ON THE CORRELATION BETWEEN THE GEOPOTENTIAL OF THE TROPOPAUSE AND THE TEMPERATURE OF THE MIDDLE TROPOSPHERE

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1. Introduction.

Until recently, the aerological observations available in the daily routine work of weather forecasting consisted mainly of the results of airplane soundings reaching an altitude of about 5 km. After the introduction of radio soundings, however, the current aerological information has been extended up to altitudes of 15—20 km. Thus it has become possible to use observations from higher levels for the benefit of forecasting.

As was shown empirically by *Hesselberg* (1) in 1913, the direction of movement of atmospheric disturbances is parallel to the air motion at the cirrus level, i. e. to the motion of the upper part of the troposphere. Recently, from theoretical considerations, *Ertel* (2) found that the movement of disturbances is parallel to the lines of equal geopotential of the tropopause, with the high tropopause to the right (northern hemisphere). Thus the geopotential of the tropopause must be expected to be a useful prognostic element.

As yet, the network of radio soundings is not dense enough to allow the drawing of detailed maps of the topography of the tropopause over Europe. In this paper, however, it will be shown that the geopotential of the tropopause may be estimated from the temperature at 5 gkm with sufficient accuracy to allow to complete the direct observations from radio soundings with estimated values from airplane soundings.

Some correlation coefficients published by other authors, gave reason to expect a rather high correlation between the geopotential of the tropopause and the temperature of the middle troposphere. Let r be the correlation coefficient, H the geometric height of the tropopause and T_n the temperature at n km. Then we have:

$r(H,T_7) = 0{,}76$. *Köppen* and *Wedemayer* (3).
$r(H,T_4) = 0{,}64$. *Dines* (4).
$r(H,T_{0-9}) = 0{,}79$. *Dines* (4).
$r(H,T_{0-9}) = 0{,}78$. *Haurwitz* (5).
$r(H,T_5) = 0{,}82$. *Palmén* (6).
$r(H,T_{0-10}) = 0{,}71$. *Zistler* (7).

Further it has been shown by *Dines* (8), *Egersdörfer* (9), and *Portig* (10) that the correlation coefficient between the simultaneous pressure and temperature at 5 km is about 0,8. On the other side, *Portig* (11) found that the correlation between pressure at 5 km and the height of the tropopause is of the order 0,7. In this indirect way we are also led to expect a rather close correlation between the temperature of the middle troposphere and the height (or geopotential) of the tropopause.

2. The material.

For our investigation we have used the following observations:

Svalbard (78° 56′ N, 11° 58′ E).

28 radio soundings made in July and August 1937 by *Tommila* and *Raunio* (12).

Tromsø (69° 39′ N, 18° 57′ E).

31 radio soundings made in April 1939 by the Weather Service at Tromsø.

Abisko (68° 21′ N, 18° 49′ E).

87 soundings in 1907—1929, published by *Rolf* (13).

The soundings of Abisko have been combined with 33 soundings at Riksgränsen (68° 26′ N, 18° 8′ E) made in 1930—1936 and published in the Annual Report (Årsbok) of Statens Meteorologisk-Hydrografiska Anstalt at Stockholm.

Thorshavn (62° 3′ N, 6° 45′ W).
36 radio soundings made in March and April 1939 and published by the Danish Meteorological Institute (14).

Ås (59° 40′ N, 10° 46′ E).
116 soundings made in 1932—1936 and published in the «Jahrbuch des Norwegischen Meteorologischen Instituts.»

København (55° 45′ N, 12° 35′ E).
30 radio soundings made from December 1938 to March 1939 and published by the Danish Meteorological Institute (14).

Sealand (53° 14′ N, 3° 0′ W).
252 soundings made in 1930—1938. Draft copies circulated by the Meteorological Office, London.

Lindenberg (52° 12′ N, 14° 7′ E).
131 soundings made in 1925—1928 and in 1932. Published by the International Aerological Commission.

3. Correlation coefficients and regression equations.

For each set of observations we have calculated the correlation coefficient between the temperature at 5 geodynamical kilometres and the geopotential of the tropopause. The result is given in Table 1, where[1])

n = number of observations,

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{n}} =$$ standard deviation of the temperature at 5 gkm,

$$\sigma_y = \sqrt{\frac{\Sigma y^2}{n}} =$$ standard deviation of the geopotential of the tropopause,

$$r = \frac{\Sigma xy}{n\,\sigma_x\,\sigma_y} =$$ correlation coefficient,

$$\varepsilon(r) = 0{,}67 \cdot \frac{1 - r^2}{\sqrt{n}} =$$ probable error of r,

$$b_1 = r \cdot \frac{\sigma_x}{\sigma_y} =$$ coefficient of regression of temperature on geopotential,

$$b_2 = r \cdot \frac{\sigma_y}{\sigma_x} =$$ coefficient of regression of geopotential on temperature.

Table 1.

Station	n	σ_x	σ_y	r	$\varepsilon(r)$	b_1	b_2	$\frac{\sigma_y}{\sigma_x}$
Svalbard .... ($\varphi = 78^\circ,9$ N)	28	2,36	0,69	0,75	0,05	2,58	0,22	0,29
Tromsø ..... ($\varphi = 69^\circ,7$ N)	31	3,55	1,08	0,82	0,04	2,73	0,25	0,30
Abisko ...... ($\varphi = 68^\circ,4$ N)	120	9,06	1,36	0,71	0,03	4,75	0,11	0,15
Thorshavn .. ($\varphi = 62^\circ,1$ N)	36	4,56	1,00	0,85	0,03	3,86	0,19	0,22
Ås .... ($\varphi = 59^\circ,7$ N)	116	8,25	1,58	0,80	0,02	4,21	0,15	0,19
København .. ($\varphi = 55^\circ,8$ N)	30	5,03	1,39	0,75	0,05	2,68	0,21	0,28
Sealand ..... ($\varphi = 53^\circ,2$ N)	252	6,87	1,43	0,82	0,01	3,90	0,17	0,21
Lindenberg .. ($\varphi = 52^\circ,2$ N)	131	7,73	1,22	0,73	0,03	4,56	0,12	0,16
Mean........		5,93	1,22	0,78	0,03	3,66	0,18	0,23

In Table 2 the mean temperature at 5 gkm and the mean geopotential of the tropopause are shown for each set of observations. Apart from those of Svalbard, the values of Table 2 lie practically on a straight line.

Table 2.

Station	Mean temperature at 5 gkm	Mean geopotential of the tropopause
Svalbard	$-19^\circ,1$	9,3 gkm
Tromsø	$-33^\circ,6$	8,2 »
Abisko......................	$-24^\circ,8$	9,6 »
Thorshavn	$-24^\circ,7$	9,6 »
Ås	$-22^\circ,1$	9,6 »
København	$-28^\circ,5$	9,2 »
Sealand	$-18^\circ,4$	10,5 »
Lindenberg	$-18^\circ,5$	10,9 »

In Fig. 1 the individual values of the temperature at 5 gkm (T_5) are plotted against the corresponding geopotential of the tropopause (Φ). The mean values for each station are indicated by a heavy dot surrounded by a circle. Through this point are drawn straight lines with the angular coefficient $\frac{\sigma_y}{\sigma_x}$.

It will be seen from the values of the correlation coefficient in Table 1 and from the figure that *there is an approximately linear relation between the tem-*

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[1]) Compare *Yule*, An Introduction to the Theory of Statistics, 4th ed., p. 171, or *Haurwitz*, loc. cit., p. 30.

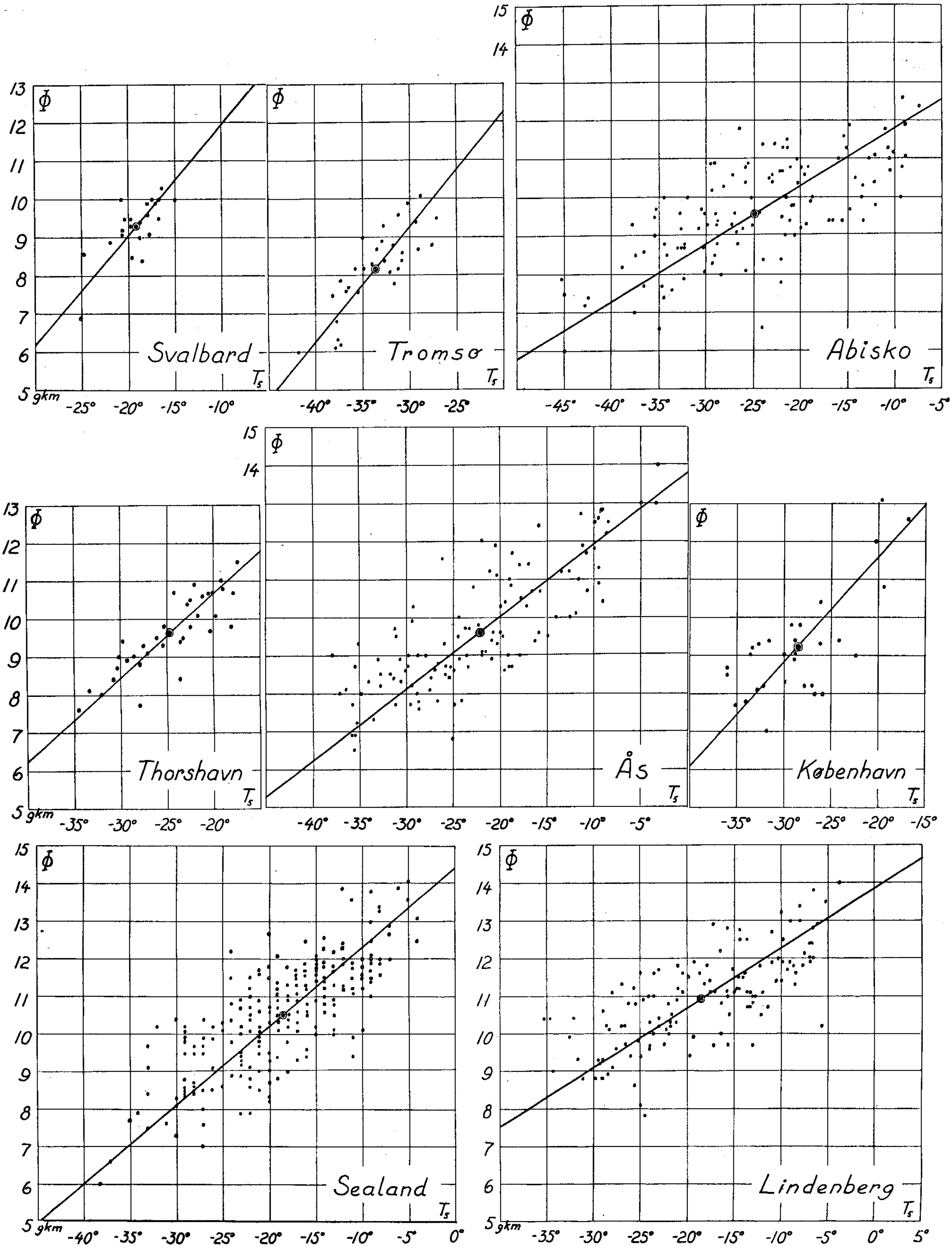

Fig. 1. Geopotential of tropopause (Φ) against temperature at 5 dynamic kilometres (T_5).

perature at 5 gkm and the geopotential of the tropopause. If $\triangle T_5$ stands for the deviation of the temperature at 5 gkm from the mean for that special station and $\triangle\Phi$ for the deviation from the mean of the geopotential of the tropopause, then $\triangle\Phi$ may be estimated from $\triangle T_5$ by means of the regression equation:

$$(1) \qquad \triangle\Phi = b_2 \cdot \triangle T_5.$$

The standard error s_y made in this estimation may be expressed thus[1]):

$$(2) \qquad s_y = \sigma_y \cdot \sqrt{1-r^2}.$$

As the value of the correlation coefficient r lies between 0,71 and 0,85, the rate of s_y to σ_y will be from 0,70 to 0,53. Thus the standard error made in estimating $\triangle\Phi$ from equation (1) will be from about $^3/_4$ down to $^1/_2$ of the standard deviation of $\triangle\Phi$.

In Table 3 are given the regression equation for each station, the rate of s_y to σ_y, and s_y. It will be seen from the table that the standard error s_y lies between 0,5 and 1,0 gkm.

Table 3.

Station	Regression equation	$\frac{s_y}{\sigma_y}$	s_y
Svalbard	$\triangle\Phi = 0{,}22 \cdot \triangle T_5$	0,66	0,5
Tromsø	$\triangle\Phi = 0{,}25 \cdot \triangle T_5$	0,57	0,6
Abisko................	$\triangle\Phi = 0{,}11 \cdot \triangle T_5$	0,70	1,0
Thorshavn	$\triangle\Phi = 0{,}19 \cdot \triangle T_5$	0,53	0,5
Ås	$\triangle\Phi = 0{,}15 \cdot \triangle T_5$	0,60	0,9
København	$\triangle\Phi = 0{,}21 \cdot \triangle T_5$	0,66	0,9
Sealand	$\triangle\Phi = 0{,}17 \cdot \triangle T_5$	0,57	0,8
Lindenberg	$\triangle\Phi = 0{,}12 \cdot \triangle T_5$	0,68	0,8

As a mean for all our stations we get as regression equation:

$$(3) \qquad \triangle\Phi = 0{,}18 \cdot \triangle T_5$$

with a standard error of about 0,7 gkm.

We may arrive at a similar result in another way. Fig. 2 is a diminished reproduction of a diagram on temperature in altitude and tropopause at Trappes in August 1938, published by *Bureau.* At the top of the diagram we have added a curve showing the variation of temperature at 5 km. The scale has been chosen so that in this curve an amplitude of 20° corresponds to 5 km in the lower part of the diagram. It will be seen that this curve of the temperature at 5 km (or 5 gkm) runs practically parallel to the curve showing the height (or geopotential) of the tropopause as a function of time; thus we get:

$$\triangle\Phi = 0{,}25 \cdot \triangle T_5.$$

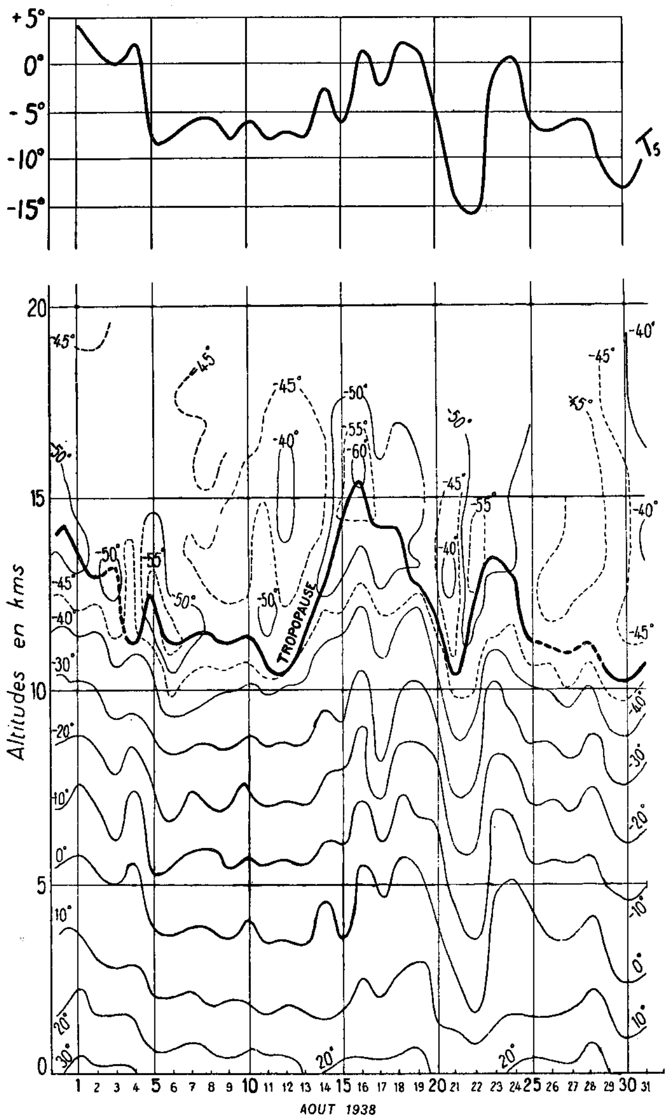

Fig. 2. Thermo-isopleths at Trappes in August 1938 (after *Bureau*). Uppermost curve: variation of temperature at 5 km.

4. The latitudinal variation of the regression coefficient.

From the values of the latitude φ and the regression coefficient b_2 in Table 1 we find that the correlation coefficient between these two sets of variables is as high as 0,79. The regression equation is:

$$(4) \qquad b_2 - 0{,}18 = 0{,}0044 \cdot (\varphi - 62{,}5), \text{ or}$$

$$b_2 = 0{,}0044 \cdot \varphi - 0{,}095.$$

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[1]) See *Yule*, loc. cit., p. 177.

In Fig. 3 the values of b_2 are plotted against φ and the regression equation is marked by a straight line.¹)

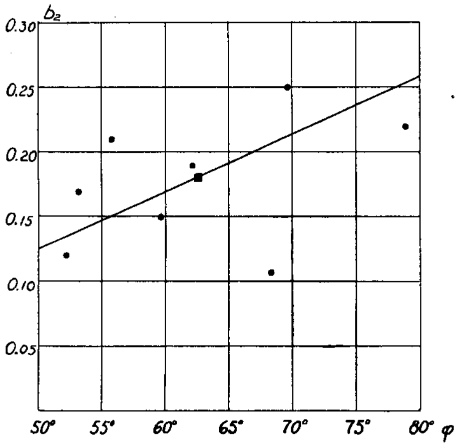

Fig. 3. The regression coefficient (b_2) as a function of latitude (φ).

As a result of the statistical treatment we may state that, on an average, the regression coefficient b_2 increases from latitude 50° N to latitude 80° N. This would mean that *in higher latitudes temperature variations at 5 gkm are correlated with greater variations of the geopotential of the tropopause than in lower latitudes.*

Taking into account that, on an average, the geopotential of the tropopause decreases with increasing latitude, it seems quite natural that the interaction between variations in the middle troposphere and variations at the tropopause should be most pronounced in higher latitudes. (In fact, a series of 64 observations at Agra ($\varphi = 27°$) gave a correlation coefficient as low as 0,36 between Φ and T_5).

5. Note on the effect of the general circulation.

In Fig. 4 the mean values of Φ and T_5 are shown as small dots and the respective regression lines are drawn in full. The broken line shows the average relation between Φ and T_5, constructed on the basis of the diagram given by *Palmén* (15) for the temperature distribution in January and July from the Pole to the Equator.

It will be seen that there is a rather good agreement between the mean values for our stations and the standard curve. At the right end of the diagram, however, there is a remarkable difference between the regression lines and the standard curve. When the height of the tropopause gets well above the mean for each special station the temperature of the middle troposphere is higher than that which corresponds to the standard curve.

Now it must be supposed that the most effective way of getting an abnormally high tropopause in middle and high latitudes, is the advection of air masses from the south which bring along their high tropopause. This supposition is supported by the fact that *Douglas* (16) found a high positive correlation between the height of the tropopause and the northward displacement of the air mass during the last 3 days.

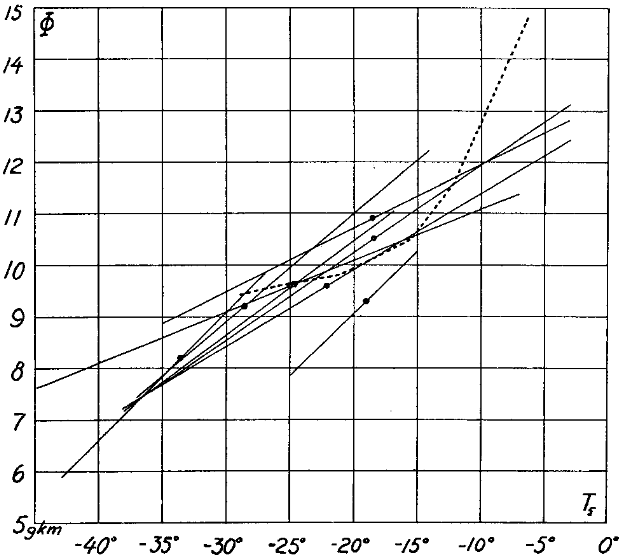

Fig. 4. The regression lines (in full) as compared whith the standard relation (broken line) between the temperature at 5 gkm and the geopotential of the tropopause.

From Fig. 4 we may then conclude that *air masses which travel northwards are often subject to heating in the middle troposphere.* As the intensity of radiation and the surface temperature decrease northwards, this heating must be due to subsidence.

6. Synoptic evidence.

For each day of the international aerological month April 1939 we have plotted three sets of maps showing (1) the synoptic situation at the ground, (2) the distribution of temperature at 5 gkm

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¹) It should be noted that the value of b_2 for Abisko—Riksgränsen is much lower than that which would be expected from the other material. If the 33 observations from Riksgränsen are treated separately, they give the rather low values $r = 0{,}59$ and $b_2 = 0{,}09$. Thus there is reason to believe that the series of observations from Abisko—Riksgränsen is less homogeneous than those of the other stations.

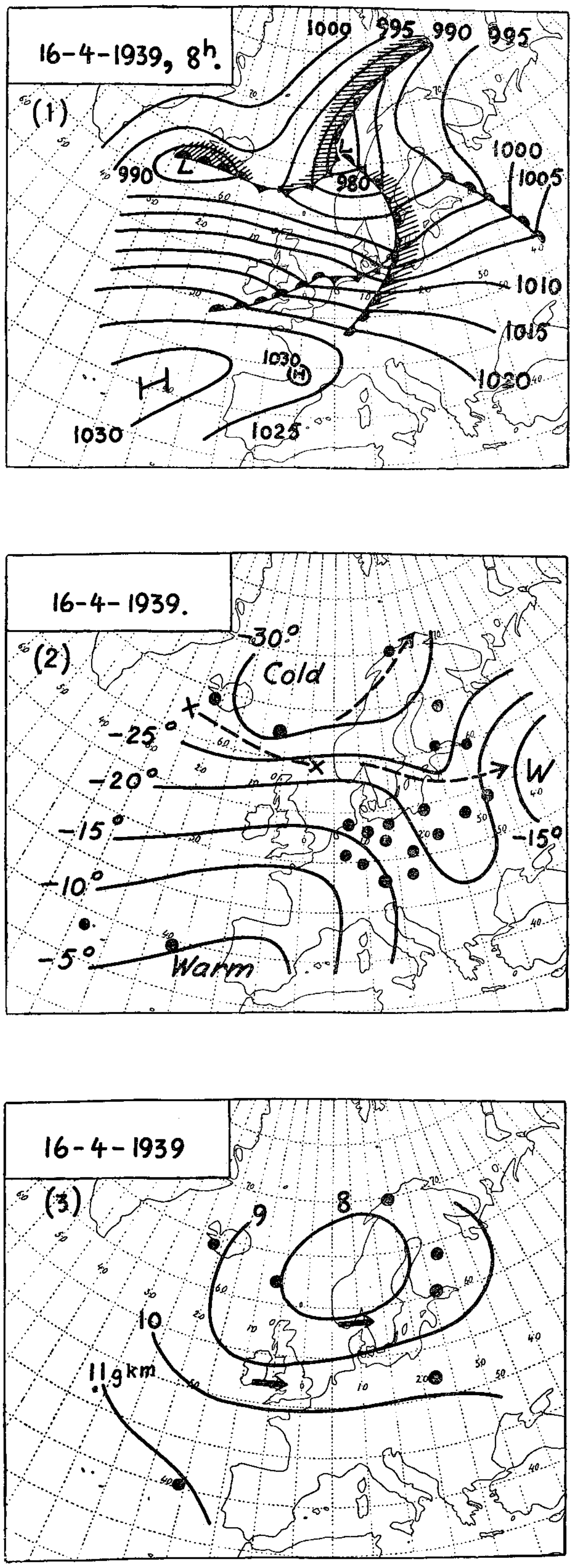

Fig. 5 a.

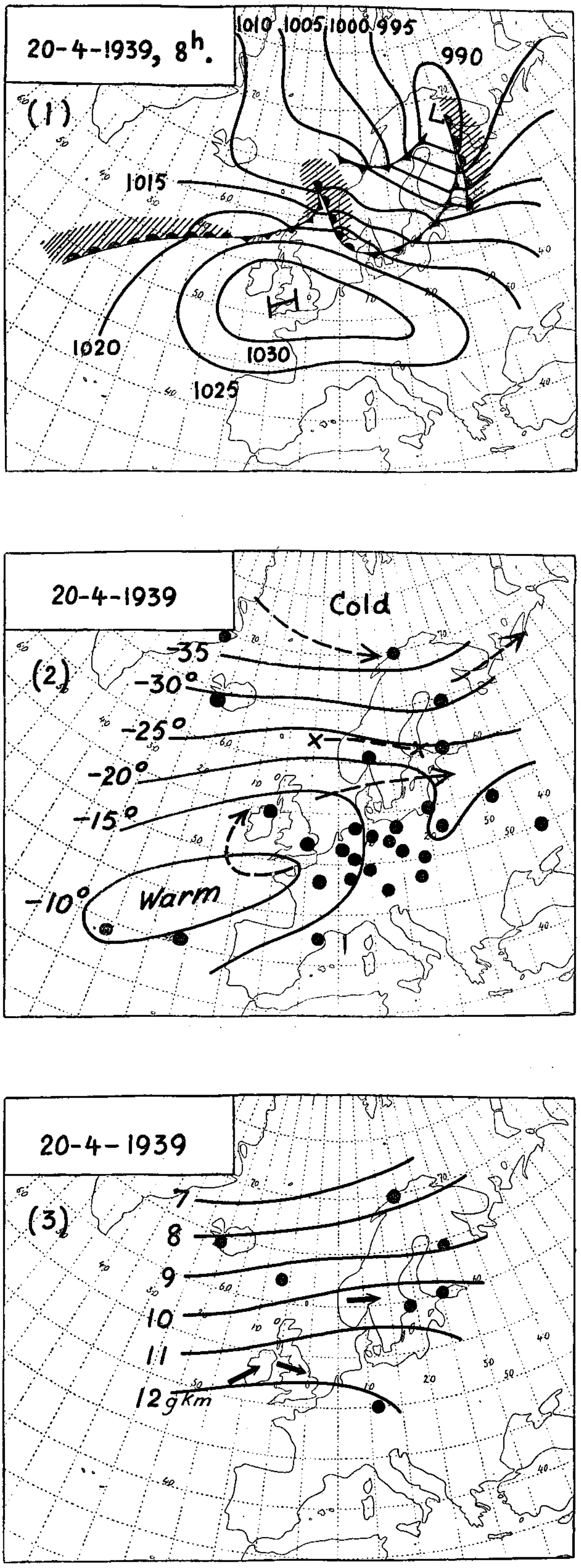

Fig. 5 b.

(1) Sy noptic situation. (2) Temperature at 5 gkm, displacement of Lows, and displacement of 24 hour isallobaric centres,

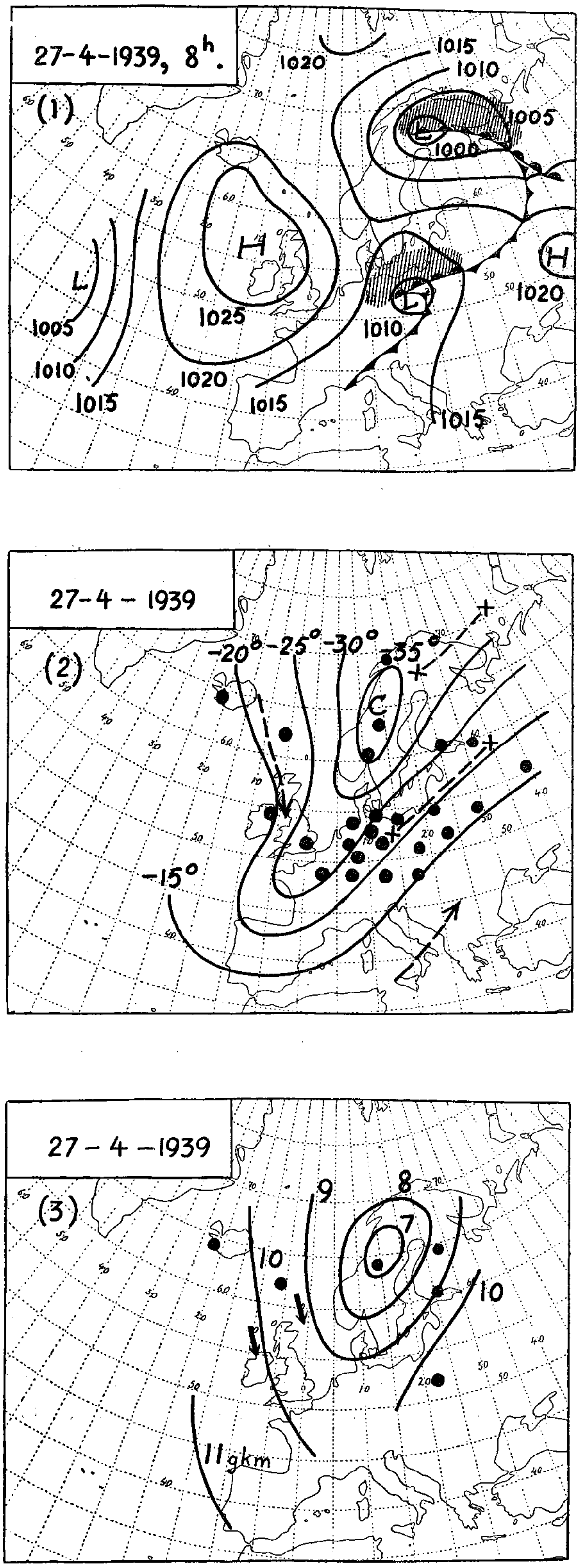

Fig. 5 c.
(3) Geopotential of the tropopause and observed motion of cirrus clouds.

and (3) the geopotential of the tropopause. As an illustration we give here the maps for April 16, 20 and 27 (Figs. 5 a–c). The isotherms at 5 gkm have been drawn for intervals of 5 degrees centigrade, and the geopotential of the tropopause is shown by lines for each geodynamic kilometre. The aerological stations whose observations have been used are marked by dots.

On comparing the maps of series (2) with those of series (3), we see that the isotherms at 5 gkm and the lines of equal geopotential of the tropopause are practically parallel. Thus our statistical result is synoptically confirmed. It should be mentioned, however, that in the series of maps for April 1939 there are also some cases where the relation between the isotherms at 5 gkm and the topography of the tropopause is more complicated.

On the maps of series (2) crosses denote the position of Lows at time of observation and 24 hours later. Broken lines between the crosses show the displacement of Lows. Arrows show the displacement in the 24 hours following the time of observation of negative isallobaric centres for intervals of 24 hours (copied from the Daily Synoptic Map of the Deutsche Seewarte).

It will be seen from these maps that *the displacement of the disturbances is practically parallel to the isotherms at 5 gkm, with the warm air to the right (northern hemisphere).*

On taking into account the close correlation between these isotherms and the topography of the tropopause, this empirical result is in perfect accordance with the theoretical statement of *Ertel* (loc. cit., p. 405) that the displacement of disturbances is parallel to the lines of equal geopotential of the tropopause, with the high tropopause to the right (northern hemisphere).

On the maps of series (3) arrows show observations of the motion of cirrus clouds. It will be seen that in all three cases *the cirrus motion is parallel to the lines of equal geopotential of the tropopause.*

In his paper on the motion of the atmosphere at the cirrus level *Hesselberg* (loc. cit., p. 33) stated that it had been impossible to find any satisfactory statistical relation between the rate of propagation of disturbances at the ground and the velocity of the air at the cirrus level. Further, according to *Ertel* (loc. cit., p. 406) the rate of propagation of the disturbance at the ground is proportional not only to the slope of the tropopause but also to the rate of the pressure at the tropopause to that at the ground, to the rate between the geopotential and the absolute temperature of the tropopause, and to the discontinuous change of the vertical temperature gradient at the tropopause. Thus, *there is no simple relation between the rate of propagation of disturbances and the slope of the tropopause (or the horizontal temperature gradient at 5 gkm).*

7. Practical method of constructing maps of the topography of the tropopause.

On the map, Fig. 6, the values without a bracket give the geopotential of the tropopause from direct measurements (radio soundings). This values are not numerous enough to allow the drawing of the lines of equal geopotential with any certainty. To be able to do this, the gaps between the measured values are filled up with approximate values, estimated from the temperature at 5 gkm (taken from airplane soundings). In Fig. 6 the estimated values stand in brackets and arrows show observations of the motion of cirrus clouds. With this additional information the lines of equal geopotential may be drawn without difficulty.

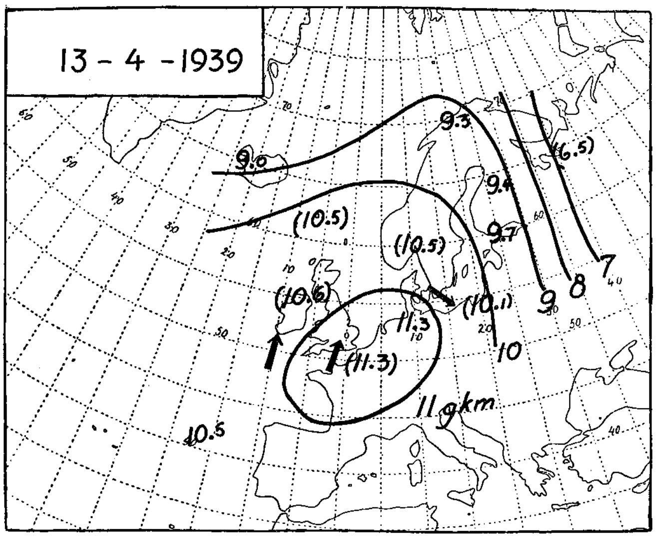

Fig. 6. Geopotential of the tropopause. Numbers without a bracket are measured values, numbers in brackets are estimated values. Arrows show the observed motion of cirrus clouds.

The approximate values of Φ may be calculated by means of the regression equation (1) in the form:

$$\Phi = \overline{\Phi} + b_2 \cdot (T_5 - \overline{T}_5).$$

For the average geopotential $\overline{\Phi}$, the average temperature $\overline{T}_5$, and the regression coefficient b_2 we may use the values in Table 4, where $\overline{\Phi}$ and $\overline{T}_5$ have been calculated from the diagram of *Palmén* (15) and b_2 from equation (4). This values have turned out to give good agreement for the observations of April 1939.

Table 4.

φ	50°	55°	60°	65°	70°
$\overline{\Phi}$	— 16°	— 18°,5	— 21°	— 23°,5	— 26°
$\overline{T}_5$......	10,5	10,1	9,8	9,7	9,6 gkm
b_2	0,13	0,15	0,17	0,19	0,21

8. Summary.

a) Observations from Svalbard, Tromsø, Abisko, Thorshavn, Ås, København, Sealand, and Lindenberg give, as a mean, a correlation coefficient of 0,78 between the temperature at 5 gkm and the geopotential of the tropopause.

b) The regression coefficient is, on an average, 0,18. It seems to be a linear function of the latitude, increasing northwards.

c) The statistical results seem to indicate that air masses which travel northwards, are generally heated in the middle troposphere.

d) Synoptic maps show cases where the displacement of disturbances at the ground is parallel to the isotherms at 5 gkm, with the warm air to the right.

e) A method is given to construct maps for the dynamic topography of the tropopause by combining direct measurements of the geopotential and approximate values, estimated from the measured temperature at 5 gkm. The motion of cirrus clouds is also taken into account.

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