THE TEMPERATURE OF MAXIMUM DENSITY IN FRESH WATERS

AN ATTEMPT TO DETERMINE ITS LOWERING WITH INCREASED PRESSURE FROM OBSERVATIONS IN DEEP LAKES

BY *KAARE MÜNSTER STRØM*

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Introduction.

The lowering of the temperature of maximum density (t_{md}) in fresh waters with increasing pressure, apart from its purely physical aspect, presents problems of the greatest interest to limnologists.

We should expect physicists to be able to furnish us with undoubted values, experimentally determined, but in reality there is a great discrepancy between the values and even between the statements given, some of them being clearly impossible in view of the best determination of the pressure needed to lower the t_{md} to 0° C (600 kg/cm^2, PUSHIN & GREBENSHCHIKOV 1923).

There is, however, a possibility for an empirical determination of the t_{md}. If a holomictic lake (a lake with seasonal circulations extending to the bottom) is cooled sufficiently down before vernal full circulation, temperatures in the deep must immediately after the establishment of incipient summer stratification very nearly correspond to the t_{md} at the various depths.

From Norwegian lakes I have a large number of observations from the deep, which could be supplemented from abroad, especially with interesting series from the deepest lake of the world, Lake Bajkal.

When giving temperatures in the text, ° C is always omitted.

For their kindness in exchange of data and thoughts I am indebted to Dr. P. W. BRIDGMAN of Harvard, Dr. J. E. FJELDSTAD, Oslo, Dr. C. JUDAY, Madison, Wis., and the Academician G. J. VEREŠČAGIN, Leningrad.

Physical Attempts at Experimental or Deductive Determination of t_{md}.

t_{md} at atmospheric pressure is conveniently given as 4, but for a long time the majority of determinations have shown values between 3.9 and 4. In different physical tables 3.92 to 3.98 are usually found. See i. a. LANDOLT-BÖRNSTEIN (ROTH & SCHEEL 1923, p. 438).

At the occasion of fixing international prototypes, very exact determinations were made (BROCH 1881, p. 61). As a mean of these the value 3.92776 is given. I have essayed a graphical interpretation from the data by BROCH, and got the value 3.94625. A similar graphical determination of the Critical Tables data (WASHBURN 1928, p. 25) gave 3.93750. Until entirely new experiments have been made, the value may be fixed at *3.94*. It is not always clear whether data refer to 0 or 1 kg/cm^2 pressure. This would make a difference of about 0.01.

A possible experimental method for an examination of the lowering of t_{md} with increased pressure is that used by AMAGAT (1893, p. 950 f). At a number of temperatures (whole degrees from 0 to 10) he determined the variations in an initial pressure needed to keep constant volume. From these data t_{md} at the various initial pressures could be found through plotting the pressure differences (ordinates) against temperatures (abscissae), and noting the abscissa for the ordinate minimum (p. 950).

This is obviously no very exact method, as one has to determine differences in pressure down to 0.06 atm. for a temperature interval as large as 1. (AMAGAT's table p. 951). Nor does the evaluation of the results conduce much to exactness. As AMAGAT says (p. 952): "... il suffit d'une bien petite erreur pour déplacer d'une façon notable le point de contact des tangentes horizontales aux courbes". The results, even if now only of historical interest, should be quoted:

atm.	t_{md}	*atm.*	t_{md}
41.6	3.3	144.8	0.6
93.3	2.0	197	a little below 0

Further values are found in a large number of works cited by BRIDGMAN (1929, p. 322) and HALBFASS (1928, p. 351, 1932, p. 163). The values of ROEBUCK (1913) were used for reference by VEREŠČAGIN (1936) in his studies on Lake Bajkal. They are probably only an elaboration of those by AMAGAT. (ROEBUCK'S original paper was not accessible to me.) All these values must now be regarded as obsolete.

A great advance was made through the investigations of water under pressure by BRIDGMAN (1912 a, 1912 b, see also 1929), even if these were for purposes other than determining t_{md}, and mainly deal with pressures from 500 atm. and upwards. The very interesting result of t_{md} being 0 at about 500 kg/cm^2 (1912 a, p. 539, fig. 40, p. 544), or exactly 500 kg/cm^2 (1912 b, p. 388) was arrived at.

The final improvement in method was made by PUSHIN & GREBENSHCHIKOV (1923). Water at a temperature constant to the nearest 0.01 is made subject to series of different pressures. Change of temperature in adiabatic compression and expansion is noted. If p_{md} be the pressure required to lower t_{md} from the value 3.94 at atmospheric pressure to the constant temperature at which the experiment is conducted, there will be adiabatic cooling during compression, warming during expansion at pressures below p_{md}, while pressures above p_{md} give warming during compression, cooling during expansion. When there is no adiabatic change in temperature, we have the p_{md}-value at the constant temperature, or vice versa the t_{md} at the pressure determined.

The numerous determinations by PUSHIN & GREBENSHCHIKOV (p. 2720) both from compression and expansion experiments, with great certainty fixed p_{md} at 0 as 600 kg/cm^2, the difference against BRIDGMAN being easily explained by the refinement in determination introduced through the adiabatic warming-cooling method.

An ingenious attempt at deduction was made by SCHIØTZ (1887), t_{md} being derived as a function of compressibility between 4 and 0. The exposition is, however, so extremely brief, that it is impossible to judge of the fitness of the formula arrived at. The experimental data from which calculations were made, were the old by GRASSI from 1851. These were also used by VAN DER WAALS in 1877, but his results (ROTH & SCHEEL 1923, p. 1231) differ widely from those of SCHIØTZ.

This attempt at fixing t_{md}-values must also be regarded as of mainly historical interest. The values have been used by me in heat storage calculations (STRØM 1932 ff.), and are practically the same as those empirically arrived at in the present paper. The lowering of t_{md} is given as 0.1087 for each 10 atm. increase in pressure. The values are (pressures given as m water column):

m	t_{md}	*m*	t_{md}
0	3.93	300	3.61
100	3.82	400	3.51
200	3.72	500	3.40

Against all merely deductive attempts for a substance so abnormal and varying in physical properties as water, the cautionary words of BRIDGMAN (1929, p. 322) must be cited: "Insonderheit is es ausgeschlossen, eine einzige Flüssigkeit durch eine allgemeine Gleichung auch in ihren kleinen Besonderheiten zu beschreiben.«

Through physical determinations we thus at present have only two t_{md}-values fixed: *3.94 at 1 kg/cm², 0 at 600 kg/cm².*

Future experimental work should be done with the methods of PUSHIN & GREBENSHCHIKOV, and with constant temperatures at 0.1 intervals from 4 to 0. This is the only way to arrive at definite t_{md}-values. All other, either by deduction or by empirical results from lakes, as essayed in the present paper, can only be of a provisional nature.

Actual Temperature in Deep Norwegian Lakes.

The data are from Schiøtz 1887, Strøm 1932 a, 1932 b, 1933, 1934, 1937, 1938 a, 1938 b, 1944. Previously unpublished are the 1939 data from Mjøsa, Tyrifjord, Tinnsjø, and Norsjø. Observations in the two last-named lakes are by O. Hassel.

Lakes marked with an asterisk are influenced by glacial waters or cold waters from surrounding montain areas. A circlet at the date marks vernal full circulation.

In one lake of the Hadeland area very low temperatures were observed at moderate depths (Vassjøtjern 30 m: Aug. 8, 1940, 3.71; July 7, 1941, 3.75; Aug. 11, 1941, 3.83), but values for dissolved salts and oxygen (Strøm 1942, 1945) clearly show this lake to be meromictic (i. e. with seasonal full circulations not descending to the bottom), the waters of the deep being so much enriched in dissolved salts that density is not determined by temperature alone. The observations thus do not bear upon the present question concerning the maximum density under pressure.

The older data from Mjøsa and Lundevatn by Huitfeldt-Kaas (1905) are not determined with sufficient accuracy to be entered into the tables, as they are only given to the nearest 0.05 and from inner evidence the correction must be ± 0.1. The lowest temperatures during summer stagnation are as given: Mjøsa 415 m (bottom): Aug. 10, 1900, 3.60; Lundevatn 280 m (bottom): June 24, 1899, 3.80.

The Schiøtz (1887) data I consider sufficiently exact to be used, the Negretti-Zambra instruments of the period (one of them an extremely good instrument, says Schiøtz), in the hands of a physicist so well versed in precision methods, can be considered accurate wihin about ± 0.04, and with a greatest probability within ± 0.02.

All the other data published from Norwegian lakes are correct within ± 0.02, greatest probability within ± 0.01, except the 1936 series from Mjøsa which is taken with a reversing thermometer read to 0.001, and thus should be correct within ± 0.002. (Strøm 1939 b). The results of the ordinary reversing thermometer used within the same frame and read to 0.01 were identical below 5 degrees.

It is evident, that apart from very large and deep basins, where currents displacing the waters of the middle deeps upwards may cause a possible distribution of temperatures below t_{md} at observation depths (*post* p. 11) temperatures of a holomictic lake will always be above t_{md} during summer stagnation. If, in regarding fig. 1, we provisionally imagine a t_{md} curve by connecting the points of minimum temperature (all lakes and depths considered), we see that all other summer temperatures experienced in all the lakes are not on this curve, but more or less to the right.

The reasons for this are somewhat different. In the first place, the lake may not have cooled down to t_{md} during winter. Such insufficient cooling will as a rule show in a nearly isothermic curve from the point where t_{md} was reached, and downwards. With such a distribution of temperatures, denser water layers might seem to rest upon less dense. But either isothermic waters will circulate very slowly from the depth corresponding to their t_{md} down to the bottom (as probable in lake Bajkal, *post* p. 11) or conditions soon become stable through the biogenetic accumulation of salts in the deep. (Strøm 1933, p. 40). This kind of stabilisation would probably be established already during winter stagnation, and not be sufficiently disturbed by spring circulation. In Hornindalsvatn waters are more or less isothermic from below 250 (probably about 300) m to the bottom. (Observations 490, 500 m). The waters during winter have evidently only been cooled to about 3.60, which is above t_{md} at 500 m. (And corresponds to my value for 300 m.)

In 1883 this was also the case at 300 m in Mjøsa, as is shown by temperatures on March 11 (Schiøtz 1887, p. 64), when the lake should be coldest.

m		*March 11*	*June 23*
50		3.01	4.13
100		3.82	3.96
150		3.78	3.88
200		3.75	3.77
300		3.67	3.68
300	**Difference against July 9, 1936**	+ 0.06	+ 0.07

In the second place, a heating from the bottom through transmission of the inner heat of the earth and oxidation of the sediments is possible. The two factors have been calculated for the sea to be 55 and 74 cal/cm² respectively p. a. (Kalle 1943, p. 13 f.). At least in Hornindalsvatn their

Table 1.

m depth	t_{md} Strøm	Mjøsa			Tyrifjord (Holsfjord)						Eikeren				
		Jun. 23 1883	July 9 1936	Aug. 21 1939	○ May 25 1933	Jun. 3 1939	Jun. 26 1930	Aug. 27 1930	Sep. 19 1936	Sep. 26 1930	○ May 18 1935	○ May 24 1935	May 27 1935	Jun. 2 1935	Min t. during summer stagnation
0.1	3.94				4.34										
1	3.94				3.97						3.61	3.95	5.67		
2.5	3.94												4.93		
5	3.93				3 92						3.52	3.86	4.59		
10	3.93				3.78						3.47	3.79	4.17	5.22	
15	3.92				3.72						3.50	3.83	4.09	4.71	
20	3.92										3.47	3.71	4.08	4.66	
25	3.91				3.71										
30	3.90										3.44	3.72	4 07	4.27	
40	3.89														
50	3.88	4.13	4.40		3.73			4.69	4.72	5.12	3.46	3.72	4.02	4.04	
60	3.87														
75	3.85					4.02									
80	3 84														
100	3.82	3.96	4.00					4.07		4.19	3.48	3 75	3.82	3.84	
120	3.80														
125	3.80										3.58				
146	3.77										3.63				
147	3.77											3.70	3.79	3.79	
150	3.77	3.88			3.75		3.88	3.90		4.00					3.77
164	3.75														
165	3.75														
200	3.71	3.77	3.84												
250	3.66						3.71	3.72		3.79					
260	3 65					3 69									
270	3.64														
275	3.63									3.68					
280	3.62				3.58				3.70						
295	3.60		3.62												
300	3.60	3.68													
370	3.52														
410	3.48		3.51												
420	3.47			3.59											
490	3.40														
500	3.39														
Area of lake km		362.4			121.3						25.7				
Max. depth of lake m		449			295						154				

Table 1 (cont.)

m depth	Lilla Le	Tinnsjø*	Norsjø	Hornindalsvatn		Breimsvatn*	Strynsvatn*	Eikesdalsvatn*	Reinevetn	Studalsvatn	Solbjørnvatn	Tennesvatn
	Sep. 16 1931	Aug. 18 1930	Aug. 15 1939	Jun. 16 1931	Aug. 25 1931	Aug. 13 1931	Aug. 27 1931	July 17 1936	July 30 1935	Aug. 9 1935	Aug. 2 1935	Aug. 7 1935
0												
1												
2.5												
5												
10									5.82			5.48
15	5.57								4.92	5.34		
20	4.90										5.45	
25									4.12			4.56
30	4.52									4.26	4.34	
40	4.34											
50	4.24	5.31			4.78	4.91	4.92	5.55				4 19
60									3.92			
75												
80											3.92	
100		4.48			4.10	4.27	4 32	4.13				
120										3.89		
125												
146												
147												
150								4.02				3.85
164		4.09	3.96									
165											3.79	
200						4 01	4.15					
250					3.73							
260												
270						3.87						
275												
280												
395												
200												
370		3.69										
410												
420												
490					3.63							
500				3.62								
	0.8	54.1	59.7	50.8		23.0	22.3	23.2	0.2	0.4	5.1	0.9
	54	445	176	514		278	209	155	69	127	171	168

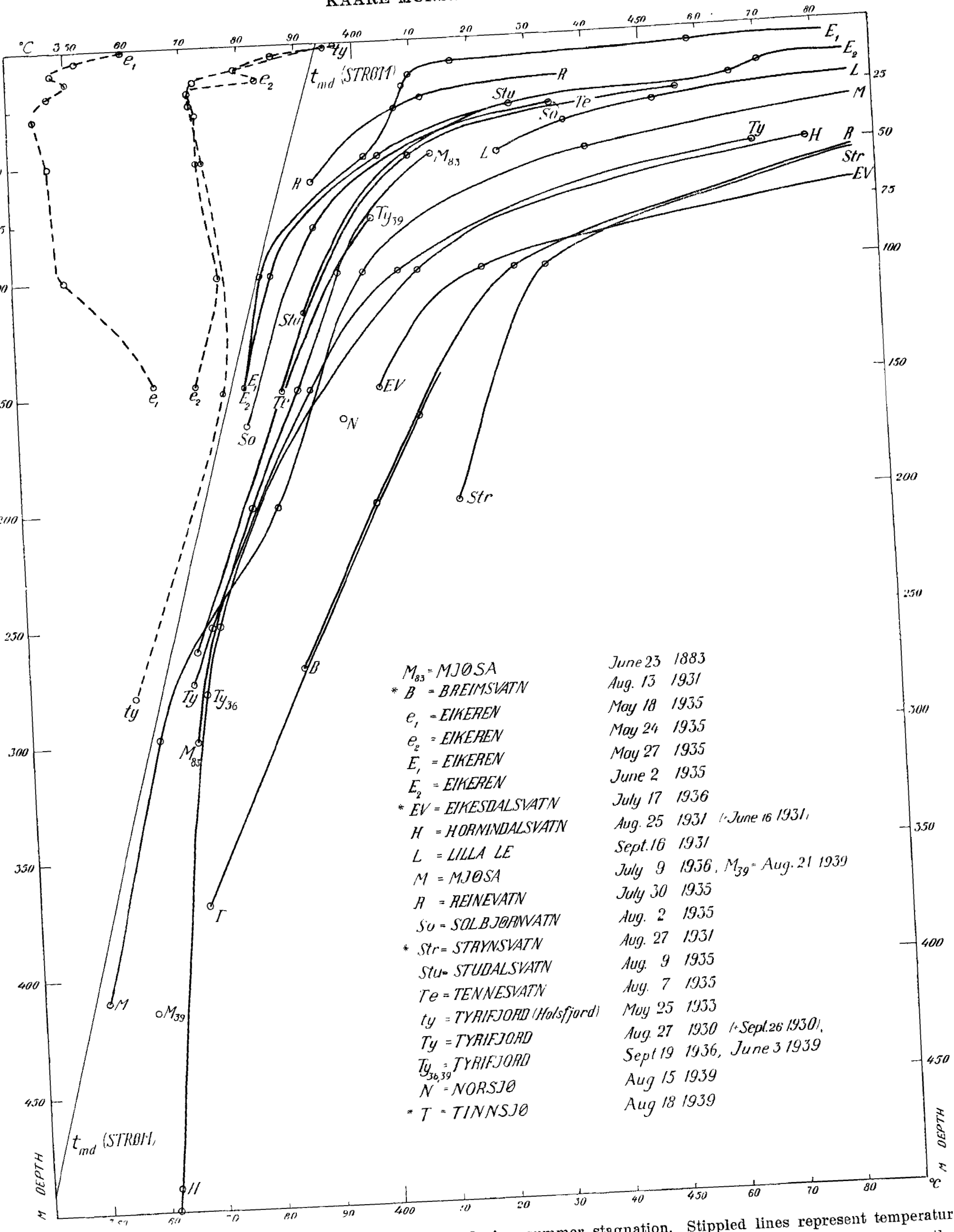

Fig. 1. — Temperatures in the deep of Norwegian lakes during summer stagnation. Stippled lines represent temperature series from the vernal full circulation period. Asterisks mark lakes that are influenced by supplies of glacial or other cold waters.

influence must be very slight, there is virtually no increase in temperature from June 16 to August 25, 1931.

Thirdly, the normally most conspicuous factor is the intermixture since vernal full circulation with waters lying above through turbulence. This leads to a heightening of temperature during the stagnation period, of which illustrative series are given for Eikeren (in STRØM 1944) where 150 m temperatures slowly rise from 3.77—3.79 at the end of May to 3.90 in November.

Intermixture towards the deep through turbulence is much facilitated if there are large supplies of cold waters during summer, or if climatical conditions preclude somewhat high summer temperatures in the surface. Bottom waters even in very deep lakes then show temperatures considerably above t_{md}. Among the lakes tabulated this is conspicuously the case with those that receive glacial waters, or cold-water supplies from high-lying mountain areas. (Strynsvatn, Breimsvatn, Tinnsjø, Eikesdalsvatn, marked with asterisks on table 1 and fig. 1.)

If possible values of t_{md} can thus only be directly obtained from deep lakes which have been sufficiently cooled down during winter, and only immediately after vernal full circulation, temperature series from other deep, holomictic lakes are nevertheless of the greatest interest in showing the trend of the curves. (Post, p. 11.)

Actual Temperature in Lake Bajkal, and in Other Deep Temperate Lakes outside Norway.

According to HALBFASS (1937, p. 264) lakes deeper than Hornindalsvatn are Bajkal (max. depth 1741 m), Tanganyika (1435), Caspian Sea (946), Nyasa (706), Issyk-kul (702), Crater Lake (Oregon, 608), Matana (Celebes, 590). Of these Tanganyika, Nyasa, and Matana are tropical, the Caspian Sea and Issyk-kul salt.

Lake Bajkal and Crater Lake are thus the only lakes deeper than Hornindalsvatn, from which data bearing upon the t_{md} question could be obtained.

In order to understand the following, a short description of Lake Bajkal must be given, mainly after FICKELER (1927, p. 84 ff.).

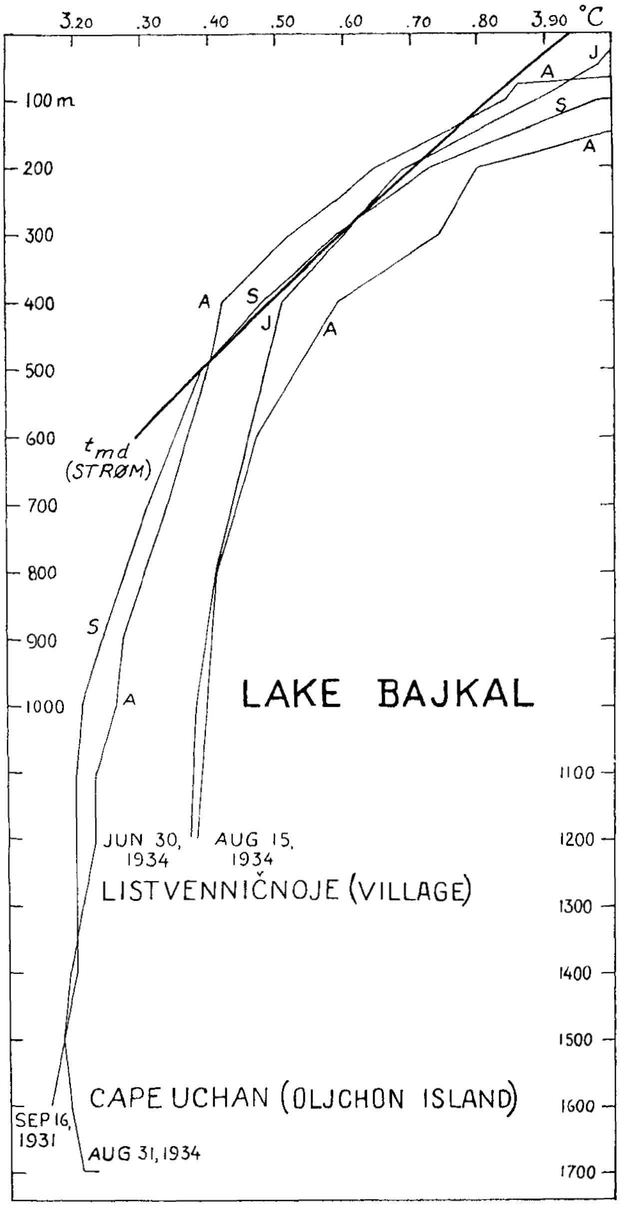

Fig. 2. — Temperatures of Lake Bajkal. Listvenničnoje is on the S basin (max. depth 1400 m), which is separated by the 150 m deep Selenga ridge from the N basin (max. depth at least 1741 m), where Oljchon Island is situated. (From data by VEREŠČAGIN 1936).

Lake Bajkal lies between about 51½ and 56° N. L. With a greatest length of 674 km, and an average breadth of 50 km (max. 74, min. 25), the lake covers an area of 33 000 km^2. Through the Selenga ridge with a depth of 150 m, a smaller S basin with a length of 200 km, and a maximum depth of 1400 m is separated from the N basin with a length of 474 km, and up to 1741 m deep. (Or even more than 1800 m, for which statement I

Table 2.

Lake Bajkal (VEREŠČAGIN 1936).

m	tmd (STRØM)	S. Basin		N. Basin	
		Off Listvenničnoje (Village)		Off Cape Uchan (Oljchon Island)	
		June 30 1934	Aug. 15 1934	Aug. 31 1934	Sep. 16 1931
0	3.94	4.45	14.54	10.80	9.35
10	3.93	4.06	12.29	10.48	8.95
25	3.91	4.00	10.78	5.48	7.85
50	3.88	3.98	6.30	4.24	5.44
75	3.85			3.86	4.33
100	3.82	3.89	4.27	3.84	3.98
150	3.77	3.79	3.99	3.75	
200	3.71	3.69	3.80	3.65	3.73
250	3.66			3.59	
300	3.60	3.60	3.74	3.52	3.59
400	3.49	3.51	3.59	3.42	3.48
500	3.39			3.40	3.39
600	3.29	3.46	3.47		
700	(3.19)			3 34	3.31
800	(3 09)	3.41	3.41		
900	(3.00)			3.27	3.24
1000	(2.91)	3.38		3.26	3.21
1100	(2.82)			3.23	3.20
1200	(2.73)	3.37	3.38	3.23	3.20
1300	(2.64)			3.21	
1400	(2.56)			3.19	3.20
1500	(2.48)			3.18	
1600	(2.40)			3.19	3.16
1697	(2.32)			3.21	
1698 (bottom)	(2.32)			3.23	

Area 33000 km^2; Max. depth at least 1741 m

Probable accuracy of observations ± 0.02

have been unable to find the reference.) Lake waters are extremely transparent (SECCHI disk values up to 40 m), and contain relatively little salts in solution. (Total solids 63—76 lmg. HALBFASS 1928, p. 351.)

Series of temperatures from the deep of Lake Bajakal have been published by VEREŠČAGIN (1927, 1936). There are a large number of series unpublished (G. J. VEREŠČAGIN in litt., Nov. 20, 1940), but these I have not seen. I was also hindered by the war in getting my high precision thermometer used on Lake Bajkal.

From Crater Lake there is only one somewhat old series of temperatures (KEMMERER, BOVARD, and BOORMAN 1924, cited from C. JUDAY in litt. June 21, 1941). New observations here would be of the greatest interest.

Table 3.

Various deep lakes (PETTERSSON 1902, KEMMERER, BOVARD & BOORMAN 1924, YOSHIMURA 1936).

m	tmd (STRØM)	Ladoga 1 July 29 1900	Ladoga 2 July 29 1900	Crater Lake Aug. 1 1913	Sikotuko Aug. 1 1922	Tasawako July 23 1931
50	3.88	4.20	3.98	4.6	4.5	5.2
60	3.87	4.05			4.2	5.1
70	3.86			4.1	4.1	
75	3.85	4.00	3.95			4 9
80	3.84				4.05	
100	3.82	3.94	3.93	3.8	4.0	4.1
125	3.80					
150	3.77	3.89	3 85		3.7	4.0
200	3.71			3.5	3.7	4.0
205–226	3.70—3 68	3.84 (bottom)	3.75 (bottom)			
250	3.66					3.9
300	3.60			3.4	3.7	3.8
363	3.53				3.6 (bottom)	
400	3.49					3.8 (bottom)
500	3.39			3.5		
602	3,29			3.5 (bottom)		
Area km^2		19000		50	76	26
Max. depth m		250		610	363	425
Probable accuracy of observations		± 0.05		± 0.15	± 0.15	± 0.15

Some temperature series from other deep lakes, but less deep than Hornindalsvatn, are also given in table 3. Lake Tahoe and several Japanese lakes could have been added, but with all summer temperatures above 4, data from these lakes are not sufficiently pertinent to the t_{md} question.

If the observations can be sufficiently relied upon, Crater Lake, and possibly Sikotuko are more or less meromictic.

Deduction of t_{md} from Reliable Experimental Determinations and Actual Lake Temperatures.

If the points 3.94 at 1 kg/cm^2 and 0 at 600 kg/cm^2 are rectilinearly connected, the gradient will be 0.066 for each 10 kg/cm^2 increase in pressure, or for each 100 m water column. (For convenience when dealing with lakes, pressure will hereafter be expressed as m water column: 10 m = 1 kg/cm^2).

A glance at figs. 1 and 2 at once makes it obvious that the lowering of t_{md} must be much more than 0.066 for each of the upper few 100 m, very many actual temperatures lying far to the left of that line. (See also fig. 3, where the 0.066/100 m-line is indicated.)

To attempt an empirical determination of t_{md}, we must have temperatures exactly determined immediately after vernal full circulation, and supply these temperatures with gradients in the deep, derived from other series.

The most important data are those from Eikeren (see table 1 and fig. 1). They include observations from May 24 and May 27, 1935, immediately before and after incipient summer stagnation. The picture (fig. 1) of the change from labile full circulation, with the irregular curves (cp. also Tyrifjord May 25, 1933), to the regularity of stabilisation, is very striking. From these data t_{md} at 100 m can with great certainty be fixed as 3.82 or immediately below, t_{md} at 150 m as 3.77 or immediately below. The other important values are from Mjøsa somewhat later in the season, and give t_{md} as below 3.62 at 295 m, below 3.51 at 410 m.

If we draw a line through the two first points, and a little to the left of the two last, we get a curve which is beautifully asymptotic to all the temperature curves in deep Norwegian lakes (except Hornindalsvatn, where the deep below 300 m has evidently not been cooled down far enough in winter, ante p. 5), and which separates the stippled lines representing temperatures during vernal full circulation from temperatures of summer stagnation.

Now we must consider the Lake Bajkal material. The curves are by no means so regular as from the Norwegian lakes, in some cases the same temperatures are given for readings at two different depths. Remarkable are the low temperatures in the middle deeps on Aug. 31, 1934 in the N basin. But in the S basin, temperatures even from June 30 of the same year are higher, and this curve, which represents conditions immediately after vernal full circulation, must be regarded as the most relevant one.

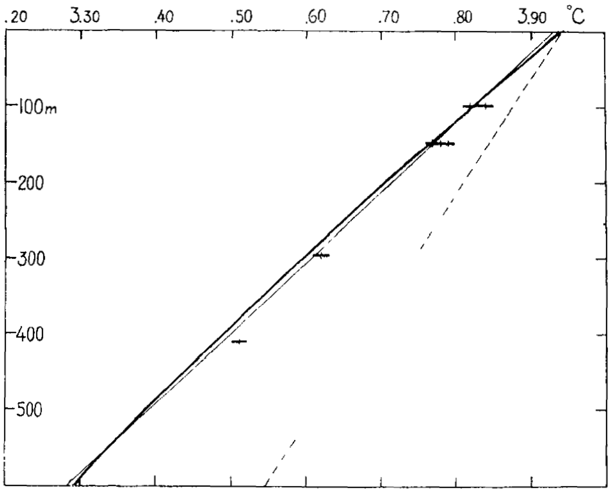

Fig. 3. t_{md} down to 600 m. Heavy line: Values arrived at. Light line: Values according to SCHIØTZ (1887). Stippled line: Rectilinear connection between t_{md} at atmospheric pressure and $t_{md} = 0$ at 6000 m. (0-value of PUSHIN & GREBENSHCHIKOV 1923). Bars: Uncertainly ranges of especially relevant temperature determinations from Norwegian lakes. (See p. 11). Cross bars: Temperatures as given.

I regard the cold middle waters on Aug. 31, 1934 as the result of upward water displacements, Cape Uchan lying very much nearer to the S than to the N end of the N basin, and prolonged winds in the basin direction could easily have this effect. (Cp. STRØM 1944, p. 4 b.) Temperatures are so near to t_{md} that there will be no measurable adiabatic change in temperature through the displacement.

As is pointed out by VEREŠČAGIN (1936) full circulation only extends to some 400 to 600 m. From there a zone of partial circulation through turbulence reaches to some 1000 m. Below 1000 m waters are stagnant, and practically isothermal throughout the year, though the relatively high O_2-values at 1600 m (6.77 lcm^3, or 70 pct. of saturation) indicate a more or less constant, even if very slow renewal.

Most important are the gradients, defined as difference in t_{md} for each 100 m, which can be derived from the curves. If there is much heat supply from above through turbulence, temperature differences for each 100 m will be larger than for he respective t_{md}'s, if bottom waters (as possibly in Mjøsa) are not quite sufficiently cooled, they will be smaller.

As a mean of 6 relevant gradients from 200 to 300 m in Lake Bajkal we get 0.11, of 8 from 300 to 400 0.11, of 3 from 400 to 500 0.10. (Temperatures from VEREŠČAGIN 1927, 1936). From Eikeren we get 0.12 between 0 and 100 m, from Mjøsa 0.10 for 100 m between 295 and 410.

These values in addition to the fixed temperature points: 0 m, 3.94; 100, 3.82; 150, 3.77; 295, < 3.62; 410, < 3.51; 6000, 0, lead to the values in table 4, which thus should be more or less secured down to the greatest depth of any immediate interest in lakes. They are graphically illustrated in fig. 3.

From 600 to 2000 m they are increasingly uncertain, but are entered in order to form bases of comparison for the deepest zone of Lake Bajkal. It should be noted that the provisional values given by me (STRØM 1939 a, STRØM 1942, p. 23) were a little different.

Table 4.

t_{md}-values arrived at:

m	tmd	Gradient 100 m	m	tmd	Gradient 100 m
0	3.94		1100	(2.82)	(0.09)
100	3.82	0.12	1200	(2.73)	(0.09)
200	3.71	0.11	1300	(2.64)	(0.09)
300	3.60	0.11	1400	(2.56)	(0.08)
400	3.49	0.11	1500	(2.48)	(0.08)
500	3.39	0.10	1600	(2.40)	(0.08)
600	3.29	0.10	1700	(2.32)	(0.08)
700	(3.19)	(0.10)	1800	(2.24)	(0.08)
800	(3.09)	(0.10)	1900	(2.16)	(0.08)
900	(3.00)	(0.09)	2000	(2.08)	(0.08)
1000	(2.91)	(0.09)	6000	0.00	

In using values from lakes, no regard could be paid to the change in t_{md} occasioned by very small amounts of dissolved salts. While these amounts appreciably change density itself, t_{md} is not lowered more than about 0.015 in Bajkal, the "saltest" of the lakes (values from KNUDSEN 1901), and less than 0.01 in the others.

Abstract.

Data for the lowering of the temperature of maximum density (t_{md}) in fresh waters with increased pressure are essential both to physicists and limnologists. A critical sifting of data physically determined leave only the values 3.94 at 1 kg/cm^2 and 0 at 600 kg/cm^2. From actual temperatures in the deep, and gradients of corresponding temperature curves during incipient summer stagnation in soft water lakes with vernal full circulation, and sufficiently cooled down in winter, the author endeavours to arrive at t_{md}-values between those two physically fixed points. These values must be regarded as provisional until physical determinations of the pressure at the introduction or release of which there under a constant temperature is no adiabatic change in temperature (method of N. A. PUSHIN & E. V. GREBENSHCHIKOV), are made at 0.1 intervals over the temperature range 4 to 0. They are mainly derived from deep Norwegian lakes (author), supplemented with observations from Lake Bajkal (G. J. VEREŠČAGIN), and are for the following moderate pressures (in kg/cm^2): 1, 3.94; 11, 3.82; 21, 3.71; 31, 3.60; 41, 3.49; 51, 3.39; 61, 3.29.

University Department of Limnology,
Blindern, Oslo, April 1945.

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