at the top with a slab of slate.

The pipe through which the water in the Niveauvariometers should communicate had to be placed in the horizontal plane and in a fixed position. The putting up of the pipe was mainly carried out by Mr. Aabrek. The work was done with much care and in the following manner. Many beams of wood (40), each supported by four legs, were placed in a vertical position between the pillars of the Niveauvariometers. By means of levelling, the position in which the pipe should be placed was marked down on all beams, and hooks on which the pipe could be laid were screwed into the beams at a proper height.

The pipe used consisted of galvanized iron pipes 2.7 cm in clear diameter. The pipes were carefully screwed together in order to make the pipe air-tight. The pipe was connected with the Niveauvariometers by means of rubber tubes. After having placed the pipe in proper position it was washed out by means of distilled water, and finally distilled] water was conducted into the pipe from both ends in order to keep clean the water nearest to the Niveauvariometers. Distilled water was conducted into the pipe in such a quantity that a mean height of the water of 2.0 cm could be obtained. The air in the pipe communicates with the atmospheric air through a hole made in the middle of the pipe.

The azimuth of the pipe was determined by means of observations of the sun, and the result was:

$$\text{Azimuth of pipe} = +\,18.^\circ 3.$$

The azimuth also has been determined by means of a magnetic needle; putting the magnetic declination at Bergen for 1934.5 equal to 10.°5 W the azimuth found was = 17.°2. The two results agree with one another as well as might be expected, but, of course the observations of the sun give the most accurate result.

The Niveauvariometers were fixed on their pillars by means of a sort of plastic-clay («Plastilina»). Experiences have shown that the Nujol at its first exposure to the air, evaporates rather quickly, and therefore the Nujol used had been exposed to the air for a long time before being used. For the protection of the Niveauvariometers a cover with a glass window was placed over each of them, and in order to keep the air at the Niveauvariometer dry, some $CaCl_2$ was placed under the cover. During the first measurements the cover was kept air-tight, but this caused even small temperature variations of the air under cover through their influence on the air pressure to have a great effect on the readings, and in order to prevent such effects, a small hole was made for the passage of the air.

The telescopes were placed on their respective pillars. On account of the very damp air in the tunnel, the wire of each of the telescopes, a spider-line, became flaccid, but was stretched out again by means of a small weight.

For the placing of the Hydrostatic Levelling Instrument a brass bolt was fixed near each of the pillars for the Niveauvariometers. At the highest pillar the bolt was placed on the top of another pillar, 50 cm high.

After having joined the Niveauvariometers by means of the pipe, the duration of a free oscillation of the water in the pipe was examined. Some distilled water was placed in the Niveauvariometer called No. 2, during the time from $13^h\,00^m\,40^s$ to $13^h\,01^m\,00^s$, and the extremes of the readings at the Niveauvariometer No. 1 and the time of their occurrence were as follows:

$13^h\,02^m\,05^s$	232.0	(Max.)
02 50	208.0	(Min.)
03 50	246.0	(Max.)

Thus the duration of one oscillation is about one minute. It must be added that the oscillations recede very quickly.

The determination of the scale value of the Niveauvariometers.

On the installation of the Niveauvariometers in the tunnel the telescopes and the scales were placed at such a distance from their corresponding Niveauvariometers that a scale value of about 1 μ/mm could be expected.

The scale values of the two Niveauvariometers have been controlled in two ways, namely by means of direct determinations of the scale value of each of the Niveauvariometers, and by means of determinations of the ratio between the scale values.

The direct determinations of the scale values have been made by means of the Hydrostatic Levelling Instrument. The water in the Niveauvariometer was brought into connection with the water in the reservoir of the instrument, and for different heights of the water level, readings of the considered Niveauvariometer were made contemporary with the determination of the height of the water level by means of the Hydrostatic Levelling Instrument.

In Table 1 below the results from these determinations are given.

Table 1. Direct Determinations of the Scale Values.

Date	Niveauvar. No. 1.		Niveauvar. No. 2.		Remarks
	Observations μ/mm	Mean μ/mm	Observations μ/mm	Mean μ/mm	
1934 June 6			1.000		
			1.012		
7			1.078		Friction
			1.046		
			0.998		
			1.015		
					Pivots adjusted
8	1.028				
	1.055	1.039			
	1.034				
15			1.107		
			1.091		
			1.071		
			1.075	1.079	
			1.066		
			1.066		
25			0.999		
26			1.012	1.008	
			1.012		
	0.982				
	0.991	0.995			
	1.012				
Mean μ/mm		1.017		1.043	

On account of an adjustment of the pivots of the Niveauvariometer No. 2 made before the commencement of the hourly readings of the Niveauvariometers the determination made on the 6th and the 7th of June can not be used. For the determination of scale values, only four series are at hand. From Table 2 it will be seen that the determinations made on the 8th and 15th of June give considerably higher values than those made on the 25th and the 26th of June. The scale values of the single series agree closely, while the *means* of the series for the same Niveauvariometer differ much from one another, and therefore the simple mean of the means of each series has been considered to give the best value. The found scale values of the Niveauvariometers are 1.017 μ/mm and 1.043 μ/mm respectively, and the following values are adopted:

No. 1.	No. 2.
1.02 μ/mm	1.04 μ/mm

On account of vertical movements of the pipe, it has been necessary now and then to change the height of the water level in the pipe and the Niveauvariometers. The same changes have also been used for the determination of the ratio of the scale values. The variation of the height of the water level will in such cases be the same at both Niveauvariometers. Therefore, calling the variations observed on the scales of the Niveauvariometers No. 1 and No. 2 Δ_1 and Δ_2, respectively, and the scale values of the Niveauvariometers α_1 and α_2, respectively, we have $\alpha_1 \Delta_1 = \alpha_2 \Delta_2$, thus

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta_2}{\Delta_1} \tag{14}$$

For the determination of the quantities Δ_1 and Δ_2 the hourly readings of the Niveauvariometers can be used. An examination has shown that a linear extrapolation from two consecutive readings to the nearest one will give the latter with an error that is so small compared with the magnitude of the quantities Δ that the below mentioned method can be adopted for the determination of these quantities.

Suppose that the height of the water-level has been changed between the hours n and $n + 1$, then the quantities Δ can be determined in the following manners:

1. From the readings for the hours $n - 1$ and n a value is derived for the hour $n + 1$ by a linear extrapolation from the first-mentioned readings, and the difference between this value and the really observed one gives an approximate value of Δ (forwards extrapolation).

2. From the readings for the hours $n + 1$ and $n + 2$ a value is derived for the hour n by a linear extrapolation, and the difference between this value and the really observed one gives another approximate value of Δ (backwards extrapolation).

In Table 2 the ratios of the scale values are given as well for forwards as for backwards extrapolation.

Table 2. Ratios of Scale Values.

Date 1934	Niveauvar. No. 1 Δ_1		Niveauvar. No. 2 Δ_2		Ratio of scale values $\frac{\alpha_2}{\alpha_1}$		
	Forw. extrapol.	Backw. extrapol.	Forw. extrapol.	Backw. extrapol.	Forw. extrapol.	Backw. extrapol.	Mean
	mm	mm	mm	mm			
June 10	84.3	85.1	83.1	82.7	1.014	1.029	1.022
11	124.3	126.5	124.0	124.0	1.002	1.020	1.011
11	129.0	129.0	126.3	128.0	1.021	1.008	1.015
13	108.4	110.4	105.6	107.3	1.026	1.029	1.027
14	112.9	107.4	112.0	105.8	1.008	1.015	1.012
18	82.7	82.2	77.7	78.8	1.064	1.043	1.053
23	148.1	148.9	142.3	143.8	1.041	1.035	1.038
							Mean:
							1.025
							$\pm$ 0.006

It will be seen that the ratios found are a little greater at the end of the period of observation than at the beginning, but in spite of this, the true ratio may have remained constant through the whole period.

The mean value of the ratio, which has been found with a comparatively small standard deviation, differs very little from the ratio between the scale values determined directly, and thus supports these latter determinations.

The readings.

After having determined the scale values of both of the Niveauvariometers, the hourly readings could commence. Four observers carried out the readings by turns, each observer making eight hourly readings during his turn. Five minutes before the full hour, the observer went down into the tunnel where in order as far as possible to avoid disturbances arising for instance from air-draught, he walked very slowly near the wall opposite to that where the Niveauvariometers were placed. Now he first read a thermometer placed some metres from the Niveauvariometer to be read. Thereafter the reading of the Niveauvariometer took place in the following manner. First the scale reflected by the fixed mirror was read, thereafter the telescope was made to point at a certain division of the mentioned scale, and finally the scale reflected from the mirror of the mirror arrangement was read. When the observer had finished the readings at the first Niveauvariometer he cautiously walked to the second one, where he repeated the observations.

The scales were illuminated by means of an electric dry-cell-lamp, but in spite of this excellent lighting the reading of the scales were often very difficult on account of the moisture in the tunnel.

Table 3. Readings.

Date	Station at SSW Niveauvariometer No. 2.			Station at NNE Niveauvariometer No. 1.		
	Time	Reading. Reference point 620.0	Temp. centigr.	Time	Reading. Reference point 140.0	Temp. centigr.
		mm			mm	
1934 June 9	20h 02m	601.3	7.°05	20h 06m	197.9	7.°00
	21 00	592.7	7.00	21 03	190.9	7.00
	22 00	586.5	7.00	22 04	184.2	6.95
	23 01	581.6	7.00	23 04	177.7	6.90
10	0 00	577.0	7.00	0 04	172.4	6.90
	0 59	574.3	7.00	1 04	167.2	6.90
	1 59	570.4	7.00	2 04	162.4	6.90
	3 00	569.4	7.00	3 04	158.0	6.85
	3 59	566.3	7.00	4 03	154.0	6.90
	4 59	560.8	7.00	5 03	149.2	6.90
	5 59	555.0	7.00	6 03	144.1	6.90
	6 59	550.0	7.00	7 03	139.3	6.90
	7 58	628.1	7.00	8 02	218.8	6.90
	9 00	623.2	7.05	9 03	213.2	6.90
	9 55	621.2	7.10	10 00	209.4	6.95
	10 56	618.2	7.05	10 59	204.7	6.95
	11 56	617.6	7.00	12 00	200.6	6.95
	12 58	612.8	7.05	13 02	196.1	6.95
	13 58	609.7	7.05	14 01	192.7	6.95
	14 58	605.6	7.10	15 02	189.0	6.95
	15 53	602.4	7.10	16 03	186.4	6.90
	16 55	596.0	7.00	17 01	182.4	7.00
	17 58	591.5	7.00	18 06	178.2	7.00
	18 56	587.0	6.95	19 02	176.6	6.95
	20 02	582.0	7.00	20 06	172.8	6.95
	21 00	577.5	7.00	21 06	168.0	7.00

Table 3 (continued).

Date	Station at SSW Niveauvariometer No. 2. Time	Reading. Reference point 620.0	Temp. centigr.	Station at NNE Niveauvariometer No. 1 Time	Reading. Reference point 140.0	Temp. centigr.
		mm			mm	
1934 June 10	22h 06m	574.0	7.°00	22h 10m	164.1	7.°00
	23 08	570.0	7.00	23 15	161.7	7.00
11	0 03	690.5	7.10	0 09	284.1	6.90
	0 57	687.0	7.00	1 01	279.9	6.90
	2 05	682.0	7.00	2 10	275.0	6.90
	3 00	679.1	7.00	3 06	272.1	6.90
	4 00	675.5	7.00	4 05	269.0	6.90
	5 01	671.4	7.00	5 07	265.0	6.90
	5 59	666.0	7.00	6 04	260.8	6.95
	7 01	660.5	7.00	7 06	255.3	6.95
	7 58	655.5	7.00	8 02	250.6	7.00
	9 00	650.6	7.00	9 03	245.0	7.00
	10 05	646.6	7.00	10 09	239.3	6.95
	11 01	645.0	6.95	11 05	235.2	6.90
	11 58	641.6	7.00	12 01	230.6	6.90
	12 59	639.4	6.95	13 03	225.4	6.90
	13 56	635.7	6.95	13 59	222.0	6.95
	15 03	630.7	6.90	15 07	216.7	6.85
	16 12	751.8	7.00	16 05	341.2	6.90
	17 02	746.8	7.00	17 11	337.0	6.90
	17 58	741.8	7.05	18 02	333.8	6.98
	19 01	736.2	7.00	19 05	329.6	6.95
	19 54	730.1	7.02	20 00	325.2	7.00
	20 55	722.8	7.02	21 02	319.8	6.98
	21 55	719.8	7.00	22 00	315.3	6.98
	22 56	712.9	7.00	23 01	309.6	6.95
	23 53	707.8	7.00	24 00	304.0	6.96
12	1 01	703.3	7.00	1 08	297.8	6.95
	2 00	699.5	7.00	2 05	293.7	6.94
	2 59	696.0	6.98	3 04	289.2	6.95
	3 58	691.3	6.96	4 03	285.4	6.95
	4 57	688.0	6.98	5 02	281.3	6.95
	5 55	682.5	6.98	6 00	276.8	6.95
	6 57	678.4	6.99	7 02	272.7	6.96
	8 00	674.0	6.99	8 05	268.5	6.90
	9 04	670.7	7.00	9 10	264.1	6.92
	10 07	667.8	7.00	10 12	259.1	6.92
	10 57	663.9	6.99	11 02	254.8	6.89
	11 57	662.5	7.00	12 03	251.0	6.90
	13 02	661.9	6.98	13 09	247.1	6.90
	13 57	659.9	6.99	14 02	244.5	6.90
	15 00	657.0	6.98	15 06	240.9	6.90
	16 06	650.4	6.97	16 08	235.1	6.96
	16 57	648.1	6.89	17 00	233.1	6.88
	18 00	644.4	6.94	18 04	230.7	6.92
	18 58	640.6	6.89	19 02	228.2	6.85
	19 53	635.8	6.92	19 56	225.0	6.92
	20 58	631.7	6.94	21 01	222.0	6.87
	21 58	628.2	6.87	22 01	218.5	6.84
	22 57	625.3	6 91	23 00	215.4	6.80

Table 3 (continued).

Date	Station at SSW Niveauvariometer No. 2.			Station at NNE Niveauvariometer No. 1.		
	Time	Reading. Reference point 620.0	Temp. centigr.	Time	Reading. Reference point 140.0	Temp. centigr.
		mm			mm	
1934 June 13	0h 01m	727.8	7.°00	0h 13m	320.0	6.°95
	0 55	723.8	7.02	1 01	316.2	6.90
	1 55	722.5	6.95	2 00	313.2	6.90
	2 57	718.8	6.95	2 59	309.0	6.90
	3 57	717.3	7.00	4 01	307.2	6.95
	4 58	714.2	7.00	5 02	304.6	6.95
	5 56	710.9	6.98	5 59	300.4	6.90
	6 56	707.9	6.95	7 00	298.3	6.85
	7 56	703.9	6.95	8 01	294.6	6.95
	9 00	701.9	6.96	9 05	292.8	6.95
	9 55	701.3	6.96	10 00	290.0	6.92
	10 59	695.0	6.98	11 04	282.4	6.94
	11 54	693.6	7.00	12 00	279.7	6.94
	12 57	692.2	6.99	13 01	276.0	6.93
	13 57	688.3	6.98	14 02	271.8	6.95
	14 58	689.0	6.98	15 02	269.3	6.93
	16 01	685.5	7.00	16 07	266.6	6.99
	17 00	680.9	7.00	17 07	262.2	6.99
	18 01	678.0	6.98	18 07	259.0	6.89
	19 02	673.3	6.99	19 08	255.2	6.85
	20 02	667.5	6.99	20 09	252.0	6.84
	21 00	662.5	7.00	21 06	248.5	6.89
	22 02	657.5	7.00	22 08	244.0	6.88
	23 00	653.3	6.99	23 05	240.5	6.90
	23 59	650.7	6.97	24 02	236.9	6.86
14	0 58	648.5	6.94	1 02	233.7	6.83
	1 58	645.7	6.89	2 01	230.3	6.83
	2 57	643.1	6.89	3 00	226.9	6.83
	3 58	640.0	6.92	4 02	222.8	6.83
	4 58	635.7	6.95	5 01	218.3	6.82
	5 58	631.8	6.92	6 01	213.8	6.82
	6 58	627.9	6.92	7 02	209.0	6.83
	8 00	622.8	6.95	8 04	204.0	6.85
	8 56	619.8	6.98	9 01	201.0	6.88
	9 58	615.8	6.98	10 02	197.0	6.90
	10 57	612.2	6.98	11 02	192.5	6.88
	11 57	609.4	6.98	12 02	188.0	6.90
	13 00	605.8	6.98	13 05	181.5	6.92
	14 00	602.8	6.98	14 04	177.6	6.90
	15 01	600.2	6.98	15 05	174.1	6.90
	16 01	597.0	6.95	16 07	169.0	6.92
	16 57	593.8	6.95	17 02	165.8	6.95
	17 59	589.0	6.95	18 05	161.1	6.95
	18 59	587.9	6.98	19 05	160.8	6.97
	19 57	585.6	6.98	20 02	160.2	6.95
	21 02	580.2	6.96	21 06	157.6	6.96
	22 00	687.4	6.95	22 05	268.1	6.95
	22 59	688.8	6.96	23 05	271.2	6.95
15	0 01	687.0	6.95	0 08	269.9	6.89
	1 02	686.8	6.98	1 06	268.0	6.88
	1 58	683.6	6.99	2 02	265.0	6.86

Table 3 (continued).

Date	Station at SSW Niveauvariometer No. 2.			Station at NNE Niveauvariometer No. 1.		
	Time	Reading. Reference point 620.0	Temp. centigr.	Time	Reading. Reference point 140.0	Temp. centigr.
		mm			mm	
1934 June 15	3h 01m	679.6	6.°98	3h 06m	261.0	6.°88
	4 01	677.7	6.99	4 06	258.0	6.88
	5 00	673.9	6.99	5 04	253.0	6.87
	6 00	669.0	7.00	6 05	249.0	6.86
	7 10	664.5	7.00	7 15	243.0	6.88
	8 06	661.4	7.00	8 12	240.5	6.86
	9 01	658.0	7.00	9 05	238.0	6.86
	10 02	661.2	7.02	10 06	242.0	6.92
	11 03	623.5	7.13	11 07	202.8	6.90
	12 00	621.6	7.02	12 05	201.0	6.90
	12 58	619.8	7.01	13 03	198.0	6.88
	13 57	622.4	7.00	14 03	200.1	6.87
	14 59	626.6	7.00	15 04	201.8	6.89
	15 54	629.8	6.98	15 58	203.0	6.90
	16 55	629.4	6.98	16 59	203.2	6.88
	17 56	629.8	6.98	18 00	203.8	6.88
	18 55	631.4	6.95	19 02	206.0	6.90
	19 55	631.2	6.95	20 00	206.2	6.90
	20 56	629.8	6.98	21 00	206.0	6.90
	21 58	627.2	6.98	22 02	205.0	6.90
	22 55	627.8	6.98	23 00	206.5	6.88
	23 57	629.1	6.98	24 04	208.4	6.95
16	0 56	628.4	7.00	1 02	208.3	6.98
	1 57	628.2	6.98	2 02	206.5	6.95
	3 00	628.4	6.98	3 07	206.2	6.94
	3 58	629.9	6.99	4 04	206.0	6.95
	5 02	629.8	6.98	5 08	206.5	6.93
	5 57	627.8	6.98	6 05	205.6	6.94
	6 59	627.9	6.99	7 07	204.9	6.95
	8 01	628.2	6.99	8 10	204.0	6.90
	8 58	625.5	7.00	9 04	202.5	6.88
	9 56	623.5	7.00	10 00	200.5	6.87
	11 00	621.2	7.00	11 05	197.8	6.87
	12 01	619.5	6.99	12 09	195.5	6.88
	13 02	617.5	7.00	13 07	193.0	6.89
	14 02	617.4	7.00	14 10	190.9	6.86
	15 02	617.0	7.00	15 07	189.6	6.88
	15 59	615.5	7.00	16 08	187.1	6.86
	16 57	613.9	7.00	17 00	184.8	6.87
	18 00	611.3	7.00	18 03	182.2	6.86
	18 59	610.4	7.00	19 03	181.4	6.86
	20 01	608.8	7.00	20 05	180.0	6.86
	20 59	606.0	7.00	21 05	178.2	6.86
	22 01	603.7	7.00	22 04	177.3	6.86
	22 59	598.9	7.00	23 02	174.9	6.86
	23 56	598.9	6.95	24 00	174.3	6.90
17	0 55	606.2	6.95	0 58	182.1	6.90
	1 56	640.2	7.00	2 01	217.9	6.90
	2 57	639.4	7.00	3 00	217.1	6.88
	3 56	641.2	7.00	4 00	216.4	6.88
	4 56	639.8	7.00	5 00	215.1	6.86

Table 3 (continued).

Date	Station at SSW Niveauvariometer No. 2.			Station at NNE Niveauvariometer No. 1.		
	Time	Reading. Reference point 620.0	Temp. centigr.	Time	Reading. Reference point 140.0	Temp. centigr.
		mm			mm	
1934 June 17	5h 58m	637.6	7.°00	6h 04m	212.8	6.°86
	6 56	636.6	7.00	7 02	211.8	6.86
	8 03	636.4	7.00	8 07	211.0	6.86
	9 06	634.5	7.00	9 09	209.4	6.86
	9 59	632.4	7.00	10 02	206.2	6.86
	10 59	631.0	7.00	11 04	206.1	6.86
	12 01	631.2	7.00	12 05	204.0	6.86
	12 57	629.1	7.00	13 02	202.8	6.86
	13 59	627.2	7.00	14 04	198.9	6.86
	15 00	622.5	7.00	15 05	193.7	6.86
	16 02	620.3	7.00	16 08	190.4	6.87
	17 00	621.3	7.00	17 05	190.5	6.88
	17 59	621.3	7.00	18 05	189.5	6.87
	19 00	621.3	7.00	19 05	190.5	6.87
	20 00	618.6	7.00	20 04	187.5	6.86
	20 59	614.6	7.00	21 04	184.5	6.87
	21 59	612.5	7.00	22 04	184.0	6.86
	22 57	611.7	7.00	23 03	184.3	6.86
18	0 02	608.0	7.00	0 07	180.6	6.88
	0 57	604.0	7.00	1 01	178.1	6.88
	1 58	601.5	7.00	2 02	174.7	6.88
	2 57	599.8	7.00	3 02	172.8	6.88
	3 59	598.7	7.00	4 05	170.6	6.88
	4 58	597.0	7.00	5 02	167.9	6.88
	5 58	594.5	7.00	6 04	165.1	6.88
	6 58	594.2	7.00	7 02	163.8	6.88
	8 06	593.0	7.00	8 10	160.6	6.90
	8 56	669.8	7.00	8 59	241.0	6.90
	9 58	667.2	7.00	10 02	238.4	6.90
	10 57	666.2	7.00	11 00	236.2	6.92
	11 58	661.2	7.00	12 01	235.8	6.92
	12 55	659.3	7.00	13 00	233.8	6.92
	13 55	657.2	7.00	14 00	231.2	6.92
	14 58	655.8	7.00	15 02	230.6	6.96
	16 02	655.2	7.00	16 08	227.8	6.95
	17 02	656.0	7.00	17 06	227.6	6.97
	17 55	657.4	7.00	18 00	227.5	6.98
	18 58	657.8	7.00	19 03	227.3	6.98
	19 58	657.7	7.00	20 02	227.2	6.98
	20 58	658.4	7.00	21 02	227.0	6.98
	22 00	657.6	7.00	22 05	227.5	6.99
	22 58	657.4	7.00	23 03	228.3	6.98
19	0 05	657.3	7.00	0 12	228.7	6.89
	1 00	657.3	6.99	1 07	230.0	6.90
	2 01	659.6	7.00	2 05	231.2	6.89
	3 00	660.3	7.00	3 04	233.9	6.89
	4 00	662.8	7.00	4 05	233.6	6.90
	4 59	664.8	7.00	5 04	234.0	6.89
	6 00	666.8	7.00	6 05	235.9	6.90
	6 57	670.6	7.00	7 01	237.8	6.90
	8 17	671.4	7.00	8 23	239.6	6.90

Table 3 (continued).

Date	Station at SSW Niveauvariometer No. 2. Time	Reading. Reference point 620.0	Temp. centigr.	Station at NNE Niveauvariometer No. 1. Time	Reading. Reference point 140.0	Temp. centigr.
		mm			mm	
1934 June 19	9h 03m	672.7	7.°00	9h 08m	240.2	6.°90
	9 57	676.0	7.00	10 02	242.7	6.88
	10 59	678.1	7.00	11 03	245.3	6.88
	11 57	679.8	7.00	12 01	247.4	6.88
	13 05	683.0	7.00	13 07	251.9	6.88
	13 59	686.0	7.00	14 03	254.7	6.88
	14 58	685.5	7.00	15 02	253.7	6.88
	15 59	687.8	7.00	16 04	255.2	6.88
	16 57	689.8	7.00	17 01	257.6	6.90
	17 59	692.8	7.00	18 04	260.5	6.90
	18 58	694.8	7.00	19 01	261.7	6.90
	20 00	697.4	7.00	20 04	263.8	6.90
	21 00	699.0	7.00	21 04	265.4	6.90
	21 59	700.9	7.00	22 03	267.0	6.92
	22 56	700.7	7.00	23 00	268.6	6.92
20	0 00	700.5	7.00	0 05	269.3	6.98
	1 00	699.2	7.00	1 06	269.4	6.99
	2 00	698.6	7.00	2 04	269.5	6.98
	2 59	697.7	7.00	3 03	268.9	6.98
	4 02	695.7	7.00	4 06	266.7	6.98
	4 58	695.9	7.00	5 03	266.0	6.98
	5 58	696.0	7.00	6 02	265.1	6.98
	6 59	695.2	7.00	7 03	263.2	6.98
	7 56	694.0	7.00	8 02	261.0	6.99
	8 59	692.3	7.00	9 03	258.7	6.98
	10 00	690.5	7.00	10 04	256.9	6.97
	11 00	686.9	7.00	11 04	253.2	6.98
	11 59	682.5	7.00	12 06	249.9	6.98
	13 00	678.9	7.00	13 07	246.5	6.98
	14 04	671.5	7.00	14 08	240.0	6.98
	15 04	667.0	7.00	15 11	235.7	6.98
	16 06	663.8	7.00	16 10	232.0	6.96
	17 13	658.8	7.00	17 18	227.4	6.96
	18 05	656.2	7.02	18 09	224.0	6.96
	19 00	652.5	7.02	19 04	221.0	6.98
	20 00	650.5	7.02	20 10	217.5	6.94
	21 00	646.8	7.00	21 07	213.5	6.95
	22 01	643.5	7.00	22 10	210.5	6.95
	23 00	640.7	7.00	23 05	207.4	6.95
21	0 00	636.8	7.00	0 05	204.0	6.95
	0 58	635.2	7.00	1 02	201.8	6.95
	1 57	631.8	7.02	2 00	200.2	6.95
	2 59	629.8	7.02	3 03	198.3	6.98
	3 58	628.8	7.02	4 01	196.7	6.98
	5 01	627.2	7.02	5 04	194.0	6.98
	6 00	626.2	7.02	6 03	192.1	6.98
	6 59	626.6	7.02	7 04	190.2	6.98
	7 56	624.6	7.02	8 02	187.6	7.00
	8 58	623.1	7.02	9 04	185.4	7.00
	10 01	621.6	7.01	10 06	182.4	7.00
	11 00	619.8	7.02	11 04	180.5	7.00

Table 3 (continued).

Date	Station at SSW Niveauvariometer No. 2.			Station at NNE Niveauvariometer No. 1.		
	Time	Reading. Reference point 620.0	Temp. centigr.	Time	Reading. Reference point 140 0	Temp. centigr.
		mm			mm	
1934 June 21	12^h 00^m	617.5	7.°02	12^h 05^m	178.5	6.°99
	12 56	613.5	7.02	13 00	176.1	6.98
	13 55	611.5	7.02	14 00	174.8	6.98
	14 56	608.9	7.02	15 00	173.6	6.98
	15 58	607.0	7.01	16 06	172.4	6.97
	17 02	605.4	7.02	17 07	171.1	6.97
	18 00	604.5	7.02	18 04	169.8	6.95
	19 00	603.0	7.02	19 05	168.8	6.95
	20 00	602.3	7.02	20 05	166.8	6.95
	21 00	601.1	7.02	21 05	165.3	6.95
	22 00	599.8	7.02	22 05	164.1	6.97
	22 59	599.3	7.02	23 02	163.7	6.97
	23 58	598.8	7.02	24 02	163.0	6.97
22	0 57	597.3	7.02	1 01	162.8	6.97
	1 57	597.4	7.02	2 00	163.0	6.97
	2 59	596.7	7.00	3 03	164.1	6.97
	—	—	—	—	—	—
	—	—	—	—	—	—
	6 02	597.4	7.00	6 07	163.8	6.97
	7 01	599.5	7.00	7 06	164.5	6.97
	8 13	601.8	7.00	8 07	164.3	6.97
	8 58	601.3	7.02	9 03	163.3	6.97
	10 05	604.8	7.05	10 08	164.6	6.97
	11 20	605.8	7.05	11 15	164.8	7.00
	11 56	604.8	7.05	12 05	164.2	7.00
	12 55	604.4	7.05	12 59	166.2	7.00
	13 59	603.8	7.10	14 05	166.4	7.00
	15 05	600.3	7.10	15 08	166.5	7.00
	15 59	600.0	7.02	16 04	165.4	7.00
	17 02	599.3	7.02	17 06	165.0	7.00
	18 00	597.7	7.02	18 04	164.2	7.00
	19 04	595.3	7.02	19 07	161.4	7.00
	20 05	593.4	7.02	20 08	158.7	7.00
	20 58	591.4	7.02	21 01	156.3	7.00
	22 02	588.5	7.02	22 07	152.2	7.00
	23 01	600.8	7.02	23 05	164.3	7.00
23	0 00	598.4	7.02	0 07	161.9	7.00
	1 00	595.6	7.02	1 06	159.8	7.00
	2 00	591.5	7.02	2 03	156.9	7.00
	2 59	588.4	7.02	3 03	153.8	7.00
	4 01	584.5	7.02	4 06	150.4	6.98
	4 59	579.0	7.02	5 03	145.2	6.98
	6 02	576.8	7.00	6 05	140.9	6.98
	7 02	573.7	7.00	7 06	136.2	6.98
	8 02	572.0	7.00	8 05	132.5	6.98
	9 00	569.6	7.02	9 04	127.7	7.00
	9 58	566.6	7.02	10 01	123.7	7.00
	11 03	705.5	7.02	11 06	267.2	7.00
	12 06	700.8	7.02	12 10	261.9	7.00
	12 59	698.3	7.02	13 03	259.4	7.00
	14 00	693.5	7.02	14 04	256.4	7.00

Table 3 (continued).

Date			Station at SSW Niveauvariometer No. 2.			Station at NNE Niveauvariometer No. 1	
		Time	Reading. Reference point 620.0	Temp. centigr.	Time	Reading. Reference point 140.0	Temp. centigr.
			mm			mm	
1934	June 23	15h 03m	690.3	7.°02	15h 07m	254.1	7.°00
		16 12	686.8	7.02	16 00	253.4	7.00
		17 00	683.5	7.05	17 03	251.2	7.00
		17 55	680.8	7.05	18 00	249.0	7.00
		18 57	678.6	7.05	19 01	247.2	6.95
		19 58	676.8	7.05	20 03	245.8	6.95
		20 59	676.6	7.05	21 03	244.1	6.95
		22 00	674.8	7.05	22 04	241.2	6.95
		23 02	674.8	7.10	23 05	240.4	6.95
	24	0 01	675.5	7.02	0 05	239.4	6.98
		1 04	673.7	7.02	1 08	239.4	6.98
		1 58	674.5	7.02	2 01	240.2	6.98
		2 58	673.9	7.02	3 01	240.4	6.98
		3 58	672.7	7.02	4 01	240.6	6.98
		4 59	670.7	7.02	5 04	239.6	6.98
		5 59	670.4	7.02	6 03	238.9	6.98
		7 00	669.9	7.02	7 03	237.3	6.98
		7 57	670.0	7.02	8 01	236.0	6.98
		8 56	670.8	7.02	9 03	234.5	6.98
		10 06	673.0	7.05	10 10	—	7.00
		10 57	675.3	7.05	11 15	232.3	7.00
		12 11	672.3	7.10	12 07	231.5	7.00
		13 07	672.5	7.10	13 13	230.0	7.10
		14 26	671.5	7.20	14 15	230.0	7.00
		14 58	672.1	7.10	14 57	230.6	7.00
		16 00	671.5	7.10	16 05	231.5	7.00
		17 03	669.6	7.10	17 03	232.0	7.00
		17 59	669.5	7.10	18 03	232.9	6.90
		19 00	667.5	7.10	19 03	231.9	6.90
		20 00	664.5	7.10	20 08	231.0	7.00
		21 01	663.8	7.15	21 06	229.4	7.00
		22 01	662.6	7.15	22 05	227.7	7.00
		23 04	662.5	7.15	23 09	226.1	7.00
	25	0 00	662.5	7.10	0 03	224.2	7.00
		0 59	662.0	7.10	1 02	224.1	6.98
		2 00	661.7	7.10	2 04	223.8	6.98
		3 00	662.0	7.15	3 04	223.9	6.98
		3 58	660.8	7.15	4 02	223.9	6.98
		4 59	659.8	7.10	5 03	223.5	6.98
		5 58	657.8	7.10	6 02	222.9	6.97
		7 00	658.4	7.10	7 03	222.9	6.97

The readings are given in Table 3. For each of the Niveauvariometers the time of observation, readings of the scale, and of the thermometer are given. Setting aside the cases when water just has been filled into the pipe, the readings during the greatest part of the time of observation show decreasing values. The sinking of the water may be owing to an instability of the beams or of the pipe. It has been tried to compensate this apparent loss of water by conducting, at a proper speed, distilled water into the pipe, but these arrangements never worked completely satisfactory.

The reading began on the 9th of June at 20^h M. E. T. and lasted to the 25th of June 7^h M. E. T., only two readings in the very early morning on the 22^{nd} of June being lost.

The computation of the height-variations.

From the readings of the Niveauvariometer No. 1 and No. 2 the relative height-variations of the pillars for the Niveauvariometers can be determined.

The height, H, of the liquid of the Niveauvariometer over a certain fixed point is given by the equation (13):

$$H = H_0 + an + f(t).$$

The temperature, t, is given in Table 3, but in order to illustrate the variations of the temperature in a better manner, means of t for periods generally of 32 hours are given in the following table:

Table 4. Means of Temperatures read in the Tunnel.

Period	Niv. No. 2	Niv. No. 1
1934 June 9 20h — June 11 7h	7.°02 C	6.°94 C
» 11 8 — » 12 15	6.99	6.93
» 12 16 — » 13 23	6.96	6.91
» 14 0 — » 15 7	6.96	6.89
» 15 8 — » 16 15	6.99	6.90
» 16 16 — » 17 23	7.00	6.87
» 18 0 — » 19 7	7.00	6.92
» 19 8 — » 20 15	7.00	6.94
» 20 16 — » 21 23	7.01	6.97
» 22 0 — » 23 7	7.02	6.99
» 23 8 — » 24 15	7.05	6.99
» 24 16 — » 25 7	7.11	6.99

Only in a few cases the readings of the Niveauvariometers have been made at full hour, and therefore values corresponding to full hours have been computed by interpolation for the rest of the readings. For all hours the quantity an has been computed, and the difference

$$\alpha_2 n_2 - \alpha_1 n_1$$

has been derived. Index 2 indicates that the quantity to which it is fixed refers to Niveauvariometer No. 2, and index 1 indicates that the quantity in question refers to Niveauvariometer No. 1.

From (13) we have

$$\alpha_2 n_2 - \alpha_1 n_1 =$$

$$H_2 - H_1 - (H_{0_2} - H_{0_1}) - (f_2(t) - f_1(t)) \qquad (15)$$

The differences $\alpha_2 n_2 - \alpha_1 n_1$ given in the below Table 5 contain the difference sought for, $H_2 - H_1$, but also the difference between the zero-point values, $H_{0_2} - H_{0_1}$, and the difference $f_2(t) - f_1(t)$. The latter two differences and their variations will be considered below.

The differences from Table 5 are given in a graph, Fig. 4.

Table 5. Differences between heights found from Niv. No. 2 (at SSW) and Niv. No. 1 (at NNE). Bergen 1934.

M.E.T.	June 9 μ	June 10 μ	June 11 μ	June 12 μ	June 13 μ	June 14 μ	June 15 μ	June 16 μ	June 17 μ	June 18 μ	June 19 μ	June 20 μ	June 21 μ	June 22 μ	June 23 μ	June 24 μ	June 25 μ	M.E.T.
0h		3.8	7.8	5.5	9.6	15.0	19.0	21.8	25.0	27.8	30.4	33.9	33.9	36.3	36.9	38.2	40.2	0h
1		6.3	8.7	7.0	10.0	15.8	20.7	21.0	25.0	26.3	29.2	32.4	34.6	35.1	36.3	36.6	39.9	1
2		7.1	7.7	7.5	11.6	16.4	20.4	22.6	23.4	27.2	30.3	31.7	32.8	35.0	35.0	36.5	39 9	2
3		10.7	8.5	8.4	12.2	17.2	20.3	23.2	23.7	27.3	28.3	31.3	32.6	33.3	34.7	35.7	40.0	3
4		11.5	7.8	7.5	12.5	18.1	21.3	25.0	26.0	28.4	31.0	31.4	33.2	(33.2)	34.3	34.2	38.8	4
5		10.7	7.7	8.2	11.9	18.2	22.5	24.5	25.9	29.4	32.7	32.3	34.2	(34.6)	33.8	33.0	38.2	5
6		9.8	6.1	7.2	12.7	18.8	21.5	23.1	25.9	29.7	33.1	33.4	35.2	34.2	36.0	33.5	36.7	6
7		9.5	6.1	7.0	11.6	19.4	22.5	23.9	26.1	30.9	34.8	34.4	37.6	35.8	37.4	34.5	37.3	7
8		9.6	5.7	6.8	11.4	19.3	22.4	25.1	26.6	32.3	34.1	35.4	38.0	37.9	39.5	36.1		8
9		10.3	6.4	7.6	11.2	19.2	21.4	23.7	26.3	30.6	34.6	36.0	38.8	38.8	41.8	38.4		9
10		12.2	7.9	9.5	13.0	19.3	21.1	23.8	27.3	30.6	35.6	36.0	40.1	41.0	42.9	41.8		10
11		14.2	10.5	10.3	14.2	19.9	21.4	24.0	26.0	31.7	35.1	35.9	40.4	41.6	40.8	44.9		11
12		17.4	11.9	12.8	15.9	21.6	21.3	24.6	28.1	27.0	34.8	34.5	39.9	41.5	41.1	43.4		12
13		17.1	14.8	15.8	18.1	24.4	22.5	25.1	27.3	27.0	33.8	34.3	38.3	39.1	41.5	44.5		13
14		17.3	14.5	16.7	18.5	25.4	23.5	27.1	29.0	27.6	33.8	33.2	37.5	38.1	39.5	43.8		14
15		16.8	14.6	17.3	21.7	26.3	26.1	28.1	29.4	26.8	34.2	32.8	36.1	34.8	38.6	43.8		15
16		15.9	14.7	16.4	20.8	27.8	27.9	29.0	30.6	28.6	35.1	33.4	35.3	35.2	36.5	42.3		16
17		13.3	12.4	16.1	20.1	28.1	27.3	29.9	31.9	30.0	34.7	32.9	35.1	35.0	34.5	39.9		17
18		12.9	10.7	14.7	20.5	27.9	27.2	29.9	32.8	31.6	35.0	33.7	35.4	34.0	33.8	38.7		18
19		10.0	9.3	13.1	19.6	27.4	26.3	29.8	31.8	32.3	35.8	33.0	34 8	34.4	33.5	37.6		19
20	3.1	8.8	6.9	11.5	16.8	25.4	26.0	29.5	31.8	32.3	36.3	34.1	36.1	35.1	33.1	35.4		20
21	1.3	8.7	5.0	10.5	15.1	22.6	24.7	28.2	30.8	33.2	36.4	34.4	36.4	35.6	34.6	36.2		21
22	1.7	9.3	6.5	10.2	14.5	21.7	23.2	26.9	29.3	31.8	36.7	34.1	36.3	36.4	35.6	36.7		22
23	3.3	7.8	5.1	10.5	13.8	20.1	22.4	(26.6)	28.0	30.8	34.7	34.4	36.3	36.9	36.5	38.2		23

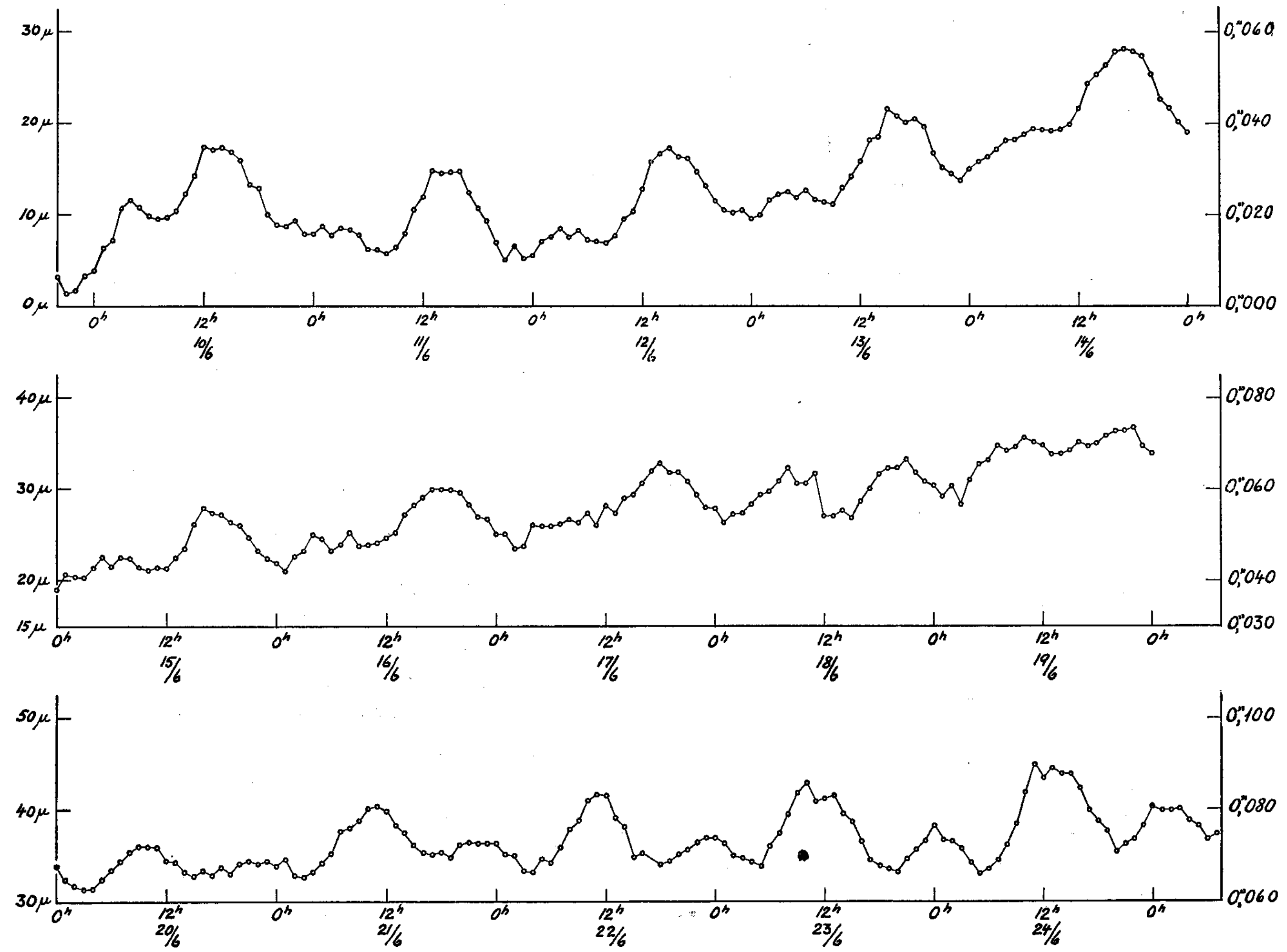

Fig. 4 Differences between heights found from Niv. No. 2 (at SSW) and Niv. No. 1 (at NNE).

Variations of the zero-point values and different height-variations of the pillars.

If the zero-point value, H_0, and $f(t)$ (13) for both Niveauvariometers are constants, the variation of the differences given in Table 5 is equal to the variation of the height difference between the two pillars (15). But the quantities H_0 and $f(t)$ may vary, and therefore it will be necessary to examine the differences $H_{0_2} - H_{0_1}$ and $f_2(t) - f_1(t)$ of equation (15) more closely.

The difference $f_2(t) - f_1(t)$ shall be considered first. The function $f(t)$ depends on variations of the dimensions of (1) the material of the Niveauvariometer, of (2) the pillar and of (3) the soil on which the pillar is resting.

On page 5 it has been shown that the height variations caused by temperature variations are small for the Niveauvariometer compared with those for the pillar.

The height variations of the soil on which the pillars rest are determined mainly by the temperature variations of the uppermost layer, because the temperature variations ordinarily are infinitely small at the dept of the tunnel, 18 metres below the surface of the earth, and at greater depths. But the temperature variations at the two pillars of the uppermost layer are so alike that the effect on the difference $f_2(t) - f_1(t)$ is neglectable.

Therefore, in order to estimate the variation of $f_2(t) - f_1(t)$, we have only to consider the variation of the difference between the height of the two pillars. The height of the pillars are 93 and 177 cm, respectively, the range of the temperature difference may be put equal to 0.°1 C (see Table 4), and the coefficient of expansion for the material considered may be estimated to 10^{-5} per centigrade, and the height variation from these data is found to be 1 μ. Thus, the order of magnitude of the range of $f_2(t) - f_1(t)$ is 1 μ for the whole period of observation.

From the variation of the readings during the time of observation, we conclude that variations much greater than those arising from the difference $f_2(t) - f_1(t)$ are found in the difference $H_{0_2} - H_{0_1}$. The difference $H_{0_2} - H_{0_1}$ can be determined approximately from the differences $\alpha_2 n_2 - \alpha_1 n_1$ of Table 5 in the following manner: We compute the mean of the differences of equation (15) for the 25 hourly readings from 12^h to 12^h the next day:

$$\frac{1}{25}\sum(\alpha_2 n_2 - \alpha_1 n_1) = \frac{1}{25}\sum(H_2 - H_1) - \frac{1}{25}\sum(H_{0_2} - H_{0_1}) - \frac{1}{25}\sum(f_2(t) - f_1(t)) \quad (16)$$

In the right-hand side of equation (16) the first mean will give a rather small value because all diurnal and semidiurnal solar and lunar variations of the tidal movements of the earth will be nearly eliminated, and consequently only long-periodic variations, which are known to have a much smaller amplitude than the must important of the first-mentioned variations, may influence the means for the different days. Also the last mean in the equation will be small because the range of $f_2(t) - f_1(t)$ is only about 1 μ for the period considered. Therefore, approximately, we have:

$$\frac{1}{25}\sum(\alpha_2 n_2 - \alpha_1 n_1) = -\frac{1}{25}\sum(H_{0_2} - H_{0_1}) \quad (17)$$

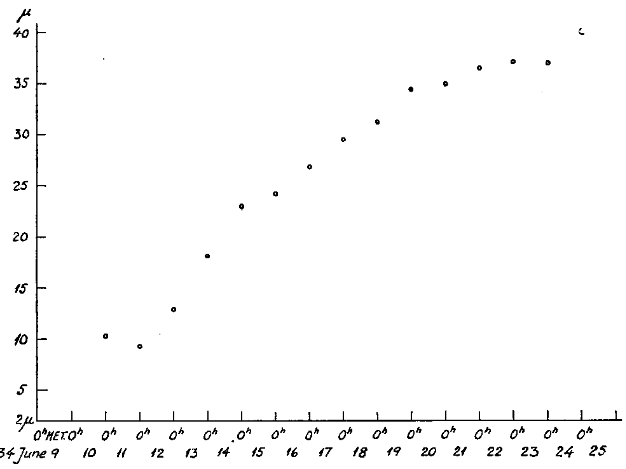

Fig. 5. Differences between the zero-point values of the two Niveauvariometers.

In Fig. 5 the mean values of (17) for every day (0^h) are represented. On the 10th of June, a more rapid variation of the difference of the zero-points may have taken place, but for the rest of the period the differences show a rather even variation. From Table 5 (Fig. 4) it will be seen that the difference between zero-point values varies from about 5 μ at the beginning to about 40 μ at the end of the period. This variation may be due as well to variations of one or both of the Niveauvariometers themselves, as to variations of the height of the instruments above the fixed points, mentioned in connection with equation (13).

The height variations of the pillars have not been determined directly, but from the determinations of the scale values it is possible to get some information concerning such variations. Under certain assumptions it is found that during the time of observation the variation of the height of the pillar at station NNE is

$$+32\mu \pm 5\mu^{1)}$$

and that of the pillar at station SSW

$$-10\mu \pm 5\mu,$$

so that the height of the pillar at station NNE has become

$$42\mu \pm 7\mu$$

greater than that of the pillar at station SSW.

A change like that mentioned above will cause an increase of the differences between the readings of the two Niveauvariometers, and the increase of 35μ of the differences of Table 5 may be explained by such changes of the pillars' height. Possibly other circumstances may have changed the difference of the zero-point values but such changes will be too small to be determined from the observations at hand.

On account of the rather even and relatively small variation of the difference of the zero-point values the determination of semidiurnal and diurnal variations may be carried outh with success, but from more reasons it is not possible to determine long-periodic variations from the present observations.

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1) The standard deviation.

RESULTS.

BY J. E. FJELDSTAD.

The tidal differences which are given in Table 5 may be attributed to different causes. The tidal forces of the sun and the moon will act on the water in the tube and cause tidal variations of the surface level. These variations can be computed according to the equilibrium theory of tides on a rigid earth. But the tidal forces will also act on the solid earth and produce elastic tides in the earth's surface and thus alter the position of the tube. These two effects will be inseparable and may be computed according to the theory of earth tides, when the rigidity of the earth is known. It is known that the effect of the yielding of the earth is to reduce the tidal variations which can be measured. Another effect is the influence of the varying load of water on the earth's surface caused by the tide on the coast, and also the varying attraction of the water.

As stated in the preceding section the tide was observed in a tube having a length of $l =$ 103 m and the direction N 18.°3 E — S 18.°3 W. Before discussing the results of the meassurements we require to compute the tidal variation in a tube of this length and direction on a rigid earth.

The tide producing potential of the moon is expressed by

$$\Omega = \frac{3}{2}\frac{M}{E}\, g\, \frac{a^2}{R^3}\, r^2 \left(\cos^2 z - \frac{1}{3}\right)$$

where M is the mass of the moon, E the mass of the earth, a is the mean radius of the earth, R the radius vector of the moon, g the accelleration of gravity and z the zenith distance of the moon. Let φ be the latitude and λ the east longitude of the place, δ the declination and θ the hour angle of the moon at Greenwich.

Then

$$\cos z = \cos(\theta + \lambda)\cos\varphi\cos\delta + \sin\varphi\sin\delta.$$

If we introduce this value of z in the expression for the potential, it takes the form

$$\Omega = \frac{3}{2}\frac{M}{E}\frac{a^2}{R^3} r^2 g \left\{\frac{1}{2}\cos^2\varphi\cos^2\delta\cos 2(\theta + \lambda) + \frac{1}{2}\sin 2\varphi\sin 2\delta\cos(\theta + \lambda) + \frac{3}{2}\left(\sin^2\varphi - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right)\right\}$$

The potential is now divided into three parts corresponding to the semidiurnal, diurnal and long-periodic tides. To obtain the equilibrium tide we have to express the condition that the disturbed surface shall be in equilibrium under the influence of the potential of gravity and the tideproducing potential. Denoting the surface elevation by ζ and the potential of gravity by V we have

$$V + \frac{\partial V}{\partial r}\zeta + \Omega = \text{const.}$$

The first term is a constant along the surface of the earth and $\frac{\partial V}{\partial r} = -g$, where g is the accelleration of gravity. The variable part of the surface elevation is then expressed by

$$\zeta = \frac{\Omega}{g}$$

The lunar tide is thus expressed by

$$\zeta = \frac{3}{2}\frac{M}{E}\left(\frac{a}{R}\right)^3 a\left(\cos^2 z - \frac{1}{3}\right)$$

To abreviate the writing we put

$$\tau = \frac{3}{2}\frac{M}{E}\left(\frac{a}{c}\right)^3$$

where c is the mean value of the radius vector of the moon R. Introducing after DOODSON

$$\frac{M}{E} = \frac{1}{81.53},\quad a = 6371.2 \text{ km},$$

$$c = 384400 \text{ km},$$

we find

$$\tau = 8.40 \cdot 10^{-8}$$

By the harmonic development of the tide producing potential a typical term will be

$$\zeta = \tau a f(\varphi) \frac{C}{2} \cos(\sigma t + n\lambda + V_0)$$

where $f(\varphi) = \cos^2\varphi$ for the semidiurnal terms and $f(\varphi) = \sin 2\varphi$ for the diurnal terms. C is a coefficient which varies slightly with the position of the lunar orbit.

Let the co-ordinates of the place in question be $x = \lambda a \cos\varphi$ and $y = a\varphi$, we want to compute the increment $\triangle\zeta$ of ζ corresponding to a variation of the coordinates $x + dx$ and $y + dy$. Let $dx = l\cos\alpha$ and $dy = l\sin\alpha$ where l is the length of the tube and α the azimuth of the tube reckoned positive from east towards north.

We then find

$$\triangle\zeta = \frac{\partial\zeta}{a\cos\varphi\partial\lambda}dx + \frac{\partial\zeta}{a\partial\varphi}dy.$$

For a semidiurnal component the result will be

$$\triangle\zeta = -2\tau l \frac{C}{2}\cos\varphi\cos\alpha\sin(\sigma t + 2\lambda + V_0)$$
$$-2\tau l\cdot\frac{C}{2}\sin\varphi\cos\varphi\sin\alpha\cos(\sigma t + 2\lambda + V_0)$$

if we put

$$R\cos\delta = -2\sin\varphi\cos\varphi\sin\alpha$$
$$R\sin\delta = -2\cos\varphi\cos\alpha$$

we obtain

$$\triangle\zeta = \tau l\frac{C}{2}R\cos(\sigma t + 2\lambda + V_0 - \delta)$$

where

$$\tan\delta = \frac{-\cos\alpha}{-\sin\varphi\sin\alpha}$$

$$R = 2\cos\varphi\sqrt{\cos^2\alpha + \sin^2\varphi\sin^2\alpha}.$$

In the present case we have $\alpha = 71.°7$ and $\varphi = 60.°3$, and get

$$\delta = 200.°9,\ R = 0.8744.$$

The expression for the difference $\triangle\zeta$ is then

$$\triangle\zeta = 2\cdot 1.03\cdot 10^4\cdot 0.4372\cdot 0.840\cdot 10^{-7}\cdot\frac{C}{2}\cdot$$
$$\cos(\sigma t + V_0 - \delta)$$
$$= \frac{C}{2}\cdot 7.570\cdot 10^{-4}\cos(\sigma t + V_0 - 200.°9).$$

For the principal semidiurnal components M_2, N_2 and S_2 the factors $\frac{C}{2}$ are 0.45426, 0.08796 and 0.21137 respectively. This gives the following values for the amplitudes which would be observed on a rigid earth

$$M_2 = 3.44\ \mu$$
$$N_2 = 0.67\ \mu$$
$$S_2 = 1.60\ \mu$$

where the amplitudes are measured in microns $1\mu = 10^{-4}$ cm.

For the diurnal components a typical term is

$$\zeta_1 = \tau a\sin 2\varphi\cdot\frac{C}{2}\cos(\sigma_1 t + \lambda + V_0)$$

correspondingly we find

$$\triangle\zeta_1 = 2\tau l\cos 2\varphi\frac{C}{2}\cos(\sigma_1 t + \lambda + V_0)\sin\alpha$$
$$-\tau l\frac{\sin 2\varphi}{\cos\varphi}\frac{C}{2}\sin(\sigma_1 t + \lambda + V_0)\cos\alpha$$
$$= \tau l\frac{C}{2}R_1\cos(\sigma_1 t + \lambda + V_0 - \delta_1)$$

where

$$R_1\cos\delta_1 = 2\cos 2\varphi\sin\alpha$$
$$R_1\sin\delta_1 = -2\sin\varphi\cos\alpha.$$

From these equations we obtain

$$\tan\delta_1 = \frac{-\sin\varphi\cos\alpha}{\cos 2\varphi\sin\alpha}$$

$$R_1 = 2\sqrt{\cos^2 2\varphi\sin^2\alpha + \sin^2\varphi\cos^2\alpha}$$

and in the present case

$$\delta_1 = 209.°4,\quad R_1 = 2\cdot 0{,}5550.$$

The amplitude of the diurnal component is then

$$2\cdot 1.03\cdot 10^4\cdot 0.840\cdot 10^{-7}\cdot 0.5550\cdot\frac{C}{2}$$
$$= 9.609\cdot 10^{-4}\cdot\frac{C}{2}$$

For K_1 the factor $\frac{C}{2}$ has the value $\frac{C}{2} = 0.26522$ and the corresponding value for the component O_1 is 0.18856. We then find the following values of the amplitudes

$$K_1 = 2.56\ \mu\quad O_1 = 1.81\ \mu.$$

To convert the values to seconds of arc of the angle of deviation of the plumb-line we assume that the measured tidal difference is h, and that it is measured in microns. The angle is then

$$\beta = \frac{h\cdot 10^{-4}}{l} = \frac{h\cdot 10^{-8}}{1.03}$$

and to convert the angle to seconds of arc it should be multiplied by

$$\frac{3600 \cdot 180}{\pi}$$

We then get

$$\beta = h \cdot 2.00 \cdot 10^{-3},$$

or the values which are found from the analysis shall be multiplied by $2 \cdot 10^{-3}$ to get the value in seconds of arc. The amplitudes which we have computed are valid for the mean value of I, the inclination of the lunar orbit. To find the values of the amplitudes which are valid for the central day, June 17th 1934, the amplitudes should be multiplied by the factor f which may be found in tables for harmonic analysis of tidal observations *e, g.* HARRIS: Manual of Tides.

Applying this factor we find the following amplitudes:

$$M_2 = 3.34\ \mu$$
$$N_2 = 0.65\ \mu$$
$$S_2 = 1.60\ \mu$$
$$K_1 = 2.78\ \mu$$
$$O = 2.07\ \mu$$

The phase angles for the semidiurnal components are 200.°9 and for the diurnal components 209.°4.

When comparing the results of the harmonic analysis with these theoretical values it should be borne in mind that the tabulated differences are SSW—NNE. This was convenient for the tabulation because all the differences are then positive. For the discussion of the results we have preferred to consider the opposite differences in order to facilitate the comparison of the observed tidal differences with the tide at the coast. When the tabulated differences have been harmonically analysed all the phase angles have been altered by 180°. The phase angles given below are found in this way. When we inspect the tabulated differences, we see that the values increase on the whole during the 15 days of observations. The differences vary also somewhat irregularly, indicating rather large errors in the single readings. When reducing the observations the effect of the systematic and casual errors must be reduced as far as possible.

Suppose the tidal differences during a certain time interval can be expressed by

$$h(t) = A + Bt + \Sigma C \cos(\sigma t - \zeta) + \triangle$$

where A stands for the mean value of the differences and Bt for the systematic error and $\triangle$ for the casual error. The effect of the casual errors will be reduced to a minimum if the analysis is made according to the rule of the least squares. A systematic error of the kind indicated above may be eliminated if we take the differences

$$h(t-3) - h(t+3) = -6B + \Sigma 2C \sin 3\sigma \sin(\sigma t + \zeta) + \triangle_1 .$$

For the semidiurnal components the factor $\sin 3\sigma$ is approximately 1, and for the diurnal components the factor is about 0,7. For the analysis of the diurnal components it is more advantageous to make the combination

$$h(t-6) - h(t+6) = -12B + \Sigma 2C \sin 6\sigma \sin(\sigma t - \zeta) + \triangle_2 .$$

In this expression the semidiurnal components are nearly eliminated. Consequently it is unnecessary to correct the diurnal harmonic constants for the influence M_2. The computation of these differences is preferably included in the scheme of daily analysis which we are about to explain.

The principles of the metod of analysis are the same as those set forth by DOODSON (P. 228).[1]) The difference is that we adhere more strictly to the rule of the least squares. Since the method is only intended to be used on 15 days of observations the amount of labour involved in the use of the multipliers cos 0° cos 15° cos 30° etc. to three decimal places is not very great, and the rather large casual errors make it desirable to use the method of the least squares as far as possible. For ordinary tidal observations DOODSON's method for the reduction of short series of observations has proved quite satisfactory.

The introduction of central time origin simplifies all formulae considerably. In the following analysis the time origin is taken to be noon on the central day, but this is not essential.

Let the semidiurnal component in question be expressed by

$$h = A \cos(\sigma t - \zeta)$$

where $\sigma = 30° - \beta$. On the kth day the expression will be

$$h_k = A \cos(\sigma t - \zeta - 24\, k\beta).$$

[1]) A. T. DOODSON: The Analysis of Tidal Observations. *Phil. Trans.* Ser. A. Vol. 227. Pp. 223—279.

In order to eliminate the obliquity of the tide curve, we form the differences

$$h'_k(t) = h_k(t-3) - h_k(t+3) = 2A \sin 3\sigma \sin(\sigma t - \zeta - 24k\beta).$$

Now we make an analysis of the hourly values for the kth day after the formula

$$\sum_{-11}^{+11} h' \cos 30^\circ t + \sum_{-12}^{+12} h' \cos 30^\circ t =$$

$$-2A \sin 3\sigma \sin(\zeta + 24k\beta) \sin 12\beta \left[\cot\frac{\beta}{2} - \cot\left(30^\circ - \frac{\beta}{2}\right)\right]$$

$$= -CA \sin(\zeta + 24k\beta) = -Y_k$$

this is equivalent to using 25 hourly values, but giving only half weight to the first and last. In the same manner we find

$$\sum_{-11}^{+11} h' \sin 30^\circ t + \sum_{-12}^{+12} h' \sin 30^\circ t =$$

$$2A \sin 3\sigma \cos(\zeta + 24k\beta) \sin 12\beta \left[\cot\frac{\beta}{2} + \cot\left(30^\circ - \frac{\beta}{2}\right)\right]$$

$$= SA \cos(\zeta + 24k\beta) = X_k.$$

The semidiurnal components which we take into consideration are M_2, N_2, S_2 and K_2. The diurnal component O_1 is not completely eliminated and so we include it in the scheme of analysis. The results of the analysis of one single day is that we obtain two quantities X_k and Y_k and these are supposed to contain contributions from the components named above. The complete expression for X_k is on the assumption above

$$X_k = S_m M_2 \cos(\zeta_m + 24k\beta_m) + S_n N_2 \cos(\zeta_n + 24k\beta_n) + S_s S_2 \cos \zeta_s + S_{k2} K_2 \cos(\zeta_{k2} + 24k\beta_{k2}) + S_0 O_1 \cos(\zeta_0 + 24k\beta_0).$$

For Y_k we get the expression

$$Y_k = C_m M_2 \sin(\zeta_m + 24k\beta_m) + C_n N_2 \sin(\zeta_n + 24k\beta_n) + C_s S_2 \sin \zeta_s + C_{k2} K_2 \sin(\zeta_{k2} + 24k\beta_{k2}) + C_0 O_1 \sin(\zeta_0 + 24k\beta_0).$$

We assume now that we have 15 days of observations and number the days from -7 to $+7$, and make the following combinations

$$\sum_{-7}^{+7} X_k, \quad \sum_{-7}^{+7} X_k \cos 24k\beta_m, \quad \sum_{-7}^{+7} X_k \sin 24k\beta_m$$

$$\sum_{-7}^{\times 7} X_k \cos 24k\beta_n, \quad \sum_{-7}^{+7} X_k \sin 24k\beta_n$$

and the corresponding combinations with Y_k.

Let $\sum \cos 24k\beta_m \cos 24k\beta_n$ be denoted by $c(m, n)$ and $\sum \sin 24k\beta_m \sin 24k\beta_n$ by $s(m, n)$. For S_2 the angle β is zero, and so

$$s(s, m) = s(m, s) = 0$$
$$c(s, m) = c(m, s) = \sum \cos 24k\beta_m.$$

We then obtain

$$\sum X_k = S_m c(m, s) M_2 \cos \zeta_m + S_n c(n, s) N_2 \cos \zeta_n + S_s c(s, s) S_2 \cos \zeta_s + S_k c(s, k) K_2 \cos \zeta_k + S_0 c(s, o) O \cos \zeta_0$$

and the corresponding equations. We remark that

$$\sum X_k \cos 24k\beta \text{ and } \sum Y_k \sin 24k\beta$$

both give expressions containing $\cos \zeta$ and

$$-\sum X_k \sin 24k\beta \text{ and } \sum Y_k \cos 24k\beta$$

give terms containing $\sin \zeta$.

Let

$$\textstyle\sum_1 = \sum X_k \cos 24k\beta_m + \sum Y_k \sin 24k\beta_m$$
$$\textstyle\sum_2 = \sum X_k \cos 24k\beta_n + \sum Y_k \sin 24k\beta_n$$
$$\textstyle\sum_3 = \sum X_k.$$

We then obtain three equations containing $M_2 \cos \zeta_m$, $N_2 \cos \zeta_n$, $S_2 \cos \zeta_s$, $K_2 \cos \zeta_k$ and $O_1 \cos \zeta_0$. $S_2 \cos \zeta_s$ and $K_2 \cos \zeta_k$ can not be separated in the short interval of 15 days, therefore, we take the sum of these to be one of the unknown, and regard the term $O_1 \cos \zeta_0$ as a correction term. Then we have three equations for the determination of $M_2 \cos \zeta_m$, $N_2 \cos \zeta_n$ and $S_2 \cos \zeta_s + K_2 \cos \zeta_k$.

In the same manner we put

$$\textstyle\sum_1' = \sum Y_k \cos 24k\beta_m - \sum X_k \sin 24k\beta_m$$
$$\textstyle\sum_2' = \sum Y_k \cos 24k\beta_n - \sum X_k \sin 24k\beta_n$$
$$\textstyle\sum_3' = \sum Y_k$$

and obtain three equations for the determination of

$$M_2 \sin \zeta_m, \; N_2 \sin \zeta_n \text{ and } S_2 \sin \zeta_s + K_2 \sin \zeta_k.$$

In the following table are given the values of the factors S_m, C_m etc. resulting from the daily analysis. The factors have been divided by 48 in order to give values near unity

	C	S
M_2	0.976	1.007
N_2	0.955	1.003
S_2	1.000	1.000
K_2	1.001	0.999
O_1	— 0.028	— 0.058

In the following table are given the values of $c\,(m, n)$, $s\,(m, n)$ etc. Theoretically we have $c\,(m, n) = c\,(n, m)$ but since the multipliers are only correct to three decimal places, there may be slight differences.

	$c\,(m,)$	$c\,(n,)$	$c\,(s,)$	$s\,(m,)$	$s\,(n,)$
m	7.622	5.298	−0.236	7.379	3.406
n	5.299	7.196	−3.060	3.407	7.804
s	−0.238	−3.058	15.000	0	0
k	−0.135	−2.972	14.835	−0.598	0.150
o	7.750	5.647	−0.811	7.209	3.856

From the tables the normal equations may easily be found, but we do not write out these equations explicity. It will only be necessary to give the solutions.

$$463\ M_2 \cos \zeta_m = \textstyle\sum_1 - 0.607 \sum_2 - 0.108 \sum_3$$
$$437\ N_2 \cos \zeta_n = -0.579 \textstyle\sum_1 + \sum_2 + 0.195 \sum_3$$
$$677\ [S_2 \cos \zeta_s] = -0.160 \textstyle\sum_1 + 0.303 \sum_2 + \sum_3$$

The correction of M_2 for the influence of O_1 is only 0.009 $O_1 \cos \zeta_0$ and may be left out of account. For the terms containing $\sin \zeta$ we find the solution

$$468\ M_2 \sin \zeta_m = \textstyle\sum_1' - 0.598 \sum_2' - 0.106 \sum_3'$$
$$444\ N_2 \sin \zeta_n = -0.575 \textstyle\sum_1' + \sum_2 + 0.195 \sum_3'$$
$$680\ [S_2 \sin \zeta_s] = -0.149 \textstyle\sum_1' + 0.285 \sum_4' + \sum_3'$$

To correct S_2 for the influence of K_2 it is necessary to assume that the ratio of K_2 to S_2 is the same as that required by the equilibrium theory, and that

$$\varkappa_s = \varkappa_k.$$

We put

$$\varkappa_s = \zeta_s + V_s$$
$$\varkappa_k = \zeta_k + V_k$$

and get

$$\zeta_k = \zeta_s - (V_k - V_s).$$

Further, we put $K_2 = r\,S_2$. Then

$$S_2 \cos \zeta_s + rS_2 \cos (\zeta_s - V_k + V_s) = [S_2 \cos \zeta_s]$$
$$S_2 \sin \zeta_s + rS_2 \sin (\zeta_s - V_k + V_s) = [S_2 \sin \zeta_s]$$

where the quantities in the brackets are the values which we find from the analysis. Let

$$\tan \psi = \frac{r \sin (V_k - V_s)}{1 + r \cos (V_k - V_s)}$$

and

$$R^2 = r^2 \sin^2 (V_k - V_s) + (1 + r \cos (V_k - V_s))^2$$

then

$$RS_2 \cos (\zeta_s - \psi) = [S_2 \cos \zeta_s]$$
$$RS_2 \sin (\zeta_s - \psi) = [S_2 \sin \zeta_s]$$

from which we find

$$\zeta_s = [\zeta] + \psi$$
$$S_2 = \frac{[S_2]}{R}.$$

The analysis of the diurnal components is carried out after the same principles. In order to eliminate the obliquity of the tide curve we first make the combination

$$h'(t) = h(t - 6) - h(t + 6) = 2A \sin 6\sigma \sin (\sigma t - \zeta).$$

The factor $\sin 6\sigma$ is then approximately 1 for the diurnal components, and nearly zero for the semi-diurnal components. Consequently, it is unnecessary to include M_2 in the scheme of analysis, and we only consider the components K_1, P_1 and O_1. The components K_1 and P_1 can not be separated in the short interval of 15 days, and so we can only find the combined effect of the two waves.

The daily process gives the results

$$\textstyle\sum h' \cos 15^\circ t = -C_k K_1 \sin (\zeta_k - 24k\beta) - C_p P_1 \sin (\zeta_p + 24k\beta) - C_0 O_1 \sin (\zeta_0 + 24k\gamma) = -Y_k$$

and

$$\textstyle\sum h' \sin 15^\circ t = S_k K_1 \cos (\zeta_k - 24k\beta) + S_k P_1 \cos (\zeta_p + 24k\beta) + S_0 O_1 \cos (\zeta_0 + 24k\gamma) = X_k.$$

Since the angle $24k\beta$ for K_1 and P_1 is only small, it is unnecessary to compute the sums

$$\textstyle\sum X_k \cos 24k\beta \text{ and } \sum X_k \sin 24k\beta$$

and we only form the combinations

$$\textstyle\sum X_k,\ \sum X_k \cos 24k\gamma,\ \sum k_k \sin 24k\gamma$$
$$\textstyle\sum Y_k,\ \sum Y_k \cos 24k\gamma \text{ and } \sum Y_k \sin 24k\gamma.$$

Let

$$\textstyle\sum_1 = \sum X_k$$
$$\textstyle\sum_2 = \sum X_k \cos 24k\gamma + \sum Y_k \sin 24k\gamma$$
$$\textstyle\sum_1' = \sum Y_k$$
$$\textstyle\sum_2' = \sum Y_k \cos 24k\gamma - \sum X_k \sin 24k\gamma.$$

Then we find for the diurnal components

$$713.4\ [K \cos \zeta] = \textstyle\sum_1 + 0.056 \sum_2$$
$$707.6\ O \cos \zeta = 0.089 \textstyle\sum_1 + \sum_2,$$
$$715.6\ [K \sin \zeta] = \textstyle\sum_1' + 0.052 \sum_2'$$
$$705.1\ O \sin \zeta = 0.089 \textstyle\sum_1' + \sum_2'.$$

To correct K_1 for the influence of P_1, we again assume that the ratio of P_1 to K_1 is the same as in the equilibrium theory and that $\varkappa_k = \varkappa_p$ consequently we put $P_1 = l \cdot K_1$ and

$$\zeta_p = \zeta_k - V_p + V_k.$$

If we then put

$$\tan \chi = \frac{l \sin (V_p - V_k)}{1 + l \cos (V_p - V_k)}$$

and

$$R_1^2 = l^2 \sin^2 (V_p - V_k) + (1 + l \cos (V_p - V_k))^2$$

then

$$\zeta_k = [\zeta_k] + \chi$$

$$K_1 = \frac{[K_1]}{R}.$$

Results of the harmonic analysis.

In the present case we find the following results:

$M_2 = 2.47\,\mu$	$M_2^\circ = 253.°8$	0.″00494
$N_2 = 0.65\,\mu$	$N_2^\circ = 241.°2$	0.″00130
$S_2 = 0.39\,\mu$	$S_2^\circ = 245°$	0.″00078
$K_1 = 1.65\,\mu$	$K_1^\circ = 209.°2$	0.″00330
$O_1 = 1.55\,\mu$	$O_1^\circ = 211.°1$	0.″00310

In the last column the amplitudes are converted to seconds of arc of the deviation of the plumb-line.

An inspection of the observed values shows that the phases of the diurnal components are very near the theoretical values, while the phases of the diurnal components have a lag of 40° to 50°.

The semidiurnal component S_2 is very small and the corresponding phase consequently rather uncertain.

The phase lag of the semidiurnal components is evidently due to the effect of the varying load of the tide on the coast. It has been shown by SCHWEYDAR that the semidiurnal tides in Europe are always more affected by the influence of the tides on the coast than the diurnal tides are. This is even more pronounced in our case than in the measurements of earth tides in Germany, where all the stations have been far away from the coast.

High water on the coast means that the load of water on the coast is increased, and since the earth's crust is elastic, it will be depressed somewhat. The depression of the earth's crust must diminish as we proceed inland. At the time of local high water the tilting of the earth will cause the water to flow towards the end of the tube which points towards the sea. The effect of the local tidal load would be to raise the surface of the water in the end of the tube which points towards the sea, and to lower the surface at the other end. The direction of our tube is N 18° E to S 18° W, and the end of the tube which is lying nearest to the sea is S 18° W. In the results of the harmonic analysis we have given the phase angles corresponding to (N 18° E) — (S 18° W). Consequently we would have «Low water» at the time of high water at the coast. The observations show that this is not the case. The effect of the tidal load is, therefore, not of local character. The direction of the tube is not well suited to find the effect of the local tidal load, since it does not differ much from the general trend of the coastline. We then arrive at the conclusion that the effect must be due to the fact that the tidal range increases rapidly when we pass along the coast of Norway from south to north. In the following table we have compiled the principal harmonic constants for a number of stations along the Norwegian coast.

	M_2	M_2°	S_2	S_2°	N_2	N_2°	K_1	K_1°	O_1	O_1°
Stavanger	14.3	278°	6.7	329°	3.3	257°	1 4	179°	1.6	8°
Bergen	43.9	298	15.6	335	8.4	269	3 3	168	2.8	17
Kjølsdal............	59.3	299	23.0	354	15.6	283				
Bodø...............	86.7	358	28.8	36	18.1	334	10,9	207	3.7	48
Kabelvaag..........	91 5	4	32.8	45	18.7	338	10.1	209	3.7	53

From this table we see that the northern part of Norway is depressed more than the southern part is. It is reasonable to assume that the tilting of the earth's surface due to the tidal load will be proportional to the gradient of the tidal range along the coast. Approximate values of the gradient may be found from the harmonic constants for Stavanger, Bergen and Kjølsdal. The geographic coordinates of the stations are

Stavanger	Lat.	58° 59′ N,	Long.	5° 44′ E
Bergen	»	60° 24′ N,	»	5° 18′ E
Kjølsdal	»	61° 54′ N,	»	5° 36′ E

The distance Stavanger—Bergen is then about 157.4 km and Bergen—Kjølsdal 166.7 km. The slight differences in longitude have been disregarded. From the harmonic constants we find the following values of the gradient for the components M_2, S_2 and N_2.

$$M_2: \quad 14.6 \cdot 10^{-7} \cos (\sigma_1 t - 303.^\circ 4)$$
$$S_2: \quad 5.3 \cdot 10^{-7} \cos (\sigma_0 t - 0.^\circ 5)$$
$$N_2: \quad 3.9 \cdot 10^{-7} \cos (\sigma_2 t - 288.^\circ 9)$$

Assuming that M_2, as observed, is composed by two components with phase angles 200.°9 and 303.°4 with unknown amplitudes R_1 and R_2, we have

$$R_1 \cos (\sigma t - 200.^\circ 9) + R_2 \cos (\sigma t - 303.^\circ 4) = 2.47 \cos (\sigma t - 253.^\circ 8)$$

comparing the coefficients of $\cos \sigma t$ and $\sin \sigma t$, we have the following relations:

$$R_1 \cos 200.^\circ 9 + R_2 \cos 303.^\circ 4 = 2.47 \cos 253.^\circ 8$$
$$R_1 \sin 200.^\circ 9 + R_2 \sin 303.^\circ 4 = 2.47 \sin 253.^\circ 8.$$

From these equations we obtain

$$R_1 = R \frac{\sin 49.^\circ 6}{\sin 102.^\circ 5} = 1.93 \, \mu$$

$$R_2 = R \frac{\sin 52.^\circ 9}{\sin 102,^\circ 5} = 2.02 \, \mu$$

R_1 then represents the amplitude of the static tide, and R_2 the tide due to the tidal load. The ratio of the computed static tide to the theoretical tide on a rigid earth is then

$$\frac{1.93}{3.34} = 0.578.$$

This value is in good agreement with the values found elsewhere.[1])

[1]) Handbuch d. Geophysik, Bd. I, Lieferung 2.

The ratio of the load tilt to the gradient of M_2 is

$$\frac{2.02 \cdot 10^{-4}}{14.6 \cdot 10^{-7} \, 1.03 \cdot 10^4} = 1.346 \cdot 10^{-2}$$

Corresponding calculations might be carried out for the components S_2 and N_2, but we may also use the ratios found above for M_2 to compute the amplitudes and phase angles which are to be expected for these components if they behave like M_2. The theoretical amplitudes of these components are in this case 1.60 and 0.65 respectively.

Using the ratios computed above for M_2, we find for S_2

$$R_1 = 0.578 \cdot 1.60 = 0.925 \, \mu$$
$$R_2 = 1.346 \cdot 1.03 \cdot 0.53 = 0.727 \, \mu.$$

If we then put

$$0.925 \cos (\sigma t - 200.^\circ 9) + 0.727 \cos (\sigma t - 0.^\circ 5) = S_2 \cos (\sigma t - \varkappa)$$

we find

$$S_2 = 0.35 \, \mu \quad \varkappa = 247^\circ.$$

The observed values are

$$S_2 = 0.39 \, \mu \quad \varkappa = 245^\circ.$$

For N_2 we find in the same manner

$$R_1 = 0.578 \cdot 0.65 = 0.376 \, \mu$$
$$R_2 = 1.346 \;\; 1.03 \;\; 0.39 = 0.538 \, \mu.$$

and if we put

$$0.376 \cos (\sigma t - 200.^\circ 9) + 0.538 \cos (\sigma t - 288.^\circ 8) = N_2 \cos (\sigma t - \varkappa).$$

We find $N_2 = 0.67 \, \mu, \quad \varkappa = 254^\circ$

against the observed values

$$N_2 = 0.65 \, \mu, \quad \varkappa = 241^\circ.$$

The difference in phase is not greater than might be expected from the short series of observations.

The observed tidal effects are thus completely explained by the hypothesis set forth above.

The diurnal tides on the Norwegian coast are everywhere small, and the corresponding tilt due to the varying tidal load will be of less consequence. This explains the fact that the observed phase angles of the diurnal components K_1 and O_1 almost agree exactly with the theoretical values.

The component O_1 is the one which theoretically should be least disturbed by influence of the oceanic tides and also by a possible temperature amplitude. The ratio of the observed value to the theoretical value is

$$\frac{1.55}{2.07} = 0.75$$

The results given for the component K_1 are more uncertain since the components K_1 and P_1 cannot be determined separately. The combined amplitude which is found from the analysis, is 2.0 while the value corrected for the influence of P_1 is 1.65 or 0.0033 seconds of arc. The ratio of the observed to the theoretical value is

$$\frac{1.65}{2.78} = 0.60.$$

The observations are not well suited for determination of the earth's rigidity, since the effect of the varying tidal load is of the same order of magnitude as the static tide caused by the direct attraction of the moon and the sun, and the separation of the two effects cannot be effected except by introduction of some hypotheses which always involve some uncertainties. On the other hand, the effect of the varying load of the oceanic tides on the earth's crust is clearly demonstrated.