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SOME STATISTICAL PROPERTIES OF THE SIGNAL FINE STRUCTURE IN IONOSPHERIC SCATTER PROPAGATION

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Summary. Some statistical properties of the signal fine structure for a VHF ionospheric scatter circuit between Northern and Southern Norway have been examined. It is shown that the background component of the signal has statistical properties in agreement with those to be expected for Gaussian noise, and that this signal therefore can be explained in terms of scattering from a large number of irregularities. The fading of the enduring meteoric bursts, on the other hand, must be explained in terms of interference between a few reflected components resulting from bending of the trail due to large scale wind gradients.

1. Introduction. In this paper an analysis will be made of some statistical properties of the signal fine structure for the ionospheric scatter circuit between Tromsø (70°N, 19°E) and Kjeller (60°N, 11°E). The path for this circuit is 1150 km, and the the frequency used is 46.8 Mc/s. Identical rhombic aerials were used at transmitting and receiving end. These were designed to give a beam width of 15° between half power points, and maximum radiation at an elevation of 6°. The transmitted power was 5 kW CW.

It is now well known that the signal in ionospheric scatter propagation is made up of sudden isolated increases of the mean signal, or "bursts" of amplitude, in addition to a background component. It is also generally agreed that these bursts are produced by reflection from trails of ionization left in the ionosphere by the meteores entering the earths atmosphere. The background component is believed to be produced, partly by scattering from irregularities formed in the lower ionosphere by turbulence, and partly by weak meteoric reflections.

Although most of the meteoric bursts last only for fractions of a second, bursts are sometimes observed which may last up to 10 seconds more. These long duration bursts are often found to develope quite deep fading of more or less random character. This fading has been explained by Lovell and Clegg [1] and Kaiser and Closs [2] by the following mechanism: After the trail is formed, wind gradients in the ionosphere make the trail curved so that it is possible to have two or more reflection conditions from the trail at the same time, and the fading is produced by interference between the resulting components.

Booker and Cohen [3] has considered an alternative explanation. They argue that the time constants involved in the turbulence are so small that after a very short time the trail will be broken up into a large number of small irregularities, and that consequently the reflection from the trail should be calculated by an incoherent scattering theory.

More recently, Manning and Eshleman [4] have criticised the Booker and Cohen paper. They argue that the observations made by Booker and Cohen can not be explained by their theory but they show good agreement with the "large scale" theory of Lovell and Clegg and of Kaiser and Closs. Manning [5] has also developed a statistical theory based on the assumption that the trail is distorted by a random wind profile, and has determined the sizes of the active irregularities to a few kilometres rather than a few metres as assumed by Booker and Cohen.

Ratcliffe [6] has shown that for scattering from a large number of irregularities or "blobs" having a Gaussian distribution of the random velocities, the statistical properties of the scattered signal should be identical with those of random noise having a Gaussian frequency spectrum centered on the carrier frequency. It therefore seems reasonable to assume that both the background signal, and the signal of the fading meteoric bursts should have essentially the same statistical properties as random noise if Booker and Cohen hypothesis were correct. The purpose of this paper is to test by various criteria the statistical properties both of the background component of the signal and the signal during the fading meteoric bursts.

2. Selection of data. In Fig. 1 examples are shown of the various types of recordings obtained. In Fig. 1a the typical background signal is shown together with a few meteoric bursts of the most common type lasting only for fractions of a second. In order to study the statistical properties of the background component of the signal, only periods were selected when no clear meteoric signal was present, as indicated in the illustration.

In Figs. 1b and 1c examples of the enduring meteoric bursts are shown. The main difference between the two types of bursts shown is that in Fig 1b the signal increases abruptly, in the same way as for the bursts shown in Fig. 1a, while in Fig. 1c the bursts are shown to start more gradually. This difference is believed to be due to the effect that in one case the point of reflection for the initially straight trail may lay inside, and in the other case outside the part of the trail which is strongly ionized or illuminated by the aerial system. The period in which the amplitude is decaying has not been used in the analysis, as indicated in the illustration, and a number of 70 bursts have been selected for analysis.

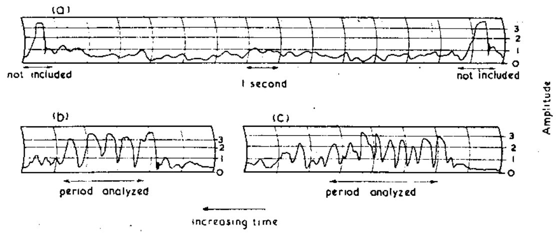

Fig. 1. Typical record samples.

3. Distribution of signal envelope and maxima of signal envelope. In this section, we shall discuss the theoretical distributions of signal envelope and of maxima of signal envelope, and compare these with the distributions found experimentally.

Distribution of signal envelope. The probability distribution of envelope of a noise signal is,

$$p(R) = \frac{R}{\psi_0} e^{\,R^2/2\psi_0}$$

and is consequently independant of the form of the frequency spectrum . ψ_0 is the total power.

In Fig. 2 the observed distributions for the background signal and for the meteoric

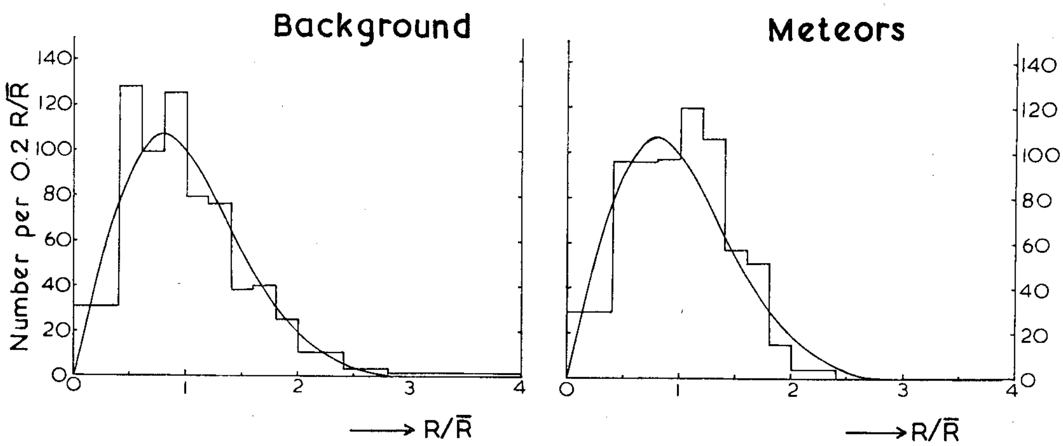

Fig. 2. Observed and theoretical distributions of signal envelope for the background and meteoric component of the signal.

signal are compared with the theoretical distribution. We see that the agreement is fair for the background signal.

Distribution of maxima of signal envelope. According to Rice [7] the probabilities for a maximum of envelope to occur within the elementary rectangle dRdt, is

$$p\,(t,\,R)\,dRdt = -\,dRdt \int_{-\infty}^{0} p\,(R,\,O,\,R'')\,R''\,dR''$$

$p\,(R,\,R',\,R'')$ is the probability density for the three dimentional distribution of R, R', R'', where primes denote differentiation with respect to time t. The expected number $\mathcal{N}$ of maxima per unit time will be,

$$\mathcal{N} = \int_{0}^{\infty} p\,(t,\,R)\,dR$$

Rice has given analytical expressions for $p\,(t,\,R)$ and $\mathcal{N}$ (eq. 3.8–11 and 3.8–12). These are dependent on the form of the frequency spectrum. The probability for maximum of envelope to occur between R and $R + dR$ will be,

$$P_{MAX}\,(R) = \frac{1}{\mathcal{N}}\,p\,(t,\,R).$$

This distribution has been computed for a Gaussian filter (using Rice's equation 3.8–11 and 3.8–12), and is compared in Fig. 3 with the curve obtained by Rice for the ideal band pass filter. The main conclusion from this comparison is that the probability distribution is not markedly different for the two filters.

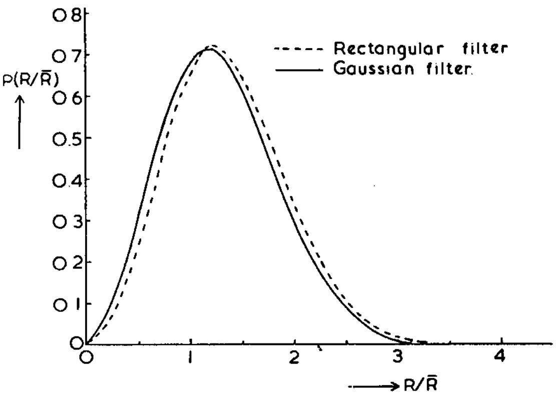

Fig. 3. Probability distribution of maxima of envelope for a Gaussian and an ideal band pass filter.

In Fig. 4 the observed distributions of maxima of envelope are shown for the background and meteoric component of the signal, and compared with the theoretical distribution for the Gaussian filter. Again we find that agreement is fair for the background component, but bad for the meteoric component.

4. Periodicity of fading. It was found in section 3 that the observed distributions of envelope and of maxima of envelope for the background component of the signal were in good agreement with those of random noise. For the meteoric signal on the

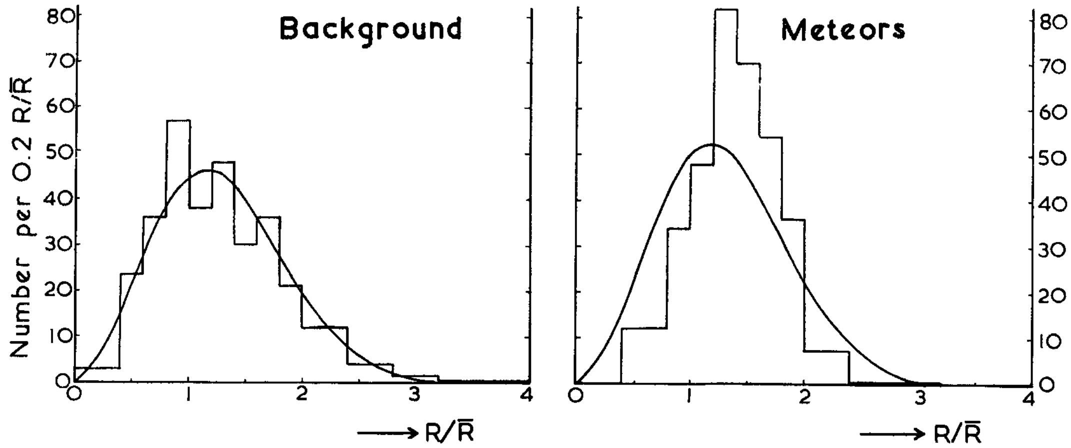

Fig. 4. Observed and theoretical distributions of maxima of envelope for the background and meteoric component of the signal.

other hand the agreement was not good. This result favours the theory of Lovell and Clegg [1] and of Kaiser and Closs [2] rather than the theory of Booker and Cohen [3] concerning the mechanism of reflection for the enduring meteor bursts. Now the fading during the bursts was produced by interference between a few reflected components, one should expect the fading to be more periodic than in the alternatived case. This is also what is observed as may be seen qualitatively from Fig. 1. We have tried to find a test from the theory of random noise by which the periodicity of the fading may be tested, but without success. We have chosen then an alternative method of finding such a distribution, namely to produce a noise signal, and then to find the distribution experimentally.

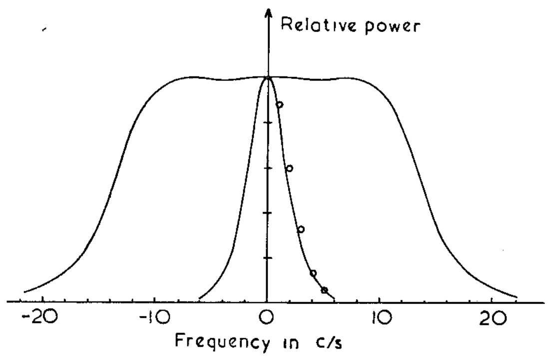

Fig. 5. The filters. Circles indicate the Gaussian curve.

Probability distribution of separation between successive maxima of signal envelope. The valve of the first stage of an amplifier was used as noise source, and the noise signal was fed through the 4 c/s and the 28 c/s filters of a Radiometer frequency analyzer to a linear detector, and the detected signal was recorded by a Brush recorder. The two filters have been measured accurately and are shown in Fig. 5. We see that the 4 c/s filter is quite a good approximation to a Gaussian filter, while the 28 c/s filter is more like an ideal band pass filter.

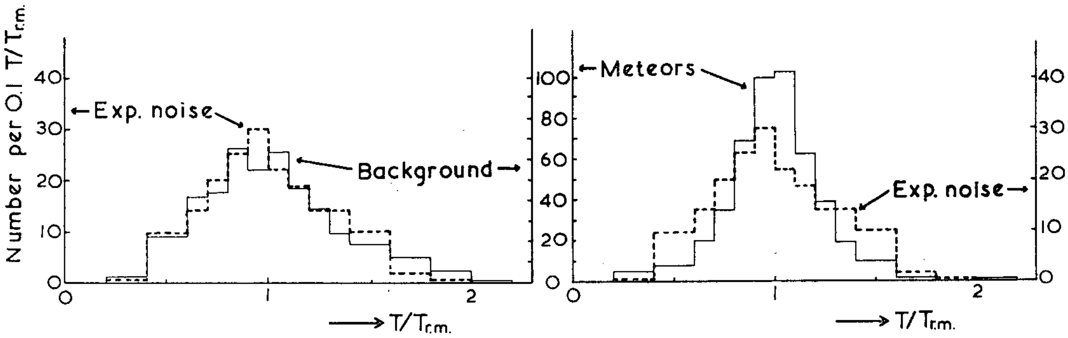

Fig. 6. Observed distribution of T/Tn for the background and meteoric component of the signal compared with the distribution found for experimental noise.

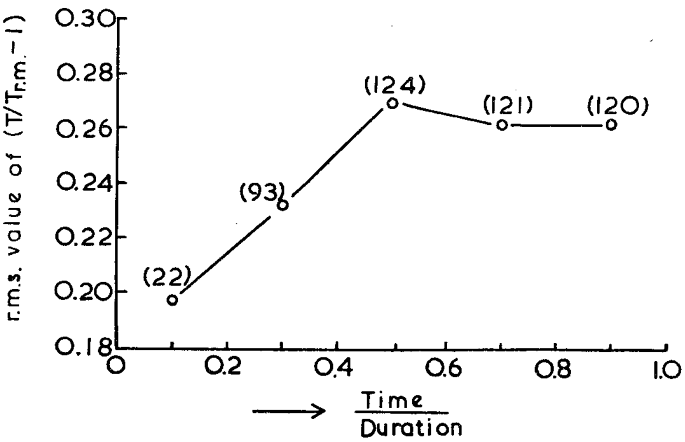

Fig. 7. Departure from periodicity as a function of time divided by duration of burst.

We will define now the period of a fading cycle as the separation between successive maxima of the envelope. In order then to test the periodicity we will find the distributions of this period divided by some mean through the sample. In order to eliminate the effect of varying width of the spectrum a three values running mean was chosen.

The probability distribution $p\ (T/T_{rm})$ of the separation between successive maxima of envelope divided by the three values running mean of T obtained for the Gaussian filter has been compared with the distribution obtained for the band "pass" filter. The comparison shows that the distribution is not markedly dependent on the form of the filter.

In Fig. 6 the observed distributions for the background and meteoric component of the signal are given in histograms and the results are compared with the distribution obtained for the Gaussian filter. We see that the agreement is good for the background component and bad for the meteoric component of the signal.

Periodicity as a function of time during the burst. We have chosen to study the quantity T/T_{rm} as a measure of the periodicity of the fading signal. For a periodic signal this quantity should be equal to 1, and the quantity $|T/T_{rm}-1|$ will therefore be a measure of departure from periodicity. In order to study whether this quantity on the average varies through the burst, all bursts were divided into 5 periods of equal length and the r. m. s. value of $(T/T_{rm}-1)$ was calculated for each period. In Fig. 7 the deduced values have been plotted as a function of time during the burst.

Fig. 7 shows that the r. m. s. value of $(T/T_{rm}-1)$ increases with time in the first of the duration, a result which indicates that so does also the number of components in the reflected signal.

5. Conclusion. It was pointed out in the introduction that for scattering from a large number of irregularities, the statistical properties of the signal should resemble random noise. If a rather limited number of components, say 6 or 7, having random phases are added, it is known that the probability distribution for the envelope becomes very nearly a Rayleigh law. In fact, even for such a limited number of components the signal would have essentially the same characteristics as random noise.

In this paper the probability distributions of signal envelope, maxima of signal envelope, and separation between successive maxima of signal envelope have been obtained both for the background component and the meteoric component of the signal, and comparison has been made with the distributions one should expect for Gaussian noise.

It is concluded that the background component of the signal gives probability distributions in good agreement with those to be expected for Gaussian noise, and that this signal therefore may be explained in terms of scattering from a large number of irregularities.

The signal during the fading meteoric burst can not be explained in terms of incoherent scattering from a large number of irregularities as suggested by Booker and Cohen [3]. The statistical properties of the meteoric signal component can be explained in terms of a large scale model.

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