# CONTRIBUTION TO THE THEORY OF CONVECTION

*BY SVERRE PETTERSSEN*

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**1. Introduction.** The word *convection* is used in meteorological literature to denote vertical currents in the atmosphere which result from static instability. The theory of convection and the study of convective weather phenomena are, therefore, closely related to the study of the criteria of stability and instability. In addition, the amount of energy which can be released from the stratification and made available for creation of convective currents has been the subject of many erudite discourses. An approach to the solution of this problem was made by *Margules*[1]) who computed the total potential and internal energy of a system in which colder air rests side by side with, or above, warmer air, the two masses being separated by a thin wall. After removal of the wall, the system assumes a stable equilibrium position with the air arranged in horizontal strata so that the potential temperature increases upward. The difference between the total energy in the original and in the final stage represents the available energy.

The assumption on which this method rests, namely, that the individual strata rearrange themselves without mixing, is not acceptable in the study of local convection. *Rossby*[2]) has shown that small scale convective currents result in fairly thorough mixing of the atmosphere, with the result that a constant potential temperature along the vertical is established. In accounting for mixing and other factors, *Rossby* was able to develop the theory of convection further. He deduced a number of theorems which have proved to be of great value in the forecasting of convective phenomena, particularly when the observations are plotted on his diagram, and when equivalent potential temperature is used consistently.

The conditions of static stability of the atmosphere and the convective weather phenomena have been studied from various points of view by several other authors. We mention here only the following: *Calwagen*[1]), *Normand*[2]) *Shaw*[3]) *Stüve*[4]), *Refsdal*[5]), and *Robitsch*[6]). Through these and other investigations, a classification of the various criteria of stability and instability has resulted, and the problem of the releasable energy has been attacked from various points of view. The results thus obtained are applied with ease in the daily forecasting practice when the data derived from aerological ascents are plotted on a thermodynamical diagram (i. e. any equal-area transformation of

[1]) *Margules*, M., Über die Energie der Stürme, Jahrbuch d. k. k. Zentralanst. f. Met. und Geodyn., Wien 1903.

[2]) *Rossby*, C.-G., Thermodynamics Applied to Air Mass Analysis, Massachusetts Institute of Technology Meteorological Papers, Vol. I, No. 3. Cambridge, Mass. 1932.

[1]) *Calwagen*, E. G., Zur Diagnose und Prognose lokaler Sommerschauer, Geof. Publ. Vol. III, No. 10.

[2]) *Normand*, C. W. B., On Instability from Water-vapour, Quart. Journ. of Roy. Meteor Soc. Vol. 64, 1938.
*Normand*, C. W. B., Energy releasable in the Atmosphere. Quart. Journ. of Roy. Meteor. Soc. Vol. 64, 1938.

[3]) *Shaw*, N., Manual of Meteorology, Vol. III, Cambridge 1930.

[5]) *Stüve*, G., Potentielle und Pseudopotentielle Temperatur, Beitr. z. Physik der freien Atmosphäre, Bd. XIII, Heft 3. 1927.

[5]) *Refsdal*, A., Der feuchtlabile Niederschlag, Geof. Publ. Vol V, No. 12.
*Refsdal*, A., Zur Thermodynamik der Atmosphäre, Geof. Publ. Vol. IX, No. 13.
*Refsdal*, A., Aerologische Diagrampapiere, Geof. Publ. Vol. XI, No. 13.

[6]) *Robitsch*, M., Die Verwertung der durch aerologische Versuche gewonnenen Feuchtigkeitsdaten zur Diagnose der jeweiligien atmosphärischen Zustände, Die Arbeiten des Preussischen Aeron. Obs. bei Lindenberg, Bd. XVI, 1930.

the Clapeyron diagram). The stability (or instability) conditions of the atmosphere are then found by letting a small parcel of air move adiabatically upwards or downwards, and considering its density (or temperature) relative to its environment. Similarly, the amount of available energy is found by measuring the area on the diagram which is enclosed by the ascent curve (the environment curve) and the curve which the parcel in question would follow on the diagram during its adiabatic, or pseudo-adiabatic, movement.

This method will be referred to later as the *parcel method*. When using this method, it is assumed that a parcel of air can ascend or descend without causing any motion in the environment. This, again, implies that the density of the environment of the moving parcel does not change. The shortcomings of this assumption and the parcel method will be discussed later. It is, however, appropriate to remark here that from the circumstance that the parcel method is inaccurate, it does not follow that it is not a valuable means for predicting convective phenomena.

The problem of the stability of the air has een attacked recently from a different point of view by *Bjerknes*[1]) who, in a special case, has discussed the influences of the changes in the environment on the stability and the amount of available energy.

This promising method of approach will be applied in this paper to all significant classes of stratifications and conditions. For reasons which will become evident later, this method will be referred to as the *slice method*. It will be shown later that the slice method renders stability criteria which in most cases are identical with those deduced by means of the parcel method. In one important case, however, the two methods give different criteria. Furthermore, it will be shown that the slice method gives more accurate values of the amount of energy which may be released from the stratification than does the parcel method. Though the slice method is theoretically more accurate than the parcel method, it will be found that the latter is more directly applicable to aerological observations than is the former.

**2. Stability and Instability.** In the same manner as stability and instability are defined for rigid systems, we may say that the air is in a stable state of equilibrium if a parcel of air, which is moved a small distance upwards or downwards and then left to itself, has a tendency to return to its original level. If such a parcel has a tendency to move further away from its original level, the air is in an unstable state of equilibrium. If the parcel considered can be at rest at any level, the air is in a neutral or indifferent state of equilibrium. Since any minute disturbance will upset the unstable systems and bring them to a stable state, it follows that unstable conditions cannot persist in the atmosphere for any appreciable interval of time. The transition from unstable to stable states of equilibrium involves a reduction of the potential energy; and all systems left to themselves will try to avoid instability and obtain a minimum of potential energy.

Thus, while it is perfectly easy to render an adequate definition of stability and instability, it is difficult to determine from actual observations whether the stratification is stable or not. This difficulty is due to several circumstances. (a) As the air is a compressible and continuous medium, the displacement of a parcel will cause compensation currents in the environment, and both the displaced parcel and its environment will change their density. As the acceleration which acts on the displaced parcel depends on the difference between its density and that of the surrounding air, it follows that this difference in density depends not only on the initial distribution of mass, but also on the kinematics of the motion. (b) When the displacement results in condensation of aqueous vapour, the latent heat of vaporization is liberated, and this causes additional changes in the density distribution. Thus, for example, when saturated air is displaced upwards in a non-saturated environment, the saturated air ascends moist-adiabatically while the environment descends dry-adiabatically. Depending on the distribution of vertical velocity, the rate of heating of the environment may then become smaller or greater than the rate of cooling of the ascending air. (c) When the degree of stability is slight, it may happen that the stratification is stable relative to small perturbations, while it is unstable relative to disturbances which are sufficiently large.

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[1]) *Bjerknes*, J., Saturated-adiabatic ascent of air through dry-adiabatically descending environment, Quart. Journ. Roy. Met. Soc. Vol. 64, 1938.

From the above it follows that the problem of determining the stability conditions of the air is not only a statical problem, but also a dynamical one. In order to determine the stability conditions of the air, it is necessary to consider the following factors:

(1) The initial distribution of mass,

(2) The nature of the perturbation, and

(3) The changes in density which result from the motion.

Having regard to the more complete discussion of the stability criteria which will be given in para. 4, it is of interest to remark that the above definition of stability and instability may be expressed as follows: If the air is at rest and an impulse is applied, the stratification is stable when the air becomes decelerated in the direction of the impulse; and it is unstable when the air becomes accelerated in the direction of the impulse.

The state of equilibrium is characterized by isobaric surfaces coinciding with the isosteric surfaces (barotropic conditions). When this condition is not fulfilled, the field is baroclinic, and accelerations develop which tend to establish barotropy. From this it follows that, when the equilibrium state is disturbed, positive solenoids must develop when there is instability; and negative solenoids must develop when stability is present.

When the parcel method is applied, it is assumed that no motion, and therefore no change in density, occurs in the environment of the displaced parcel. The stability criteria thus deduced, depend only on the initial distribution of mass along the vertical.

It is of interest to note that any displaced parcel is supposed to assume the pressure of its environment; the difference in density between the displaced parcel and its environment depends then only on the distribution of temperature and humidity.

**3. The Parcel Method.** We introduce the following symbols:

$T$ = tempereture,
$\theta$ = potential temperature,
$T_w$ = wet-bulb temperature,
$\theta_w$ = wet-bulb potential temperature,
$T_e$ = equivalent temperature,
$\theta_e$ = equivalent potential temperature, as defined by Rossby (loc. cit. p. 5).
$z$ = distance along the vertical,
$\gamma = -\frac{\partial T}{\partial z}$ = the actual lapse-rate of temperature,
$\gamma_d$ = the dry-adiabatic lapse-rate,
$\gamma_m$ = the moist-adiabatic lapse-rate.

(*A*) *Dry Air.* We consider a parcel of air $P$ (Fig. 1) and three hypothetical distributions of

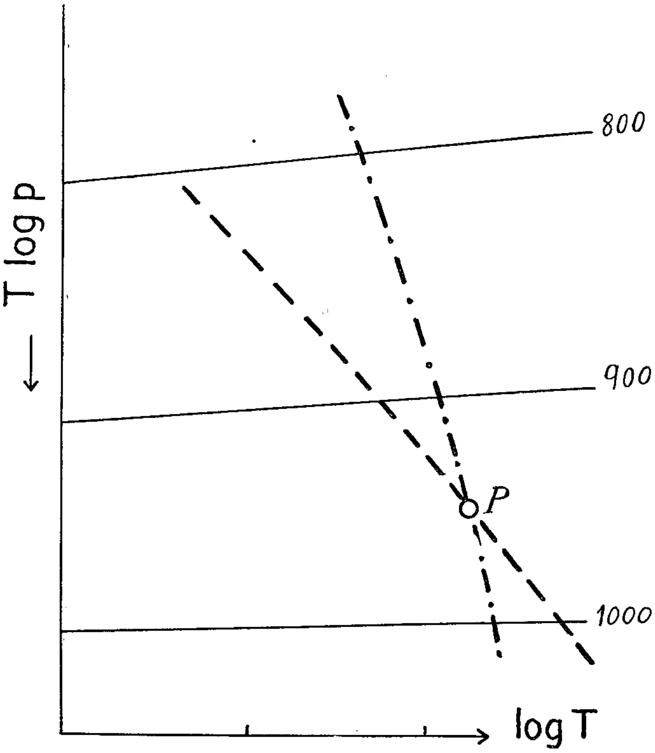


Fig. 1. Broken line indicates dry-adiabat; broken dotted line indicates moist-adiabat.

temperature along the vertical characterized by (1) $\gamma < \gamma_d$, (2) $\gamma = \gamma_d$, and (3) $\gamma > \gamma_d$. If the parcel $P$ were displaced a small distance upwards or downwards, it would change its temperature dry-adiabatically, and if no changes occurred in its environment, it is readily seen that the following conditions hold:

$$\gamma < \gamma_d \text{ or } \frac{\partial \theta}{\partial z} > 0 \text{ stable equilibrium,}$$

$$\gamma = \gamma_d \text{ or } \frac{\partial \theta}{\partial z} = 0 \text{ indifferent equilibrium,}$$

$$\gamma > \gamma_d \text{ or } \frac{\partial \theta}{\partial z} < 0 \text{ unstable equilibrium.}$$

(*B*) *Saturated Air.* In the discussion of saturated air one may refer to the pseudo-adiabatic or the moist-adiabatic lapse-rate, the former referring to the conditions when the condensed water is immediately precipitated from the air, and the latter referring to the conditions when the condensed water remains in the air. For all practical purposes it suffices to consider the two lapse-rates as identical. By displacing a saturated parcel ($P$) of air *upwards*, it is readily seen from Fig. 1 that saturated air is stable, indifferent, or unstable according to whether the lapse-rate is less than, equal to, or greater than the moist-adiabatic rate. We may then write:

$$\gamma < \gamma_m, \text{ or } \frac{\partial \theta_w}{\partial z} > 0 \text{ stable equilibrium,}$$

$$\gamma = \gamma_m, \text{ or } \frac{\partial \theta_w}{\partial z} = 0 \text{ indifferent equilibrium,}$$

$$\gamma > \gamma_m, \text{ or } \frac{\partial \theta_w}{\partial z} < 0 \text{ unstable equilibrium.}$$

It should be borne in mind that these criteria are not as general as are the corresponding ones for dry air. If $\gamma_d > \gamma > \gamma_m$, and if the air is saturated but not filled with water droplets, the above conditions hold only for upward displacements, because a parcel of air that is displaced downwards will be heated dry-adiabatically, and it would then be accelerated back to its original level. However, when the air contains a sufficient amount of liquid water which evaporates so quickly that the air is kept in a saturated state while it is displaced downwards, the above conditions hold both for upward and downward impulses.

($C$) *Non-saturated Moist Air.* Owing to the influence of the moisture content on the specific heat of air, the adiabat of moist non-saturated air differs slightly from that of perfectly dry air; but, for all practical purposes, the two may be regarded as identical. It follows then that the criteria deduced for perfectly dry air are valid also for non-saturated moist air as long as the motions set up by the perturbations do not surpass the limit where condensation occurs within the displaced air. Thus, while the criteria for perfectly dry air are applicable also to moist unsaturated air when the perturbations are small, they fail when the movements surpass the condensation limit. As the movements occurring within the troposphere often exceed this limit, it will be necessary to deduce criteria which hold also for large displacements in moist air.

Let $ACED$ in Fig. 2 represent the ascent curve for non-saturated air, and let the parcel of air of unit mass at $A$ be displaced upwards; it cools dry-adiabatically to $B$ where it becomes saturated. From the level $B$ it cools moist-adiabatically as shown by the curve $BCD$. The area represented by $ABC$ measures the amount of energy which must be supplied to the unit mass in order to raise it from the level $A$ to the level $C$. Below the level $C$, the lifted parcel of air is colder, and therefore also denser, than the surrounding air; and, if it

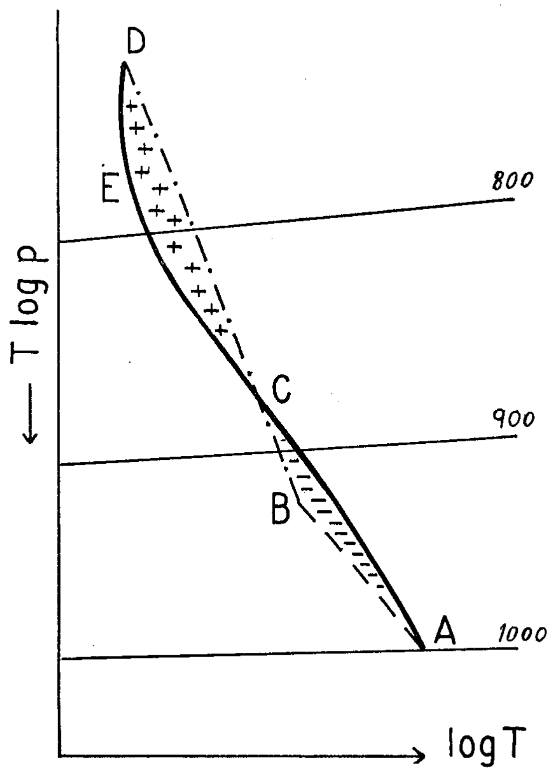


Fig. 2. Illustrating conditional instability. The diagram used here is an «aerogram», which is an equal-area transformation of the Clapeyron's diagram.

were left to itself, it would be accelerated back towards its original level (stability). However, if it were lifted further, beyond the level $C$, its representative point in the diagram would follow the line $CD$. Above the level $C$, therefore, the parcel would be warmer and less dense than its environment; it would be accelerated upwards until it reaches the level $D$. The area $CDE$ expresses the amount of energy which is gained by a parcel of air of unit mass while it moves from the level $C$ to the level $D$. This energy is used by the parcel to increase its kinetic energy and to overcome friction.

From the above it follows that the parcel $A$ is stable relative to small perturbations (which do not bring it beyond the level $C$), but is unstable with respect to perturbations which are large enough to bring it above the level $C$ (the level of free convection). The net gain of energy of the parcel of unit mass at $A$, when brought beyond the level $C$, would then be area $CED$ minus the area $ABC$.

The above example shows that instability may be released by *upward* impulses in moist non-saturated air when the impulse is strong enough to bring the parcel of air beyond a certain level. It is easily seen, however, that with the vertical distribution of temperature indicated in Fig. 2, only upward displacements could release instability.

Owing to the difference in slope between the dry and the moist adiabatic lines (see Fig. 1), a great variety of conditions will be observed. *Normand*[1]) and *Rossby*[1]), distinguish between three principal cases, viz., absolute stability, absolute instability, and conditional instability.

(1) *Absolute stability* is characterized by a lapse-rate of temperature which is less than the moist-adiabatic rate, or $\gamma < \gamma_m$. It is readily seen (e. g. from Fig. 1) that a parcel of air is then in a stable state of equilibrium with respect to upward or downward impulses, however large the displacements may be.

(2) *Absolute instability* is characterized by a lapse-rate of temperature which is greater than the dry-adiabatic rate, or $\gamma > \gamma_d$. The equilibrium is then unstable with respect to all upward or downward impulses, however small the displacements may be.

(3) *Conditional instability* is characterized by:

$$\gamma_d > \gamma > \gamma_m.$$

In this case the equilibrium is stable relative to all downward impulses. As regards upward impulses, it depends on the distribution of humidity along the vertical whether instability will be released or not.

(a) *The stable type.* Consider the ascent curve $ABC$ (Fig. 3) and the curve $A'B'C'$ which represents the corresponding distribution of the wet-bulb temperature. Let the parcel of air at $A$ ascend dry-adiabatically to its lifting condensation level $a$, whence it ascends moist-adiabatically along $aa'$. The curve $A'aa'$ represents the moist-adiabat corresponding to $A$, or the wet-bulb potential temperature line of the parcel $A$. Consider next a parcel of air at $B$ whose wet-bulb temperature is $B'$; its wet-bulb potential temperature line is then the moist-adiabat $B'bb'$. None of the wet-bulb potential temperature lines intersect the ascent curve. It will then be seen that throughout the lifting, any parcel of air belonging to the ascent curve will be colder than the environment. By definition, then, the equilibrium is a stable one.

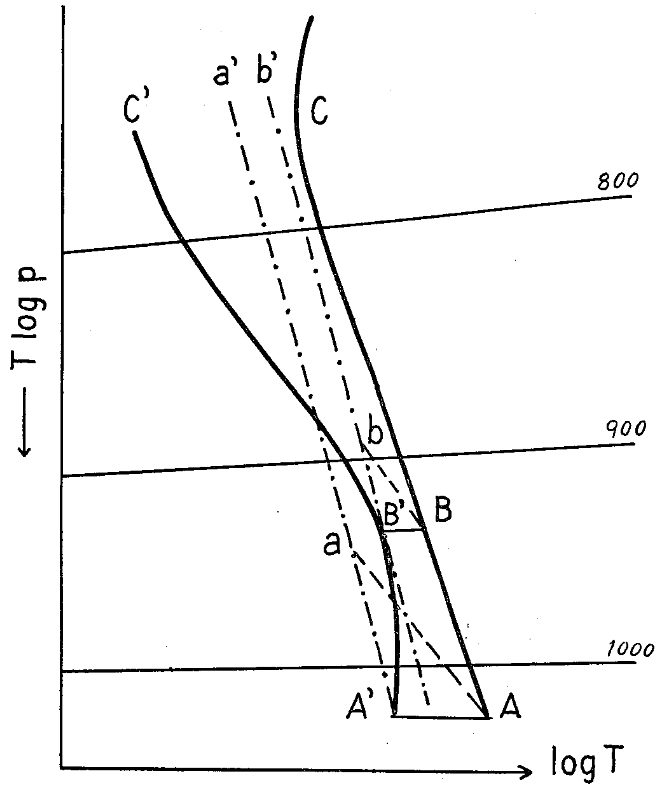


Fig. 3. Illustrated conditional instability of stable type.

(b) *Latent instability.* Consider next the ascent curve $ABCDE$ (Fig. 4), and the corresponding wet-bulb temperature curve $A'C'D'$. Suppose the parcel at $A$ is moved upwards; it will follow the dry-adiabatic $Aa$ to its lifting condensation level $a$, whence it will move along the moist-adiabatic running through the point $A'$. Below $B$ the parcel is denser than the environment, and it is stable with respect to small impulses; but as it passes the level $B$, it will be warmer and less dense than the surrounding air, and hence be accelerated upwards till it arrives at $E$. Therefore, by lifting the parcel $A$ from the level $A$ to the level $B$ or further, instability, latent in the stratification, will be released. The difference betwen the areas $BCDE$

[1]) loc. cit. p. 5.

and $AaB$ represents the amount of available energy pertaining to a unit mass at $A$.

*Normand* distinguishes between two cases of latent instability, viz.,

(i) The area $BCDE$ is larger than the area $AaB$ (see Fig. 4). In this case, more energy is released by the displacement than is used to overcome the resistance against the initial displacement. The process thus leads to a net gain of energy which may be used to overcome frictional forces and to create kinetic energy. This case is called *real latent instability.*

(ii) The area $BCDE$ is smaller than the area $AaB$. The net gain of energy is negative. This case is called *pseudo-latent instability* (see Fig. 4).

To the above classification it should be remarked that the overweight of positive area relative to the negative area is not always significant. The upper positive area only yields energy after the negative area has been overcome. Therefore, the net amount of available energy is important only when the negative area is so small that the perturbations are able to lift the air above the level where free convection commences.

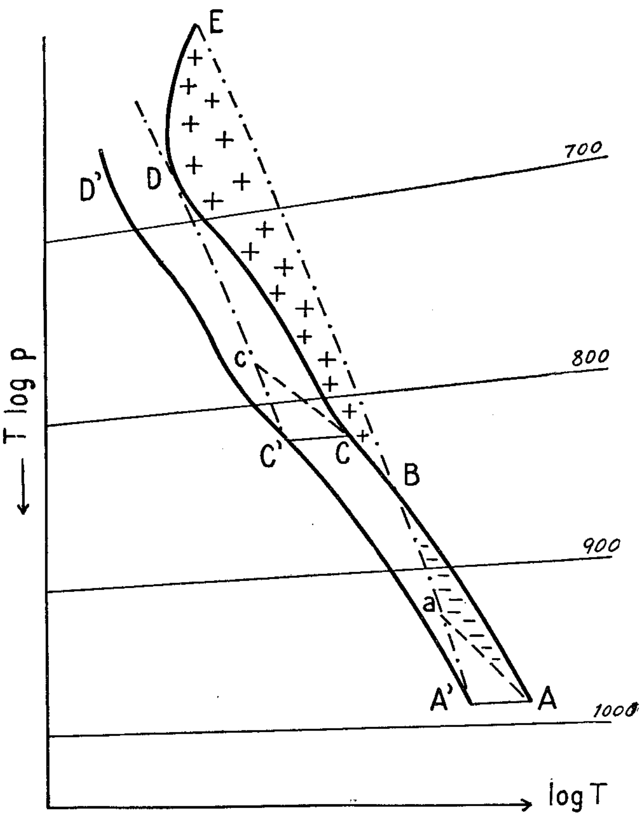


Fig. 4. Illustrating conditional instability of the *real latent type*. If the area $BCDE$ were less than the area $AaB$, the type would be *pseudo-latent.*

For later reference the various criteria deduced by means of the parcel method are summarized in Table 1.

**4. The Slice Method.** The criteria of stability and instability summarized in the previous paragraph were deduced on the assumption that an isolated parcel of air could ascend or descend and change its density adiabatically without causing any change in the density of its environment. However, the space occupied by an ascending parcel of air must immediately be filled by air from the environment; and the space which an ascending parcel of air is going to occupy, must be evacuated. An ascending current must therefore have its counterpart in a descending current, and vice versa. But as the descending air also changes its density, it follows that the assumption of an undisturbed environment does not hold. It remains therefore to discuss how the co-existence of adjacent upward and downward currents modify the criteria of stability and instability.

We consider a portion of the atmosphere which is large enough to contain several upward and downward currents, and make the following assumptions:

(a) The horizontal motion does not at any level maintain any net inflow to, or outflow from, the region under consideration.

(b) The conditions are barotropic at the initial moment.

(c) The temperature changes are adiabatic.

The assumption (a) involves equal transports of air upward and downward through any arbitrary isobaric surface, or through any isobaric slice of air of unit thickness, the slice remaining stationary while the air passes through it. We may then write:

$$(1) \qquad M'w' = -Mw,$$

where $M'$ denotes the mass of ascending air, and $M$ the mass of descending air contained within the slice, $w'$ and $w$ denote the upward and downward velocities respectively.

The assumption (b) involves coincidence of isobaric surfaces with the isosteric surfaces at the initial moment; and assumption (c) means that the

Table 1.

*Criteria of Stability and Instability Deduced by Aid of the Parcel Method.*

| | *Non-Saturated Air:* Displaced parcel not becoming saturated. | *Saturated Air:* Displaced parcel remaining saturated. |
|---|---|---|
| Absolute Stability | $\gamma < \gamma_d$ or $\frac{\partial \theta}{\partial z} > 0$ | $\gamma < \gamma_m$ or $\frac{\partial \theta_e}{\partial z} > 0$ or $-\frac{\partial T_w}{\partial z} < \gamma_m$ or $\frac{\partial \theta_w}{\partial z} > 0$ |
| Indifferent State | $\gamma = \gamma_d$ or $\frac{\partial \theta}{\partial z} = 0$ | $\gamma = \gamma_m$ or $\frac{\partial \theta_e}{\partial z} = 0$ or $-\frac{\partial T_w}{\partial z} = \gamma_m$ or $\frac{\partial \theta_w}{\partial z} = 0$ |
| Absolute Instability | $\gamma > \gamma_d$ or $\frac{\partial \theta}{\partial z} < 0$ | $\gamma > \gamma_m$ or $\frac{\partial \theta_e}{\partial z} < 0$ or $-\frac{\partial T_w}{\partial z} > \gamma_m$ or $\frac{\partial \theta_w}{\partial z} < 0$ |

| | | *Non-saturated Air:* Displaced parcel becoming saturated. | |
|---|---|---|---|
| Absolute Stability | | $\gamma < \gamma_m$ | |
| Conditional Instability | Stable Type | $\gamma_m < \gamma < \gamma_d$ | and also no wet-bulb potential temperature line intersecting the ascent curve (Fig. 3). |
| Conditional Instability | Pseudo-latent | $\gamma_m < \gamma < \gamma_d$ | and also some of the wet-bulb potential temperature lines intersecting the ascent curve, and the positive area smaller than the negative area (Fig. 4). |
| Conditional Instability | Real-latent | $\gamma_m < \gamma < \gamma_d$ | and also some of the wet-bulb potential temperature lines intersecting the ascent curve, and the positive area larger than the negative area (Fig. 4). |
| Absolute Instability | | $\gamma > \gamma_d$ or $\frac{\partial \theta}{\partial z} < 0$ | |

temperature changes per unit time within the slice may be expressed by the following equations:

$$\frac{\partial T'}{\partial t} = w' (\gamma - \gamma_d) \tag{2}$$

in ascending non-saturated air,

$$\frac{\partial T'}{\partial t} = w' (\gamma - \gamma_m) \tag{3}$$

in ascending saturated air,

$$\frac{\partial T}{\partial t} = w (\gamma - \gamma_d) \tag{4}$$

in descending non-saturated air,

$$\frac{\partial T}{\partial t} = w (\gamma - \gamma_m) \tag{5}$$

in descending saturated air.

In a later paragraph the influence of the non-fulfilment of these assumptions will be discussed.

If the air were originally at rest, and an impulse affecting the mass $M'$ is applied, the isotherms, which originally were horizontal[1]), will not remain so, and solenoids will be created. The heat gained per second within the slice is expressed by:

[1]) Strictly speaking, parallel to isobars.

(6) $$\frac{\partial Q}{\partial t} = c_p \left( M \frac{\partial T}{\partial t} + M' \frac{\partial T'}{\partial t} \right)$$
$$= c_p (M + M') \frac{\partial T}{\partial t} + c_p M' \left( \frac{\partial T'}{\partial t} - \frac{\partial T}{\partial t} \right)$$

The first term on the right hand side represents a uniform heating (or cooling) of the entire slice, at the rate $\frac{\partial T}{\partial t}$, equal to the heating (or cooling) of the descending mass; while the last term represents the excess heating (or cooling) of the ascending air, at the rate $\left(\frac{\partial T'}{\partial t} - \frac{\partial T}{\partial t}\right)$, relative to the environment. Only the last term, which may be called the *solenoid producing* term, contributes to the production or consumption of kinetic energy.

(A) *Non-saturated air.* In this case above equations (1), (2), and (4) apply. It will be seen that when $\gamma > \gamma_d$, an ascending current causes a local increase in temperature, and a downward current causes a local decrease in temperature. When $\gamma < \gamma_d$, the reverse is true. Substituting from eqs. (1), (2), and (4) into (6), we obtain:

(7) $$\frac{\partial Q}{\partial t} = - c_p \left(1 + \frac{M'}{M}\right) M'w' (\gamma - \gamma_d)$$
$$+ c_p \left(1 + \frac{M'}{M}\right) M'w' (\gamma - \gamma_d) = 0$$

Thus, the vertical velocities do not cause any net gain or loss of heat within the slice, but the isotherms will be perturbed, and the solenoid producing term is expressed by:

(8) $$c_p \left(1 + \frac{M'}{M}\right) M'w' (\gamma - \gamma_d)$$

If the air were initially at rest ($w' = 0$), and a perturbation affecting the mass $M'$ applied, it follows that a parcel of air would be accelerated in the directon of the impulse when $\gamma > \gamma_d$; it would be retarded in the direction of the impulse when $\gamma < \gamma_d$, and no accelerations would occur when $\gamma = \gamma_d$. These conditions are independent of the magnitude of the impulse and the mass affected. Therefore, the stability criteria derived on the assumption of an undisturbed environment (the parcel method) hold without restriction when the air remains non-saturated. However, as will be shown later, the two methods give different values for the amount of available energy.

The fact that the sign of the solenoid producing term is independent of $M'$ and $M$ shows that non-saturated air reacts in the same way whether the horizontal dimensions of the perturbations are large or small: We may say that *there is no selection of impulses.*

(B) *Saturated air.* We shall next consider the case when both the ascending and the descending air are saturated. This occurs when the air is filled with droplets which evaporate so quickly that the descending air remains saturated. In this case both the ascending and the descending masses follow the moist-adiabatic, and there is the same symmetry in the process as in the case described above. We may therefore substitute $\gamma_m$ for $\gamma_d$ into eq. (7). This gives:

(9) $$\frac{\partial Q}{\partial t} = - c_p \left(1 + \frac{M'}{M}\right) M'w' (\gamma - \gamma_m)$$
$$+ c_p \left(1 + \frac{M'}{M}\right) M'w' (\gamma - \gamma_m) = 0$$

As in the previous case there is no net gain of heat within the entire slice; but the isothermal surfaces, which were originally horizontal, will not remain so. As above, the term

(10) $$c_p \left(1 + \frac{M'}{M}\right) M'w' (\gamma - \gamma_m)$$

is the solenoid producing term. If the air were originally at rest ($w' = 0$) and a perturbation affecting the mass $M'$ is applied, it follows that the parcel of air would be accelerated in the direction of the impulse when $\gamma > \gamma_m$, and it would be retarded in the direction of the impulse when $\gamma < \gamma_m$. Thus, the criteria deduced on the assumption of an undisturbed environment hold without restriction when both the ascending and the descending currents are saturated. As in the previous case, the two methods give different values for the amount of available energy.

The fact that the sign of the solenoid producing term is independent of $M'$ and $M$ shows that saturated air reacts in the same way whether the horizontal dimensions of the perturbations are large or small: We may say that *there is no selection of impulses.*

(C) *Saturated ascent through a dry-adiabatically descending environment.* In this case, which has been discussed by *J. Bjerknes* (loc. cit. p. 6), the eqs. (1), (3), and (5) apply. Substituting from them into eq. (6), we obtain:

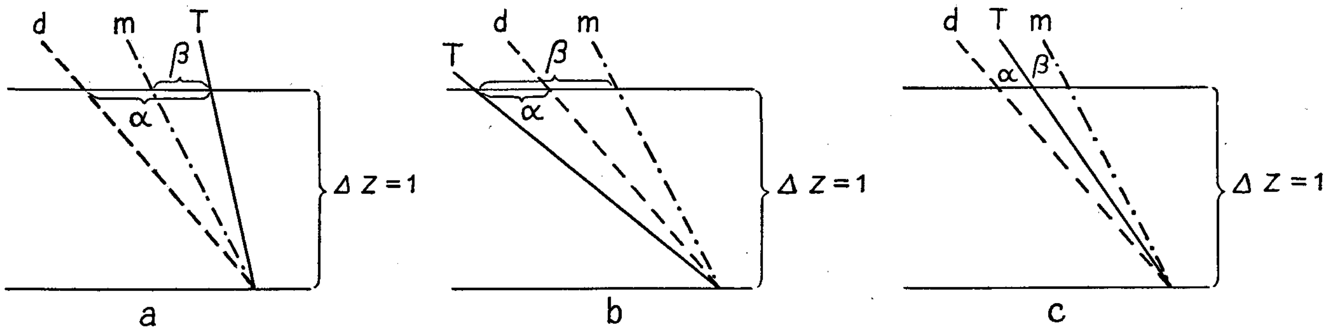


Fig. 5. Showing the definition of $\alpha$ and $\beta$. $d$ = dry-adiabat; $m$ = moist-adiabat; $T$ = temperature curve.

$$(11) \qquad \frac{\partial Q}{\partial t} = c_p w' M' (\gamma_d - \gamma_m)$$

$$= c_p w' M' \left(1 + \frac{M'}{M}\right)(\gamma_d - \gamma) + c_p w' M' \left(\gamma - \gamma_m - (\gamma_d - \gamma)\frac{M'}{M}\right)$$

Putting $\gamma_d - \gamma = \alpha$, and $\gamma - \gamma_m = \beta$, as shown in Fig. 5, we obtain:

$$(12) \qquad \frac{\partial Q}{\partial t} = c_p w' M' (\alpha + \beta)$$

$$= c_p w' M' \left(1 + \frac{M'}{M}\right)\alpha + c_p w' M' \left(\beta - \alpha\frac{M'}{M}\right)$$

It is important to note that at any level, or rather at any point on a thermodynamical diagram,

$$(13) \qquad \alpha + \beta = \delta = \text{constant} > 0,$$

so that $\alpha$ can only increase as much as $\beta$ decreases, and vice versa. Similarly, since $M' + M$ represents the mass contained in an isobaric unit slice over the area considered, it follows that

$$(14) \qquad M' + M = M_0 = \text{constant}$$

since the considered area is constant.

The term $c_p w' M' (\alpha + \beta)$ expresses the heat set free per second through condensation of aqueous vapour in the mass $M'$ when its upward velocity is $w'$. It is directly proportional to the upward velocity and the mass affected. It is also proportional to $(\alpha + \beta)$, or the angle between the moist-adiabatic and the dry-adiabatic. It is seen from any thermodynamical diagram that it decreases with decreasing temperature because the two adiabatics approach one another as the temperature decreases.

The term $c_p w' M' \left(1 + \frac{M'}{M}\right)\alpha$, which represents the uniform heating (or cooling) of the entire slice, is directly proportional to $w'M'$, and it increases with $\alpha$, or the angle between the ascent curve and the dry-adiabatic. When $\alpha > 0$ (see Fig. 5), the uniform heating term is positive and when $\alpha < 0$, it is negative. This term, which under normal conditions is positive, consumes a considerable part of the liberated heat.

The term

$$(15) \qquad c_p w' M' \left(\beta - \alpha\frac{M'}{M}\right)$$

which is the solenoid producing term, represents *the portion of the liberated heat which is available for production or consumption of kinetic energy.* From the definition of stability and instability it follows then that the equilibrium is stable when

$$(15\text{ a}) \qquad \beta < \alpha\frac{M'}{M};$$

it is indifferent when

$$(15\text{ b}) \qquad \beta = \alpha\frac{M'}{M};$$

and it is unstable when

$$(15\text{ c}) \qquad \beta > \alpha\frac{M'}{M}.$$

As the lapse-rate of temperature is always important, we may distinguish between four cases (refer to Fig. 5 and eq. (13)), viz.,

(i) $\beta < 0$ and $\alpha > 0$. In this case $\gamma < \gamma_m$. From eq. (15) it follows that kinetic energy is consumed. If the air is at rest, and then given an impulse, it would be retarded in the direction of the impulse: the stratification is by definition a stable one. This applies irrespectively of $M'$ and $M$, which shows that the air behaves in the same manner regardless of the horizontal extent of the impulse.

(ii) $\alpha < 0$ and $\beta > 0$. In this case $\gamma > \gamma_d$. From eq. (15) it follows that the sign of the solenoid producing term is positive, and the air would be accelerated in the direction of the impulse: the stratification is unstable. This applies irrespectively of $M'$ and $M$.

In the cases discussed above there is no selection of impulses, and the criteria of stability and instability conform with the ones deduced on the assumption of an undisturbed environment, which, therefore, hold without restriction. As before, the amount of available energy determined by means of the parcel method differs from that found by aid of the slice method.

(iii) $\beta = 0$ and $\alpha > 0$. In this case $\gamma = \gamma_m$. From eq. (15) it follows that kinetic energy is consumed. The equilibrium is indifferent only when eq. (15 b) holds; which implies that $\gamma > \gamma_m$.

In this case, then, the criterion of indifferent equilibrium does not conform to the one deduced on the assumption of an undisturbed environment (i. e. by the parcel method).

(iv) $\beta > 0$ and $\alpha > 0$. In this case $\gamma_d > \gamma > \gamma_m$. It is seen from eq. (15) that the sign of the solenoid producing term is no longer uniquely determined by $\beta$ and $\alpha$ (or $\gamma$), but it depends also on the ratio $\frac{M'}{M}$. If the air were initially a rest, and an impulse is applied in an upward direction, it follows from eq. (15) that the sign of the solenoid producing term would be positive only when the horizontal dimensions of the mass affected are such that

$$\frac{M'}{M} < \frac{\beta}{\alpha} \tag{16}$$

If this limit is exceeded, the mass $M'$ would be retarded in the direction of the impulse. The meaning of the above condition is perhaps best illustrated by the following example: Suppose that the slice of air considered is subject to a series of perturbations of variable horizontal extents. It would then react as an unstable medium relative to such perturbations as satisfy the inequality (16), whereas it reacts as a stable medium relative to such perturbations as do not agree with that condition. In this way the atmosphere may «pick and choose» amongst the numerous perturbations which it receives from below. The air is then *selectively unstable.* As this case ($\alpha > 0$ and $\beta > 0$, i. e. $\gamma_d > \gamma > \gamma_m$) occurs frequently in nature, it will be of interest to study this selection of perturbations in greater detail.

**5. The Available Energy.** As in the last paragraph we consider saturated air ascending through a dry-adiabatically descending environment. The solenoid producing term is then the last term in eq. (12). Substituting from eqs. (13) and (14) into the solenoid producing term, we obtain:

$$c_p\, w' M' \left(\beta - \alpha \frac{M'}{M}\right) = c_p\, M_0\, \delta\, w'\, S\,, \tag{17}$$

where

$$S = \frac{\frac{M'}{M}}{1 + \frac{M'}{M}} \cdot \frac{\frac{\beta}{\alpha} - \frac{M'}{M}}{1 + \frac{\beta}{\alpha}}. \tag{18}$$

As the solenoid producing term is directly proportional to $w'$ (which by definition is positive), and as $c_p$, $M_0$ and $\delta$ in any given case may be regarded as constants, it suffices to discuss the non-dimensional quantity $S$, which, multiplied by $w'$, expresses the solenoid producing term in relative units.

To facilitate the discussion we suppose that the air initially is at rest, and that the portion $M'$ of the slice is given an upward velocity $w' = 1$. Fig. 6 then shows the distribution of $S$ as a function of the ratio $\frac{M'}{M}$ for various values of $\frac{\beta}{\alpha}$. The diagram may be devided for convenience into three main portions, viz.,

(a) *The area of absolute stability* represented by the lower hatched portion of the diagram. This area is characterized by $\beta < 0$ and $\alpha > 0$, or $\gamma < \gamma_m$. The sign of the solenoid producing term is negative independent of $M'$ and $M$. This means that the kinetic energy of all impulses is consumed.

(b) *The area of absolute instability* represented by hatching in the top part of the diagram. This area is characterized by $\alpha < 0$ and $\beta > 0$, or $\gamma > \gamma_d$. When $\gamma$ approaches $\gamma_d$, $\frac{\beta}{\alpha}$ approaches $-\infty$. As the lapse-rate normally does not appreciably exceed $\gamma_d$, only the portion above and close to the line indicated by $\pm\infty$ is of actual interest. The sign of the solenoid term is positive independent of $M'$ and $M$, which means that kinetic energy is produced by all impulses.

(c) *The area of selective instability* represented by the unhatched portion of the diagram. This area is characterized by $\alpha > 0$ and $\beta > 0$, or

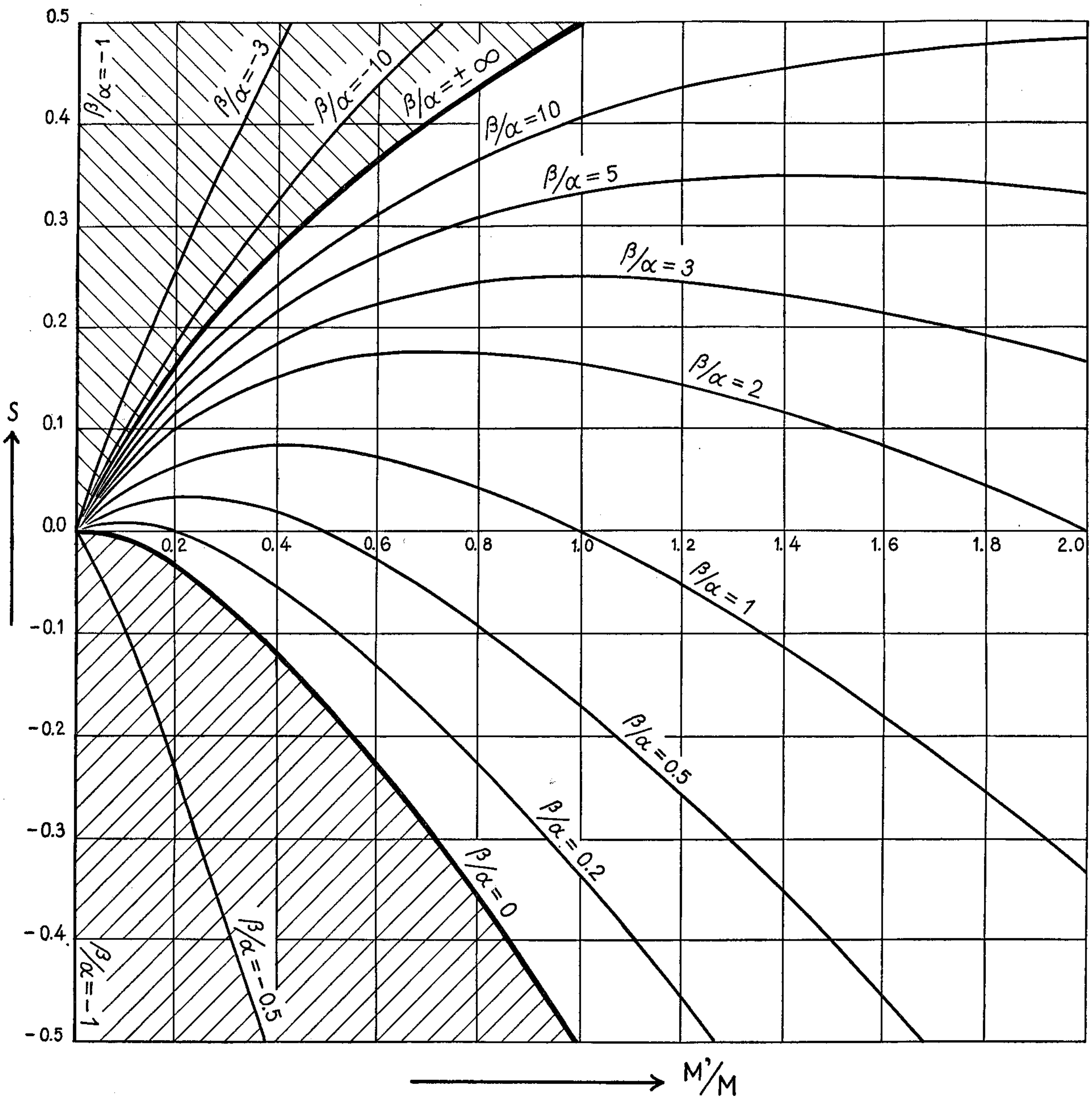


Fig. 6. Showing the distribution of $S$ as a function of $\frac{M'}{M}$ for various values of $\frac{\beta}{\alpha}$. $S$ multiplied by $w'$ is the variable factor in the solenoid producing term.

$\gamma_d > \gamma > \gamma_m$ (comp. Fig. 5). In this case, the sign of the solenoid producing term is not uniquely determined by the lapse-rate $\left(\text{or } \frac{\beta}{\alpha}\right)$ but it depends largely on the horizontal dimensions of the perturbations $\left(\text{or } \frac{M'}{M}\right)$. When $\frac{\beta}{\alpha}$ is small (i. e. $\gamma$ is close to $\gamma_m$) only such perturbations which cause $\frac{M'}{M}$ to be exceedingly small can produce kinetic energy, and the amount of energy to be gained is then small. The air is then «highly particular» about its selection of energy producing impulses. On the other hand, when $\frac{\beta}{\alpha}$ is large (i. e. $\gamma$ is close to $\gamma_d$), the air reacts in an unstable manner relative to a great variety of impulses. The ratio $\frac{M'}{M}$ may vary within wide limits, and the amount of energy to be gained is then large. This, interpreted, means that energy producing convective clouds can only occupy a small portion of the sky when $\gamma$ is close to $\gamma_m$, and the spaces between individual clouds must be large.

On the other hand, when $\gamma$ is close to $\gamma_d$, the self-supporting individual clouds may occupy a considerable portion of the sky; large and small clouds may co-exist, and the distances between the individual clouds may vary within wide limits.

The above deductions agree with observations: When the lapse-rate above the condensation level is close to $\gamma_m$, the self-supporting convective clouds occupy only a small portion of the sky, and they are fairly uniformly spaced. However, when $\gamma$ approaches $\gamma_d$, large and small clouds co-exist; the sky is more chaotic; the clouds may fill a considerable portion of the sky, and wind squalls and strong or violent gusts occur frequently.

**6. Selection of Perturbations.** As in the last paragraphs we consider the conditions in an isobaric unit slice when saturated air is given an upward impulse so that it is made to ascend in a dry-adiabatically descending environment. The conditions for production of kinetic energy is expressed by (15 c), viz.,

(19) $$\beta > \alpha \frac{M'}{M}$$

It has already been shown that all impulses are suppressed when $\beta > 0$, and that all impulses create kinetic energy when $\alpha < 0$. It remains to discuss in greater detail the frequently occuring case when $p > 0$ and $\alpha > 0$, or when $\gamma_d > \gamma > \gamma_m$.

Let us first remark that on account of the roughness of the ground, the unequal heating of the underlying surface, and other factors, the lower atmosphere is continuously subjected to a multitude of impulses of varying intensities and varying horizontal dimensions. But when the air is selectively unstable, it will react in an unstable manner only for such impulses as satisfy the above inequality. Moreover, it follows from eq. (17) that the solenoid producing term is directly proportional to the upward velocity $w'$ of the impulse. Out of the multitude of the varying impulses, the air will «pick and choose» according to the intensity and the horizontal spacing of the impulses.

(a) *Selection by intensity.* From what has been said above it follows that the stronger the impulse the greater are the upward acceleration in the ascending mass ($M'$) and the downward acceleration in the descending mass ($M$). If two (or more) upwards impulses co-exist near one another, the stronger one will have a tendency to suppress the weaker one, because the latter will be counteracted by the downward acceleration created by the former. Thus, at the initial moment, there is a tendency for the stronger impulse to start convection and to suppress neighbouring weaker impulses: What happens at the initial moment is controlled by the law of the survival of the fittest.

(b) *Selection by horizontal extent.* Other conditions being equal, the stratification is more unstable relative to such impulses which cause $\frac{M'}{M}$ to be small than relative to such impulses which cause $\frac{M'}{M}$ to be large. That this is so is readily shown in the following manner: Let $w'_1 M'_1$ represent one impulse, and let $w'_2 M'_2$ represent the other impulse; and suppose that

$$w'_1 M'_1 = w'_2 M'_2.$$

The ratio ($r$) between the solenoid producing terms is then

$$r = \frac{c_p w'_1 M'_1 \left(\beta - \alpha \, {M'_1}/{M_1}\right)}{c_p w'_2 M'_2 \left(\beta - \alpha \, {M'_2}/{M_2}\right)} = \frac{\beta - \alpha \, {M'_1}/{M_1}}{\beta - \alpha \, {M'_2}/{M_2}}$$

Hence, $r > 1$ when $\frac{M'_1}{M_1} < \frac{M'_2}{M_2}$. Thus, there is a tendency for the impulse which cause the ratio $\frac{M'}{M}$ to be a minimum to gain over other neighbouring impulses[1]). It should be borne in mind that this condition depends only on the ratio $\frac{M'}{M}$, and not on the absolute magnitudes of $M'$ and $M$. It is therefore not permissible to conclude that there is a preference for narrow cloud towers to exist.

(c) *Preference of existing circulations.* It is reasonable to assume that the cross-section distribution of the vertical velocity resulting from an impulse is in principle as shown in Fig. 7. Let us

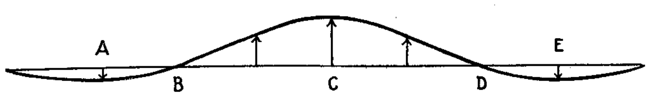


Fig. 7. Diagrammatical distribution of vertical velocities in and around a cylindrical ascending current which satisfy eq. (1).

suppose that a second upward impulse is applied somewhere between $B$ and $D$. This impulse would

[1]) A similar result has been obtained by J. *Bjerknes* (loc. cit. p. 6).

add to the solenoids already produced; the upward velocity would be increased accordingly, and so would the descending velocity in the surrounding air.

Let us next suppose that the second impulse is applied somewhere near $A$ or $E$. Here the impulse would be counteracted by the already existing downward velocities, and the impulse would be suppressed if it were not sufficiently intense to overcompensate the field of solenoids already created. Thus, an already existing upward energy producing circulation has a tendency to suppress neighbouring impulses, and to feed on the impulses which occur within its own domain.

(d) *Horizontal growth of energy producing circulations.* Let us now suppose that a second impulse is applied at $B$ or at $D$. Here the vertical velocities are slight or even nil. Additional impulses will then create kinetic energy, and the field of solenoids will grow and spread laterally. Thus, the circulation initially established by the strongest impulse has a tendency to grow laterally, and to make its suppressing influence on more distant impulses felt over a wider region.

In this way, large cumulus clouds are built up of a great number of impulses the horizontal extent of each of which may be quite small compared with the horizontal dimensions of the entire cloud. That this is so is corroborated by observations of the growth and the structure of cumulus clouds. It would therefore be erroneous to assume that a cumulus cloud results from one single impulse, or from one uniform ascending current.

(e) *Limit of horizontal growth when* $\frac{\beta}{\alpha} < 1$. To go a step further, we assume that at the initial moment there are several impulses over a wide area which start energy producing circulations: None of these, then, clash with the inequality (19) above. On account of the great multitude of impulses which the air receives from below, the existing circulations have a tendency to increase horizontally, and, if this continues, the state may be reached when the ratio $\frac{M'}{M}$ approaches the ratio $\frac{\beta}{\alpha}$. As the ratio $\frac{M'}{M}$ thus increases from a value which is small compared with $\frac{\beta}{\alpha}$ to the final value equal to $\frac{\beta}{\alpha}$, it is seen from Fig. 6 that it must pass the value at which there is a maximum of production of kinetic energy, and it must gradually approach the state when no more solenoids are available.

The above is not exact inasmuch as the convective currents have a tendency to change the lapse-rate, so that the limit set for the final value of $\frac{M'}{M}$ is determined not by the initial value of $\frac{\beta}{\alpha}$, but by the value that gradually develops. Moreover, as will be shown later, not all the available energy is used to create kinetic energy. It is important to bear in mind that when $\frac{\beta}{\alpha} < 1$, the ratio $\frac{M'}{M}$ must, in virtue of the inequality (19), remain less than unity. This involves that the descending currents must be weaker than the ascending currents, as indicated by eq. (1).

(f) *Limit of horizontal growth when* $\frac{\beta}{\alpha} > 1$. As above we assume that widely scattered energy producing circulations start and grow horizontally. As $\frac{\beta}{\alpha} > 1$, the *stratification* would allow the ratio $\frac{M'}{M}$ to increase and approach the ratio $\frac{\beta}{\alpha}$. But it is also necessary to consider the continuity of the motion. As $\frac{M'}{M}$ approaches 1, the descending currents must be as strong as the ascending ones. When this state is reached, it is obvious that only excessive impulses which affect the mass $M$ could overcompensate the descending velocity; and, in such cases, the circulation would be reversed so as to cause the already ascending air to descend. It follows, therefore, that the horizontal growth of energy producing upward circulations will not surpass the limit set by $M' = M$, i. e. when the upward currents occupy half of the total area. This implies that *the energy producing circulations must have an asymmetrical distribution of vertical velocity.*

It is of interest to note that the ratio $\frac{M'}{M}$ is not expressive of the cloud cover as observed from the earth's surface. Thus, if the clouds were cylindrical and 1000 metres high with their bases about 1000 metres above the observer, a cloud cover of about 7/10 to 9/10 would be noted when $M' = M$.

(g) *Lower limit of* $\frac{M'}{M}$. In discussions on convective clouds it has been stated that the most favourable conditions exist when $\frac{M'}{M}$ approaches zero; because the ascending motion would then be spread over a wide area, and the environment of the ascending currents would then remain sensibly unchanged. However, it is hardly reasonable to assume that such conditions can exist in the atmosphere. If an upward impulse affects the mass $M'$, there seems no reason to justify the assumption that the descending compensation current would affect an area which is infinitely larger than that of the ascending mass. What regulates the lower limit of the ratio $\frac{M'}{M}$ we do not know; it is plausible that the stratification is a determining factor. Thus, while it is certain that the distribution of vertical velocity in energy producing circulations must be asymmetrical (and therefore $M' < M$), it is plausible that there must also exist a lower limit of the ratio $\frac{M'}{M}$. This question can only be decided by further research. However, if $\frac{M'}{M}$ could approach zero, it would imply that energy producing circulations of saturated air could be started as soon as $\gamma$ slightly exceeds $\gamma_m$. This follows from the inequality (19) above. Experience gained from the analysis of ascent curves seem to show that energy producing clouds do not start unless $\gamma$ is noticeably greater than $\gamma_m$. The exact limit is not known; and, with due allowance for this, we may say that the lower limit for $\frac{M'}{M}$ is about at 0.1 or 0.2. The upper limit is set by $\frac{\beta}{\alpha}$ when $\frac{\beta}{\alpha} < 1$; and by $\frac{M'}{M} = 1$ when $\frac{\beta}{\alpha} > 1$. Thus, when the air is selectively unstable, all possible developments of energy producing clouds should occur in the unhatched portion of Fig. 6 between the lines indicated by $\frac{M'}{M} = 0.1$ and $\frac{M'}{M} = 1$. The rest of the unhatched area is only of theoretical interest.

**7. The Parcel Method and the Slice Method.** The outstanding difference between the slice method and the parcel method is that the former takes into account the changes which occur in the environment of the ascending air, while these changes are disregarded in the latter. Evidently, the slice method is more exact than the parcel method. However, in practice the parcel method has certain advantages, and a comparison between the two methods is therefore given below.

(1) *Non-saturated air.* According to eq. (6) the slice method gives the following solenoid producing term

$$c_p M' \left(\frac{\partial T'}{\partial t} - \frac{\partial T}{\partial t}\right).$$

If the environment remained unchanged (i. e. by the parcel method), $\frac{\partial T}{\partial t} = 0$, and the solenoid producing term would be:

$$c_p M' \frac{\partial T'}{\partial t}$$

From the eqs. (2) and (4) it follows that the parcel method would give a solenoid producing term which is numerically too small both when $\gamma > \gamma_d$ (absolute instability) and when $\gamma < \gamma_d$ (absolute stability). The parcel method, therefore, underestimates both the stability and the instability of unsaturated air. From both methods it follows that the air is stable, indifferent, or unstable according to whether $\gamma \lesseqgtr \gamma_d$.

(2) *Saturated air.* In the same manner as above it follows from the deductions in para. 4 that the solenoid producing term, when determined on the assumption of an undisturbed environment, is numerically too small both when $\gamma > \gamma_m$ (absolute instability) and when $\gamma < \gamma_m$ (absolute stability). This applies when the descending air remains saturated. From both methods it follows that the air is stable, indifferent, or unstable according to whether $\gamma \lesseqgtr \gamma_m$.

(3) *Saturated ascent through a dry-adiabatically descending environment.* From eqs. (3), (4), and (6) in para. 4 it follows that the following is true of the solenoid producing term as deduced on the assumption of an undisturbed environment:

(a) When $\gamma > \gamma_d$, the solenoid producing term comes out with too small a positive value.

(b) When $\gamma_d > \gamma > \gamma_m$, the solenoid producing term comes out with too large a positive value.

(c) When $\gamma < \gamma_m$, the solenoid producing term comes out with too small a negative value.

From both methods it follows that the air is unstable when $\gamma > \gamma_d$, and stable when $\gamma < \gamma_m$. When $\gamma_d > \gamma > \gamma_m$, the stability criteria of the slice method differ from those of the parcel method as has been shown in para. 4.

From the above and from para. 3 it follows that when the air is conditionally unstable (i. e. when $\gamma_d > \gamma > \gamma_m$) the parcel method under-estimates the resistance against lifting, and it over-estimates the available energy (see Fig. 2). On the other hand, it under-estimates the available energy when the air is absolutely unstable.

It is important to note that the error due to the assumption of an undisturbed environment vanishes when $\gamma$ approaches $\gamma_d$; this follows from eqs. (4) and (6) in para. 4. Similarly, from eqs. (1), (4), and (6) in para. 4 it follows that this error decreases when the ratio $\frac{M'}{M}$ decreases, and it becomes insignificant when the cloud towers are very narrow relative to the width of the descending currents.

**8. Discussion of the Assumptions.** The deductions in para. 4—6 were based on the three assumptions quoted at the beginning of para. 4. As none are strictly fulfilled in the atmosphere, it is of interest to investigate how the non-fulfilment of these assumptions affect the results deduced above.

We consider first assumption (a). If there is convergence towards the area under consideration, the descending velocity would be less than it would be if there were no convergence. Conversely, if there is divergence from the area under consideration, the descending velocities would be increased relatively to the ascending ones. It follows then from eqs. (4), (5), and (6) in para. 4 that horizontal convergence would increase the solenoid producing term in all cases when the solenoid producing term is positive, and diminish the solenoid producing term when this term is negative. The reverse is true for horizontal divergence. Horizontal convergence near the earth's surface is an important factor in releasing and maintaining convection. It has been shown by *Rossby* (loc. cit. p. 5) that when the air is convectively unstable (i. e. when the wet-bulb potential temperature or the equivalent potential temperature decreases along the vertical), the ascending motion which results from horizontal convergence will cause the stratification to become unstable when the air becomes saturated.

As regards assumption (b) it suffices to remark that if the conditions are not strictly barotropic at the initial moment, the initial solenoids would add to those created by the vertical displacements.

If the assumption (c) is not fulfilled, it is necessary to consider whether the non-adiabatic changes act in such a way as to increase, or decrease, the instability forces.

**9. Convective Currents.** Let us first consider convection in a layer between two rigid boundaries. The isothermal surfaces are horizontal initially, and the lower portion of the layer is unstable while the upper portion is stably stratified (see Fig. 8).

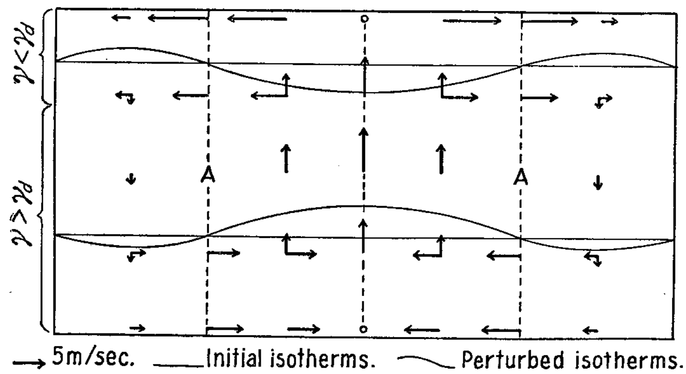


Fig. 8. Diagrammatical distribution of vertical and horizontal velocity in a cylindrical convective current when $M'/M = 1/8$.

If an impulse is applied in an upward direction, the air in the unstable layer would be accelerated in the direction of the impulse, and the air in the stable layer would be retarded in the direction of the impulse. The isotherms resulting from the vertical motion set up by the impulse are shown in Fig. 8. The vertical motions must vanish at the boundaries and attain a maximum somewhere between them. Kinetic energy is produced in the unstable layer and consumed in the stable layer. As continuity must be established, there must be horizontal convergence in the lower portion of each ascending current, and horizontal divergence in the upper portion of each ascending current. In the descending currents the reverse is true. Fig. 8 shows the horizontal velocities, computed from the equation of continuity, which result when the vertical velocities are as indicated in the diagram.

We consider next the circulation round the point $A$ in Fig. 8. The velocity of revolution round the point $A$ varies with the distance from this point, and, as a result, the layers cannot arrange themselves in such a way that the potentially coldest air comes to the bottom of the layer, and the potentially warmer air to the top of the layer. It is readily seen that the currents established will maintain solenoids until the various layers are thoroughly mixed. The final result of convection will, therefore, be an adiabatic lapse-rate throughout the layer.

*Rossby*[1]) has shown that the above holds also when there is no upper rigid boundary to the convective layer. Part of the energy which is released in the unstable layer is then used to establish an adiabatic lapse-rate in the stably stratified air above.

In Fig. 8 there is no general horizontal velocity superimposed on the convective currents. Fig. 9

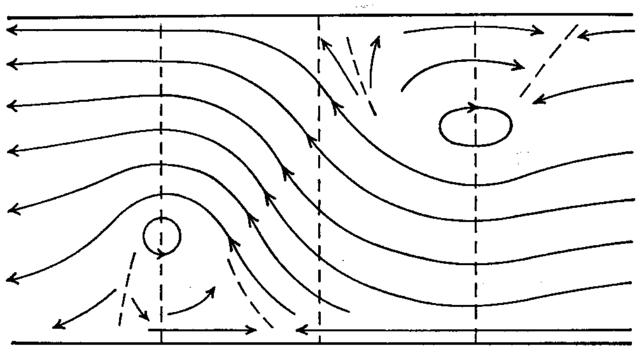

Fig. 9. Stream lines when a convective current as shown in Fig. 8 is superimposed on a slight translatory movement. The eddies in the stream lines vanish when the translatory movement increases.

shows the resulting stream lines when a slight translatory velocity (4 m/sec.) is superimposed on the currents shown in Fig. 8. It should be noted that the superimposition of a translatory movement changes nothing but the stream line picture. The wind shift at the ground, which in Fig. 8 is found in the centre of the ascending current, is displaced in a forward direction when the convective system is superimposed on a translatory motion. It should be noted that, when the translatory movement is slight, the convective cloud may be stationary while the air blows through it.

What has been said above of the convective currents should only be regarded as vague suggestions. Further observations as well as dynamical and kinematical investigations will be needed to render an adequate picture of these complex phenomena.

It will be seen from eq. (7) that the solenoid producing term in non-saturated air is directly proportional to $(\gamma - \gamma_d)$. As non-saturated air is unstable relative to all impulses when $\gamma > \gamma_d$, and as perturbations are always present in abundant amounts, it follows that convection is released as soon as $\gamma$ exceeds $\gamma_d$. As convection and vertical mixing tend to make $\gamma$ approach $\gamma_d$, it follows that $\gamma$ will not appreciably exceed $\gamma_d$. The solenoid producing term will therefore remain small as long as condensation does not occur. The «dry convection», therefore, does not normally produce much kinetic energy. Only when the air is heated very rapidly by the underlying surface will strong wind gusts result (e. g. desert wind squalls).

When condensation occurs in the ascending current, $\gamma$ may be considerably larger than $\gamma_m$. It follows then from eq. (15) that the solenoid producing term may be quite large. Strong wind gusts, therefore, often accompany convective clouds.

Referring again to Fig. 8, it will be seen that if condensation occurs in the ascending current, the cloud will have a tendency to spread out in the top part of the ascending current. Here, then, the ratio $\frac{M'}{M}$ will increase; and if the stratification is selectively unstable $\left(\text{i. e. } \beta > \alpha \frac{M'}{M}\right)$, the growth of the ratio $\frac{M'}{M}$ will tend to create stability. The cloud will flatten out; it ceases to grow upwards, and its upper portion will consume kinetic energy. *The divergent outflow from the top part of the ascending current is a stabilizing factor of great importance.*

**10. Convective Clouds.** All clouds of the cumulus family form as a result of instability, either in the cloud layer or in the layer under the

[1]) loc. cit. p. 5.

cloud. The various forms which these clouds may take depend on the degree of instability, the depth of the unstable layer, and the distance from the condensation level to the top of the convective layer. In addition, one must take into account the processes which contribute to the maintenance and frustration of instability, namely: vertical movement of the air mass as a whole, advection of air aloft, mixing, radiation, and exchange of heat and moisture between the air and the underlying surface. The practical aspects of the problem of forecasting convective phenomena will be discussed elsewhere[1]). Here, we shall discuss the types of convective clouds[2]) on the basis of the stability and instability conditions which actually exist.

(a) *Cumulus humilis.* These clouds form when the convective condensation level is close to the upper limit of the layer affected by the convective currents. They are sometimes entirely embedded in a layer of air which is absolutely stable (i. e. $\gamma < \gamma_m$). In such cases the clouds form in the upper, energy consuming part of the ascending currents. The cloud air is then colder than the environment, and the cloud feeds on the energy producing dry convection below the condensation level.

At other times, the cumulus humilis form in selectively unstable air (i. e. $\gamma_d > \gamma > \gamma_m$) just under a layer of absolute stability. The cloud is then embedded in the upper part of the ascending current where the cloud, on account of the horizontal divergence, has grown laterally to such an extent that it has become stable $\left(\text{i. e. } \beta < \alpha \frac{M'}{M}\right)$. In this case, too, the cloud air is colder than the environment, and it consumes kinetic energy. Cumulus humilis that form in the upper portion of a selectively unstable layer will usually grow upwards until the dome reaches into a completely stable layer ($\gamma < \gamma_m$). These clouds, therefore, develop when the condensation level is slightly above an unstable layer and, at the same time, not far from the layer where $\gamma = \gamma_m$. Fig. 10 a shows the typical features of a cumulus humilis or fair weather cumulus. It has a «dead» appearance and no active towers or protuberances. Fig. 11 shows the vertical distribution of temperature which is typical of clouds of this type. Related to the cumulus humilis are the cumulo-stratus and the cumulus undulatus both of which usually form under an inversion as a result of convection in the air below.

(b) *Cumulus congestus.* These clouds form in deep layers of air which are either absolutely unstable or else selectively unstable. The condensation level may be below or above the limit between absolute instability and selective instability. If the

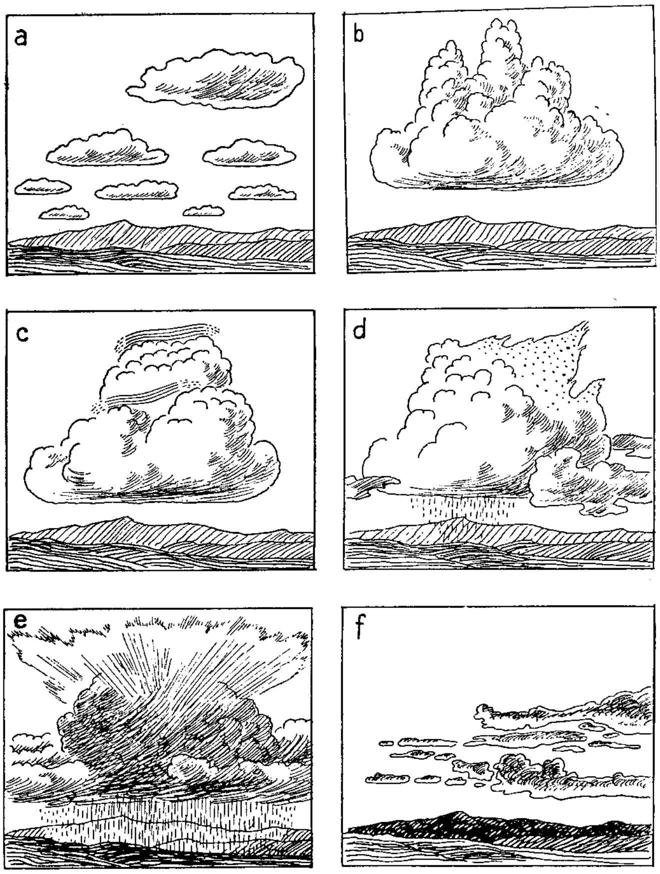


Fig. 10. Types of convective clouds.

condensation level is in absolutely unstable air, the rising masses will reach saturation while $\gamma$ is close to $\gamma_d$. It follows then from Fig. 6 that great amounts of kinetic energy will be released: towers and protuberances grow upwards rapidly, and the cloud grows also horizontally. As the towers grow upwards into the air that is selectively unstable, only those perturbations which cause $\frac{M'}{M}$ to be less than $\frac{\beta}{\alpha}$ will produce kinetic energy. Other pertur-

[1]) *Petterssen*, S., Weather Analysis and Forecasting (not yet published).

[2]) See International Atlas of Clouds and States of Sky, Paris 1930.

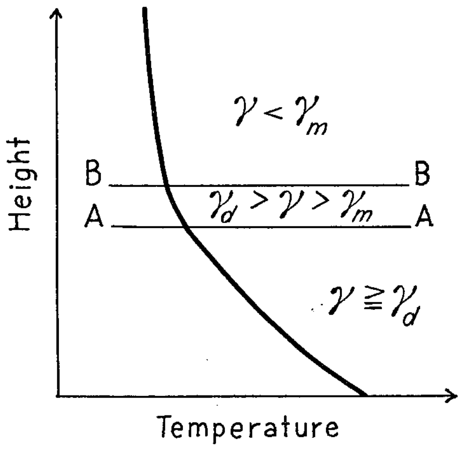


Fig. 11. Vertical distribution of temperature typical of cumulus humilis. Condensation level close *BB*.

bations are suppressed. If $\gamma$ decreases upwards, the towers must become narrower relative to the descending currents around them. Eventually, the towers may reach up to heights where there is either absolute stability ($\gamma < \gamma_m$), or where the divergent outflow from the upper portion of the ascending current is sufficiently intense to make the cloud spread out horizontally, whereby the ratio $\frac{M'}{M}$ becomes too large for further production of kinetic energy.

The typical features of a developing cumulus congestus are shown in Fig. 10 b, and the temperature distribution typical of such development is shown in Fig. 12. It will be seen that the size of the convective elements in the cloud decreases upwards, and the top of the cloud has a typical cauliflower structure, showing that the selective instability decreases with elevation.

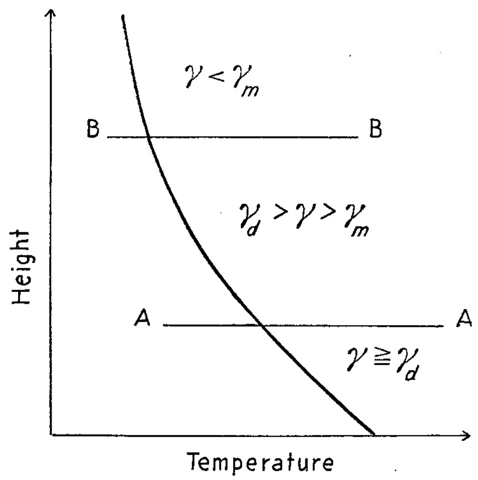


Fig. 12. Vertical distribution of temperature typical of cumulus condestus. Condensation level close to *AA*.

The condensation level may sometimes be situated slightly above the bottom layer which is absolutely unstable. In such cases, the ascending currents from the earth's surface will have to ascend through a layer which consumes part of the kinetic energy. The convention is then, as a whole, not so intense as in the case discussed above.

It is sometimes observed that the towers of cumulus congestus break through minor inversions in the upper atmosphere. In such cases a veil (pileus) forms around or above the towers (see Fig. 10 c). This condition is due so the circumstance that turbulence has transported so much moisture up to the air under the base of the inversion that, when the air is lifted by the ascending towers, it becomes saturated. The cloud is then called cumulus pileus. The veil (or the «scarf») usually consists of water droplets, but, if the temperature is well below freezing, it may consist of ice crystals.

As the cumulus congestus grows further, several developments are possible: (i) It grows up into layers where the wind increases rapidly with height. The towers are then blown asunder and dissolve by mixing into the air above. (ii) It grows into a stable layer and flattens out in its upper portion before it reaches temperatures which are sufficiently low for ice to form. In such cases the cloud usually does not produce precipitation[1]). (iii) It grows into layers where the top part of the cloud changes into ice or snow crystals. It has then developed into a cumulo-nimbus: a shower cloud, or a thunder cloud.

(c) *Cumulo-nimbus.* These clouds develop from cumulus congestus (or pileus). When the cloud loses its cauliflower structure, the towers and protuberances become less pronounced, the crevices gradually disappear, and the upper portion of the cloud changes gradually into a tangled web

[1]) According to *Bergeron* (On the Physics of Clouds. Memoire présenté a l'Association de Météorologie de l'U. G. G. X, I., Lisbon, 1933) the colloidal instability caused by the co-existence of ice and water in the cloud is the all-important factor in the release of precipitation from clouds. This theory is corroborated by numerous observations in high and middle latitudes. In low latitudes, showers are observed quite frequently without the cloud towers reaching above 0° C isothermal level.

with only slight traces of convective elements. During this process precipitation is usually released from the cloud. This type of cloud is called cumulo-nimbus calvus: Its principal features are shown in Fig. 10 d.

The calvus type usually develops into cumulo-nimbus capillatus or cumulo-nimbus incus whose principal features are shown in Fig. 10 e. The top part of a such cloud consits of ice crystals, and is called cirrus densus or cirrus nothus. It resembles a tangled web which surrounds the top of the water cloud. Such clouds give heavy precipitation in the form of showers or squalls, and frequently also thunderstorms. The

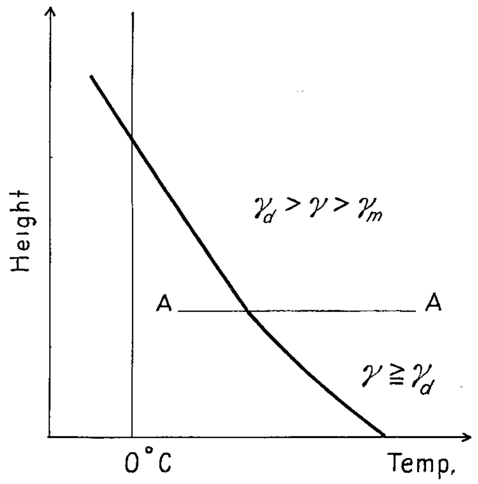


Fig. 13. Vertical distribution of temperature typical of cumulo-nimbus. Condensation level close to *AA*.

cumulo-nimbus clouds develop from cumulus congestus when the unstable layer is sufficiently high for the cloud to grow considerably above the zero isothermal level. The typical structure of the air is shown in Fig. 13.

It is possible with practice to forecast the weather for a few hours ahead by looking at the cumulus clouds and observing their development. It is then most important to observe the changes in the upper part of the clouds. If there are no towers (as in Fig. 10 a), there is no chance of precipitation. If there are towers (as in Fig. 10 b or c), it is possible that precipitation may develop. If some of the clouds show signs of presence of ice crystals, precipitation is sure to be released soon. Fig. 10 c often occurs as a transition from Fig. 10 b to Fig. 10 d, but Fig. 10 d usually develops directly from Fig. 10 b.

The cumulus clouds sometimes dissolve by general shrinking and disappear gradually. This usually occurs when a sheet of high clouds (cirro-stratus or alto-stratus) develops above them. In dissolving in this way they pass through the state of fair weather cumulus (Fig. 10 a). Most frequently the cumulus clouds flatten out into rolls or bulging layers resembling strato-cumulus: this development is shown in Fig. 10 f. This often occurs in the evening when the atmosphere is settling down after the diurnal heating (strato-cumulus vesperalis). In any case, the dissolution of cumulus clouds shows that the atmosphere is developing towards a stable stratification.

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