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STUDIES ON THE EXCITATION OF AURORA BOREALIS
II. THE FORBIDDEN OXYGEN LINES

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Summary. Excitation and deactivation of the forbidden oxygen lines is studied theoretically with emphasis on excitation by secondary electrons and discharge mechanisms. The computed value of the intensity ratio I (5577)/I(4709) for excitation by secondary electrons is of the same order of magnitude as observed in aurorae. Discharge mechanisms are probably not able to contribute significantly to the auroral excitation, but the possibility that the horizontal currents responsible for the magnetic perturbations associated with aurorae contribute to some extent cannot be dismissed. Other indirect excitation mechanisms and deactivation mechanisms for the excited atoms are briefly reviewed.

The intensity ratio $I(5577)/I(4709)$ in aurorae was measured with a photoelectrical photometer and found to be constant within $\pm$ 25 % in all the forms measured. Rapid fluctuations in the intensities of the green [OI] line and the First Negative N_2^+ bands have been studied, and the lifetime τ of the $O(^1S)$ atoms has been computed by various methods. The lifetime appears to be 0.7 $\pm$ 0.1 sec. The detailed interpretation depends on the true value of the lifetime of the undisturbed $O(^1S)$ atoms, but the available evidence suggests that collisional deactivation sometimes occur, the lifetime of the undisturbed atoms being about 0.75 sec, and the probability of deactivation being in the range 0—1/sec. This implies a probability of deactivation per gaskinetic collision of the order of magnitude 10^{-4}. The importance of the various excitation mechanisms is discussed in the view of the measured lifetimes. Excitation by secondary electrons seems to be consistent with the observations.

The red doublet is briefly discussed. There are not necessarily any inconsistencies between theory and observations, but the relevant data are too inaccurate to justify detailed discussions and definite conclusions. New observations on aurorae can yield only limited information until our knowledge about the basic processes is improved by theoretical or laboratory investigations.

1. Introduction. The forbidden transitions in the oxygen atom, ^1D — ^1S and ^3P — ^1D, give rise to the green line of wavelength 5577,3 Å and the red doublet of wavelength 6300.3 Å and 6363.9 Å. The latter of the two transitions should yield a triplet, the third line having the wavelength 6391.7 Å, but this line has never been observed because it has a much lower transition probability than the other two. These [OI] lines are among the strongest features in the auroral spectrum, and it appears that they were first observed by ÅNGSTRØM in 1867 (cf. STØRMER 1955).

Many years elapsed before the origin of these lines was found. They were finally identified through the work by MC LENNAN and SHRUM (1925), BABCOCK (1923), VEGARD and HARANG (1934, 1936) and others. The lines have been extensively studied with spectrographs, notably by VEGARD and collaborators (cf. VEGARD, BERGER and NUNDAL 1958, and references therein), who particularly studied the variations in the intensity ratio between the green line and the red doublet.

HARANG (1958) has also studied the height distribution of the [OI] lines relative to the First Negative N_2^+ bands in various auroral forms. With a photoelectric photometer, the time delay of the green [OI] line relative to the N_2^+ bands in rapidly fluctuating aurorae has been studied, in order to determine the lifetime of the oxygen atoms in the ^1S term (OMHOLT and HARANG 1955, OMHOLT 1956). Photoelectric intensity measurements have also been made by others, particularly by HUNTEN (1955).

The relevant transition probabilities have been computed theoretically by PASTERNACK (1940) and later by GARSTANG (1951, 1956) with improved methods. EMELEUS, SAYERS and BAILEY (1950), SAYERS and EMELEUS (1952) and HERMAN and HERMAN (1954) have made attempts to determine the lifetime of the ^1S term experimentally. The experimental values are in the range 2 to 0.3 sec, but the radiation measured appears to be the green line accompanied by a band characteristic to argon—oxygen mixtures.

There has been much speculation about the nature of the excitation processes responsible for the [OI] emissions (cf. SWINGS 1948). The work by SEATON (1953, 1956) on electron exchange collisions between oxygen atoms and free electrons of moderate energy (2 — 20 eV) made it clear, however, that this process may be very effective in populating the ^1D and ^1S terms. Free electrons of sufficient energy may be produced through the ionization processes which obviously take place in the atmosphere during aurorae. Clearly the rate of excitation of the [OI] lines depends on the energy distribution of the secondary electrons, which in turn depends on the nature of the primary processes. One should therefore, for example, expect that impact by fast particles and local discharges in the atmosphere yield rather different relative rates of excitation of the [OI] lines and the N_2^+ bands.

It has not yet been possible to form a correct and detailed picture of what is actually going on in the atmosphere during aurorae, and how the ^1D and ^1S terms of the oxygen atoms are populated. Many of the ambiguities which still exist are probably due to the fact that the $O(^1D)$ and $O(^1S)$ atoms are metastable, having lifetimes of about 110 and 0.7 sec respectively, if undisturbed by collisions (GARSTANG 1951). This makes collisions with other atoms and molecules (and with the walls of the vessel in the case of laboratory experiments) important, and makes the interpretation more difficult.

As regards the relative intensity between the green line and the N_2^+ bands observed in aurorae, the results obtained by various authors are somewhat contradictory. It is likely that this is partly due to the difficulties encountered in the measurements. The green line is very strong, and on a spectrogram of the visual region it is therefore difficult to compare it with anything but the red doublet and the First Negative N_2^+ bands $\lambda 3914$ and $\lambda 4278$ ($B^2\Sigma$ — $X^2\Sigma$, 0—0 and 1—1). These N_2^+ bands are subject to severe absorption in the atmosphere, which again introduces large errors. In addition comes the general difficulty in comparing lines with bands in low dispersion spectrograms and in comparing emissions of considerable wavelength differences recorded by detectors whose sensitivety varies strongly with the wavelength.

In this paper we shall give the results of some observations of the green [OI] line and the First Negative N_2^+ bands made with a photoelectric photometer. These observations have been made to study the excitation of the green line and the possible collisional deactivation of the oxygen atoms in the ^1S term. Rapidly fluctuating as well as quiet auroral forms have been studied. Before we describe the observational work, we shall present some theoretical considerations which is the basis of our observations and the interpretation of these. Also the red [OI] doublet will be discussed.

2. Excitation of the [OI] lines.

a. Excitation by secondary electrons. From the work by SEATON (1953, 1956) it appears likely that secondary electrons may be responsible for most of the excitation of the [OI] lines. As we shall see later this view is also supported by the observations. We shall therefore discuss this process in some detail. The processes in consideration are

$$O(^3P) + e \rightarrow O(^1D, {}^1S) + e, \qquad (2.1\ D,\ S)$$

in which one of the bound electrons in the atom is exchanged with the free electron. SEATON computed theoretically the cross section for these processes as a function of the energy of the free electron. The cross sections fall off rapidly with increasing energy above a value which is about twice the excitation energy. The rates of the processes (2.1 *D, S*) are therefore negligible for the primary electrons in question. Experimental results (cf. LANDOLT-BÖRNSTEIN 1952, Vol. 1, part 5, page 343) show that about two thirds of the total ionization due to fast electrons is performed by secondary electrons.

Obviously these must also have ample energy to excite the ^1D and ^1S terms (excitation potentials 1.96 and 4.17 eV respectively). This is in agreement with theoretical work by BATES, MCDOWELL and OMHOLT (1957). If therefore aurorae are caused by fast primary particles, the processes (2.1 D, S) may be important. It is of great interest to get at least an order of magnitude estimate of the rates of excitation of the ^1D and ^1S terms by these processes, and to compare them with the rate of excitation of the First Negative N_2^+ bands. The latter are excited directly from N_2 by the primary particles as well as by the secondary electrons.

The simplest way to approach the problem seems to be to consider the energy distribution of all active electrons, i.e. those having sufficient energy to perform electron excitation ($E \geqq 2$ eV). Let $n\, f(E)\, dE$ be the number of electrons per cm³ per sec ejected from the air molecules and atoms, the initial energy being in the range dE centered at E ($\int_0^\infty f(E)\, dE = 1$), and let $u(E) = w(E)\, v(E)$, $w(E)$ being the total number of secondary electrons per cm³ within the range dE, and $v(E)$ the electron velocity. We then have, with sufficient accuracy, that

$$n\, f(E) + \Sigma_j N_j Q_j(E + \triangle E_j)\, u(E + \triangle E_j) = \Sigma_j N_j Q_j(E)\, u(E), \tag{2.2}$$

where the Q_j's are the cross sections for the various kinds of inelastic collisions, the $\triangle E_j$'s the energy losses per collision of the electrons, and the N_j's the concentrations of the appropriate atoms or molecules. The left and right hand parts of equation (2.2), when multiplied by dE, represent respectively the number of electrons per cm³ per sec entering into and leaving the interval dE centered at E. For ionization, $Q(E + \triangle E_j)$ $u(E + \triangle E_j)$ should certainly be replaced by an integral over the energies of the tertiary electrons, but with sufficient accuracy we may use the adopted form, $Q_i(E)$ being the total ionization cross-section and $\triangle E_i$ the sum of the average ionization energy and he average ejection energy of the tertiary electrons.

Elastic collisions are certainly unimportant for electron energies above 2 eV and are therefore neglected. A more detailed comparison of the cross sections and the energy losses in various processes suggests that we can neglect rotational excitation of N_2, and probably also vibrational excitation, without introducing any major error (cf. ANDERSON and GOLDSTEIN 1956). At all energies above 2 eV the energy loss of the electrons due to rotational and vibrational excitation is probably less than 15 per cent of the loss due to electronic excitation and ionization.

The number of secondary excitations or ionizations per ejected secondary electron (the specific rate S_j for the particular process designated by j) is given by

$$S_j = \frac{1}{n} \int_{E\,\min}^{\infty} N_j Q_j(E)\, u(E)\, dE, \tag{2.3}$$

where $E_{\min}$ is the excitation or ionization energy.

$u(E)$ may be derived from equation (2.2) by starting at energies which are so high that $u(E)$ is vanishingly small and estimating $u(E + \triangle E_j)$ by trial and error methods. The distiibution function $f(E)$ will depend on the energy of the primary particles and is not accurately known. But it is convenient to adopt the form

$$f_a\,(E) = a\,e^{-aE}, \tag{2.4}$$

the average initial energy of the secondary electrons being $1/a$. An actual distribution $f(E)$ may perhaps be described with sufficient accuracy by a linear combination of functions of the form (2.4) with different a's, $u(E)$ and the S_j's will than be the same linear combination of the $u(E)$'s and S_j's computed for the individual $f_a(E)$'s. This procedure is at least possible for the electron distributions computed by BATES *et al* (1957) for proton impact on neon, and it has the advantage that it is possible to present S_j as a function of a single parameter a.

Computations were carried out under the following simplified assumptions: Ionization of all air molecules and atoms were represented by one process, in which the cross section was taken to be equal to that for total ionization of N_2 (TATE and SMITH 1932) and with $\triangle E_i = 19$ eV. Optically allowed excitation processes were similarly represented by one process, having a cross section function $Q_e(E)$ of the same shape and magnitude as that for ionization, but starting at 9.5 eV and with $\triangle E_e = 9.5$ eV. Excitation to the metastable ^1S and ^1D terms was taken into account, adopting the cross sections computed by SEATON (1953) and assuming complete dissociation of oxygen, but all other processes were neglected. Some results of importance are presented in Fig. 1. For the computation of S_j for ionization of N_2 followed by emission of a photon in the λ4709-band of the First Negative system of N_2^+ (S(4709)) STEWART's (1956) cross sections were adopted. The unit for a Fig. 1 is $1/I_H$,I_H being the ionization potential for hydrogen. S_i, S_{12} and S_{13} are for total ionization and excitation of the ^1D and ^1S terms respectively.

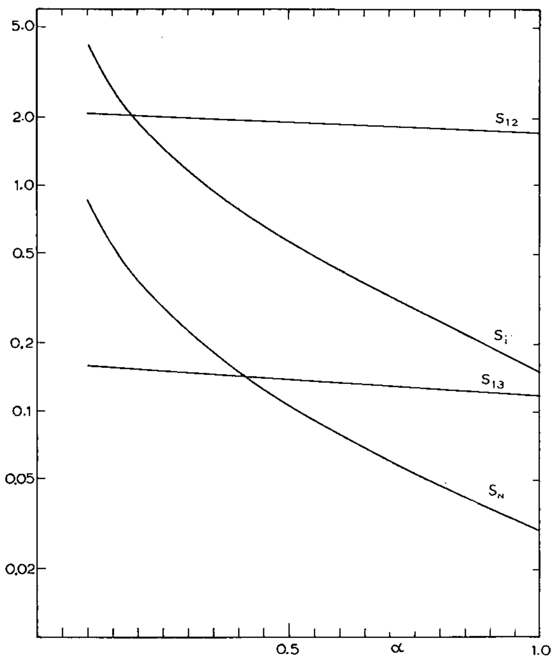

Fig. 1. Specific rates of excitation and ionization by secondary electrons. The unit for a is $1/I_H$. $S_N = 100\,S(4709)$ (cf. the text).

The average of the total energy spent per ionization, $\overline{\triangle E'}$ (primary + secondary)is $(a^{-1} + 1.4\,I_H)/(S_i + 1)$, $(a^{-1} + 1.4\,I_H)$ being the aveiage total energy spent on $(S_i + 1)$ ionizations. Our result yields $\overline{\triangle E'}$ to be about 2.2 I_H or about 30 eV, being almost independent of a. The experimental value is about 32 eV (cf. LANDOLT-BØRNSTEIN, Vol. I, part 5, p. 344). Computations were also carried out with twice as high value of $Q_e(E)$, and $\overline{\triangle E}$ was then increased to about 2.6 I_H or about 35 eV. The value of S_i and $S(4709)$ were then reduced by a factor close to 1.4, whereas the change in S_{12} and S_{13} was insignificant. This change was almost independent of α.

Computations show that if we assume that excitation of N_2 to the states $A\,^3\Sigma$ and $B^3\Pi$ is the dominating process in the energy range 6.2 to 9.5 eV, then S_{12} and S_{13} are reduced by factors close to 1.6 and 3.2 respectively, for all values of α, whereas S_i and $S(4709)$ remain almost unchaged. It should be noted that the S_j's depend on the relative values of the (N_jQ_j)'s and not on the absolute values or the atmospheric density.

Experimental work shows that the total rate of ionization by electrons in air is about 3 times the rate of pi imary ionization (for electron energies between 20 and 100 keV.) or a value of 2 for S_i (cf. LANDOLT-BØRNSTEIN, Vol. I, part 5, p. 343). This corresponds to a value of α about $0.2/I_H$, a reasonable value also from the theoretical results by BATES *et al* (1957), although their distributions are for neon and are better expressed as linear combinations of various $f(a_n)$'s.

Adopting $S_{12} = 2.0$, $S_{13} = 0.15$, $S_i + 1 = 3$ and $S'(4709)/(S_i + 1) = 2.0 \times 10^{-3}$, $S'(4709)$ including the excitation by the primary processes, we obtain the result that for each emitted $\lambda 4709$ quant the number of excitations to the ^1S and ^1D terms are about 25 and 330 respectively. When excitation of N_2 to the $A^3\Sigma$ and $B^3\Pi$ states is taken into account as mentioned above, these figures are reduced to 8 and 200 respectively.

The number of secondary electrons with energies above 4.2 eV $\left(\int_{4,2\,\text{eV}}^{\infty} w(E)\,dE\right)$ is about $10^9\, n/N$ (sec/cm^3), n being the rate of primary ionization, and N the total particle density in the atmosphere. The quantity $\int_{4,2\,\text{eV}}^{\infty} w(E)\,dE/n$ has dimension time, and gives the order of magnitude of the time that elapses between the primary ionization and the excitation of the ^1S term. The time delay between the primary and secondary ionization is negligible, so that it also gives the time delay between the emission of the $\lambda 4709$ quanta and the excitation of the ^1S term. Below about 140 km (density N above 10^{11}/cm^3) this time delay is of the order 10^{-2} sec or less and is thus negligible. For the ^1D term the time delay is only slightly higher.

It is evident that the computations presented here are based on very crude approximations, and serve only as a guide to the order of magnitude of the correct values. The deduced values of $S_{13}/S(4709)$ are, as we shall see in section 6, very close to the value derived from the observations, but this very close agreement is certainly no proof of the correctness of the theory.

b. Discharge mechanisms. The suggestion that local discharge mechanisms in the atmosphere may be partly or wholly responsible for aurorae has been put forward by several authors. More recently CHAMBERLAIN (1956) and LEBEDINSKY (1956) have discussed discharge mechanisms in more detail. CHAMBERLAIN considers the possibility that auroral rays are due to local discharges along the magnetic lines of force, whereas homogeneous aurorae are caused by fast indicent particles. LEBEDINSKY proposes that auroral arcs may be due to currents across the magnetic field, along the visual auroral arc, whereas rays are caused by currents along the magnetic lines of force.

It is difficult to see how electric discharges along the magnetic field can occur, since the electron density is rather high in the ionosphere. The influx of energy required to produce an aurora of international brightness III is probably about 6×10^{13} eV/cm² sec (cf. OMHOLT 1959a, sec. 7). With 10 keV electrons the electron flux will be 6×10^{9} /cm² sec. The electron density in an aurora of brightness III is close to 10^{6}/cm³ (cf. OMHOLT 1959b), and higher up in the atmosphere it is at least 10^{4}/cm³. The drift velocity of the electrons needs thus to be only 10^{6} cm/sec, probably much less, to compensate for the influx of electrons[1].

It is evident from CHAMBERLAIN's work (1956) as well as from the experimental results obtained by CROMPTON and SUTTON (1952) that the electric field necessary to produce this drift velocity is only about 10^{-2} V per mean free path and that no detectable emission would be produced. Even with an appreciably stronger field the red [OI] doublet would be the dominating emission and the discharge would not produce any significant ionization, and thus no N_2^+ emission.

In view of what is said above it is difficult to see how the field necessary to produce visible aurorae by a discharge along the magnetic lines of force, can be created by accumulation of the charges incident on the atmosphere. If such fields nevertheless were created by this or electrodymanic mechanisms, there would almost certainly be a profond variation with height in the spectrum of the emission. The brightness of a long ray vary very little with height; this means that the rate of excitation per atmospheric particle must increase strongly with the height as the density decreases. If the current is approximately constant along a ray, the average energy of the electrons must increase considerably with the height. (With a constant current and energy the rate of excitation per atmospheric particle would be constant with height.) But as demonstrated by CHAMBERLAIN, and this can also be seen from general arguments, this would lead to strong variations in the relative rates of excitation of emissions with widely different excitation potentials. The intensity of the N_2^+ bands would increase strongly with the height compared to the green [OI] line.

LEBEDINSKY (1956) assumes that the currents responsible for the magnetic disturbances associated with visual aurorae are due to ions and concentrated in the aurorae, being responsible for the excitation of the aurorae. He computes the mobility of electrons and positive ions across the magnetic lines for force. The formulas he uses are not consistent with those of ALFVEN (1950) but the computed mobilities are roughly correct.

However, LEBEDINSKY neglects the electron and ion currents in the direction $\mathbf{E} \times \mathbf{H}$ (the Hall current) and also that the mobilities depend on the temperature of

[1] The view that aurorae are mainly caused by fast electrons was advocated in an earlier paper (OMHOLT 1959a) and is supported by rocket measurements (MCILWAIN 1958, MEREDITH, DAVIES, HEPPNER and BERG 1958).

of the ions and electrons. When the electric and magnetic fields are crossed at right angle, the specific conductivity σ_1/n in the direction of **E** is

$$\frac{\sigma_1}{n} = \frac{e^2 \tau}{m\,(1 + \omega^2 \tau^2)}, \quad \omega = \frac{eH}{mc} \tag{2.5}$$

(ALFVEN 1950), where τ is the mean time between two collisions, n the number density and m the mass of the electrons or ions in question, ω the gyrofrequency and the other symbols have their usual significance. The maximum value of σ_1/n is obtained where $1/\tau^2$ is equal to ω^2. For a magnetic field of 0.5 gauss[1] the maximum value of σ_1/n for molecular ions occurs where the particle density $\mathcal{N}$ is approximately $10^{12}/\mathrm{cm}^3$ and for electrons where $\mathcal{N}$ is approximately $10^{15}/\mathrm{cm}^3$, provided the ion or electron temperature is about 270° K. The maximum value of σ_1/n is about 1.6×10^{-11} A cm²/V.

The electric field causes also currents in the direction of **E** × **H**, the specific conductivity being given by

$$\frac{\sigma_2}{n} = \omega\,\tau\,\frac{\sigma_1}{n} \tag{2.6}$$

The electron and positive ion currents are here in opposite direction, the ω's having different signs. $|\sigma_2/n|$ increases with increasing τ, being equal to σ_1/n where the latter has its maximum. For electrons above 100 km τ is very large compared to $|1/\omega|$ and $|\sigma_2/n|$ is close to its maximum value ($\tau = \infty$) which is about 3.2×10^{-11} A cm²/V, twice the maximum value of σ_1/n for ions. As the height increases $|\sigma_2/n|$ for the ions increases, and ultimately the currents produced by ions and electrons become equal in magnitude but opposite in direction. Thus the ionized matter drifts in the direction of **E** × **H**, causing no net current.

With low electric fields, so that the electron and ion temperature do not differ appreciably from the gas temperature, the currents produced by ions and electrons at auroral heights may be comparable in magnitude, but differ in direction. The detailed picture depends on the electron and ion distribution and on the formation of negative ions. At higher electric fields the electron and ion temperature may increase considerably, and this may alter the picture completely, particularly for the ions, where $|\omega|$ and $1/\tau$ are comparable.

According to McNISH (1938) the current necessary to produce a dip of 500 γ in the horizontal magnetic field at the earth's surface is of the order 3×10^{-9} A/cm², if we assume that the thickness of the current sheet is about 30 km, a reasonable value for the vertical extension of auroral arcs and thus for the associated ionization (HARANG 1945, cf. also OMHOLT 1959b). With an electron and ion density of $2 \times 10^5/\mathrm{cm}^3$, which may be regarded as typical for aurorae (cf. OMHOLT 1959b), the electric field E must be of the order 10^{-3} V/cm, and the drift velocities of the order 2×10^5 cm/sec. This drift velocity corresponds to an electron drift energy of about 10^{-5} eV, which is negligible, and an ion drift energy of about 0.6 eV, which is much larger than the

[1] For our purpose the magnetic field can be regarded as constant, independent of the height.

gaskinetic energi. It is noteworthy that this drift velocity is of the same order of magnitude as those found from radar investigations of auroral ionization (cf. KAISER 1958).

It is beyond the scope of this paper to discuss the electric field and current, but we may investigate whether the ions can contribute significantly to the excitation of aurorae. At the height of maximum conductivity the energy gained from the electric field, per ion per sec, is $\sigma_1 E^2/n$, or about 100 eV/sec. The collision frequency ν is then equal to the gyrofrequency ω, or about 170 per sec for molecular ions. In general, the average energy gained between two collisions is close to the drift energy 0.6 eV where ν is equal to or less than ω. Where ν exceeds ω the energy gain is limited by the mean free path λ. The total energy absorbed per sec is of course proportional to σ_1/n (cf. ALFVEN 1950). Since the ions loose, on the average, about half of their energy in each elastic collision the average energy of the ions cannot exceed the average energy gain between two collisions very much.

The estimates made here are, of course, very crude, and we cannot exclude that the average energy of the ions is an order of magnitude greater or smaller. With an ion density of $2 \times 10^5/\text{cm}^3$ the number of collisions between ions and oxygen atoms is of the order $10^7/\text{cm}^3$ sec where σ_1/n has its maximum, whereas the rate of emission of the green [OI] line is about $3 \times 10^3/\text{cm}^3$ sec in aurorae of brightness II (cf. OMHOLT 1959b). An excitation efficiency of 10^{-4} per collision would make ion excitation important. It seems rather unlikely that ions can be so efficient in exciting the lower, metastable levels of the main atmospheric constituents, since this involves transitions that are optically forbidden. The rate of vibrational and rotational excitation of molecules may be considerable, and the ions themselves may be excited from their ground levels or metastable levels in such collisions.

Recently HUNTEN (1958) has pointed out that the excitation of the Meinel bands is probably more complex than the simple electron or proton ionization of N_2. It is perhaps possible that the accelleration of the N_2^+ ions can lead to excitation of the Meinel bands by the process

$$N_2^+ (X^2\Sigma) + M \rightarrow N_2^+ (A^2\Pi) + M \tag{2.7}$$

at an appreciable rate. The height distribution of the Meinel bands may also be consistent with this (OMHOLT 1957a).

The accelleration of negative ions may be an effective way of destroying these by collisions, and the accelleration of positive atomic ions may increase the rate of charge-exchange to O_2 or atom interchange processes (cf. section 2.4). Further, the currents may be active in the re-distribution of the ionized matter that apparantly takes place during magnetic storms.

c. Other excitation processes. A number of other, indirect excitation mechanisms for the [OI] lines have been proposed by various workers. Reviews of some of these processes have been given by SWINGS (1948) and BATES, MASSEY and PEARSE (1948).

The only processes that Bates *et al.* found possible to be of any importance were

(2.8) $$N_2\,(A^2\Sigma) + O(^3P) \rightarrow N_2\,(X^1\Sigma) + O(^1S)$$

proposed by Vegard (1933),

(2.9) $$N_2^+ + O^- \rightarrow N_2\,(B^3\Pi) + O(^1S),$$

proposed by Mitra (1946) and Gosh (1946), and

(2.10 *P, D, S*) $$O_2^+ + e \rightarrow O' + O'' \text{ (terms } ^3P,\ ^1D,\ ^1S),$$

proposed by them. The last process is probably the one responsible for the nightglow emission of the red lines whereas the green line in the nightglow may be excited by

(2.11) $$O + O + O \rightarrow O_2 + O(^1S)$$

(cf. Dalgarno 1958). This last process will not be important in aurorae as the dissociation of oxygen is not likely to increase appreciably during aurorae.

If the processes (2.9) and (2.10) are important, there will be a time delay of at least one sec between the ionization and the excitation of the oxygen lines. The apparent recombination coefficient during aurorae is 10^{-6} cm^3/sec or less and the electron density is less than 10^6/cm^3 (cf. Omholt 1959b). Thus the mean lifetime of the free electrons is at least one sec, and the processes (2.9) and (2.10) take place on the average at least one sec after the ionization. The lifetime of O^- adds to the delay of the process (2.9). Process (2.8) will be delayed relative to the excitation of the $N_2(A^2\Sigma)$ level by an amount corresponding to the effective lifetime of the metastable N_2-molecules. This lifetime is not known, but it may be appreciable. To reduce the lifetime towards collisions to 0.1 sec throughout an ordinary aurora requires that the probability of deactivation is of the order 0.01—0.1 per collision, a rather high value. If any of the processes (2.8), (2.9) or (2.10) contribute appreciably to the excitation of the green [OI] line there should thus be a detectable increase in the time delay between the emission of the N_2^+ bands and this line.

It is, however, less likely that the rates of the two first processes are sufficient to make them important. Thus Bates does not mention them in a later review paper (Bates 1955a). If process (2.10) is the one that is responsible for the red doublet in the nightglow it should also be important in aurorae. The rate of excitation should be roughly proportional to the square of the electron density, which increases rather much during aurorae. Direct comparison with the nightglow emission is difficult because this probably takes place well above the height of the main auroral luminosity (cf. Seaton 1958). If effective charge transfer from O^+ to O_2 is dominating in removing O^+ ions (cf. Bates 1956), about 20 % of the free electrons may disappear through the processes (2.10 *P, D, S*). The total rate of these processes will then be about 20 % of the total rate of ionization or about 5 times the actual rate of emission of the green [OI] line (cf. Omholt 1959a, sec. 7 and this paper, sec. 6, compared with Fig. 1). We do not know the specific rates of the individual processes (2.10 D) and (2.10 S), but they could be of importance.

Dissociation of O_2 may also produce $O(^1S)$ and $O(^1D)$ atoms by

(2.12) $$O_2\,(X^3\Sigma) + X \rightarrow O_2\,(B^3\Sigma) + X \rightarrow O\,(^3P) + O\,(^1D) + X,$$

and

(2.13) $$O_2\,(X^3\Sigma) + X \rightarrow O\,(^3P) + O\,(^1S) + X$$

(cf. BATES 1955a). The particle X may be the primary particles but also the secondary electrons have sufficient energy, as only about 10 eV is required. Absorption of ultraviolet auroral radiation by

(2.14) $$O_2\,(X^3\Sigma) + h\nu \rightarrow O_2\,(\Upsilon) \rightarrow O' + O'' \text{ (terms } P,\ ^1D,\ ^1S)$$

should also be considered, but ultraviolet emissions from N_2 would probably result in ionization of O_2 rather than dissociation (cf. LEE 1955 compared with the energy level diagram for N_2, HERZBERG 1950).

The time delay between the primary ionization and the processes (2.12) and 2(.13) would be negligible. This will also be the case for process (2.14) unless ultraviolet radiation from the metastable levels or recombination processes contribute appreciably to this process. The rates of these processes are difficult to estimate, and this deserves further investigation, but evidently the relative importance of these processes increase strongly with decreasing height in the O_2 — O transition region.

We may expect that an appreciable fraction of the O^+ ions formed are in the metastable ^2P or ^2D terms rather than the ground term ^4S. By charge transfer processes the metastable ions may be transformed to O atoms in the terms ^1S and ^1D as well as the ground term ^3P. The only ones of such processes involving O_2, N_2 or N that can produce $O(^1S)$ atoms are

(2.15) $$O^+(^2P) + O_2\,(X^3\Sigma) \rightarrow O\,(^1S) + O_2^+(^2\Pi) + 2.2 \text{ eV},$$

and

(2.16) $$O^+\,(^2D) + O_2\,(X^3\Sigma) \rightarrow O\,(^1S) + O_2^+\,(^2\Pi) + 0.5 \text{ eV}.$$

The latter of the two processes involves $\triangle L = 2$ for the oxygen atom and thus a re-arrangement of the bound O^+ electrons, and the process may therefore be less effective than the first one.

There are a number of similar processes, deactivating the $O^+(^2P)$ and $O^+(^2D)$ atoms by charge transfer to O_2, N_2 or N, which can lead to formation of $O(^1D)$ atoms, but the greatest number of possible processes lead to formation of atoms in the ground term.

BATES (1955b) believes that atom interchange processes (i.e. processes in which one of the atoms in the molecule is interchanged with the atomic ion) are much more likely to occur than pure charge transfer processes. Formally, such processes involving O^+ and O_2 can be written similarly to equations (2.15) and (2.16).

It is difficult to estimate the rate of formation of $O(^1S)$ and $O(^1D)$ atoms by such processes, but we believe that only a small fraction of all O^+ ions are transformed to $O(^1D)$ atoms and a vary small fraction to $O(^1S)$ atoms. The rate of formation of O^+ ions (including ions in the ground term) is probably of the order 0.2 times the total rate of ionization, which again is about 25 times the rate of emission of the $\lambda 5577$ line (cf. OMHOLT 1959a, sec. 7 and this paper, sec. 6, compared with Fig. 1). On the other hand, if O^+ ions are formed in the metastable terms in appreciable amounts, the weakness of the corresponding emissions (cf. CHAMBERLAIN and MEINEL 1954, OMHOLT 1957a) show that collisional deactivation is important. It is thus not unlikely that the rate of formation of $O(^1D)$ atoms by the discussed processes is of the same order of magnitude as the total rate of formation of $O(^1S)$ atoms, whereas probably only a minor part of the $O(^1S)$ atoms are formed by these processes.

The time delay between the ionization process and the charge transfer or atom interchange processes is also difficult to estimate. The lifetimes of the metastable ions towards radiation are about 5 sec (^2P) and 3 hour (^2D) (GARSTANG 1956). According to BATES (1955b) the rate coefficient for the atom interchange process

$$O^+ + O_2 \rightarrow O_2^+ + O \qquad (2.17)$$

may be as high as 10^{-10} cm^3/sec. With this rate coefficient the lifetime of O^+ is 10^{-2} to 1 sec in the main auroral region ($n(O_2) = 10^{10}$ to 10^{12} /cm^3). The absence of the line 7325 Å (^2D—^2P) in most auroral spectra (cf. OMHOLT 1957a) may indicate that the rate coefficient for process (2.17) with $O^+(^2P)$ ions actually is of the order 10^{-10} to 10^{-11} cm^3/sec.

3. Deactivation mechanisms.

a. Deactivation of $O(^1S)$ *atoms.* The earlier work by OMHOLT and HARANG (1955) and OMHOLT (1956) indicated that collisional deactivation of the excited $O(^1S)$ atoms is of importance in the lower ionosphere. Deactivation by

$$O\,(^1S) + O_2\,(X) \rightarrow O\,(^1D \text{ or } ^3P) + O_2\,(Y) \qquad (3.1)$$

has been studied in the laboratory by KVIFTE and VEGARD (1947). They found γ, the probability of process (3.1) per gaskinetic collision between the two particles involved, to be about 3×10^{-6}. At an altitude of 100—105 km the appropriate collision frequency is only of the order 10^2/sec, so that from KVIFTE and VEGARD's result it follows that the probability of deactivation by process (3.1) is less than 10^{-3}/sec. at this height and above.

SEATON (1958) proposed the mechanism

$$O\,(^1S) + N_2\,(X^2\Sigma, v''{=}0) \rightarrow O\,(^1D) + N_2\,(X^2\Sigma,\ v' \neq 0). \qquad (3.2)$$

It may be worth while to note that with $v' = 8$ this process is in almost exact energy resonance, although it is doubtful whether this is of any importance. Attempts by

VEGARD and KVIFTE assisted by the author (unpublished) to obtain the green [OI] line in discharges through O_2 — N_2 mixtures failed to give results. This many indicate that the rate coefficient for process (3.2) is much higher than that for process (3.1).

It does not appear likely that other processes are important for the deactivation of the $O(^1S)$ atoms (cf. SEATON 1958).

b. Deactivation of $O(^1D)$ *atoms.* As has been pointed out by SEATON (1954, 1958) it seems not possible to explain the altitude distribution of the red [OI] doublet without invoking collisional deactivation of the $O(^1D)$ atoms. This may occur through

$$O\,(^1D) + O_2\,(X^3\Sigma,\ v'' = O) \rightarrow O\,(^3P_2) + O_2\,(b^1\Sigma,\ v' = 2), \tag{3.3}$$

which is in almost exact energy reasonance (SEATON 1954, BATES and DALGARNO 1954). SEATON (1958) has discussed this process in some detail, considering also the effect of the reverse process. The effective deactivation probability by process (3.3) together with other collision processes is

$$d_2 = \beta\, n\, (O_2)\, \frac{A'}{A' + \beta' n\, (O)} + \beta_M\, n\, (M), \tag{3.4}$$

where $n(X)$ is the density of the particles denoted by X, $\beta n(O_2)$ and $\beta_M n(M)$ the probabilities that an $O(^1D)$ atom will be deactivated by respectively process (3.3) and by collisions with other molecules, $\beta'\, n(O)$ the probability that an excited O_2 molecule will be deactivated by the reverse process, and A' the probability that the same molecule will radiate its energy (SEATON 1958). SEATON shows that $(\beta'/\beta) = 5/3$. We let A' include the probability of additional collisional deactivation of the O_2 molecules, which makes equation (3.4) more general. SEATON did not take such processes into account.

By choosing suitable values for the rate coefficients involved, SEATON was able to reproduce the variation of the intensity ratio $I(6300)/\,I(5577)$, which he derived from various observations. Below 140 km $\beta' n(O)$ will be much higher than A', which means that process (3.3) and the reverse process proceed approximately at the same rate and much faster than the spontaneous radiation or other processes included in A'. If A_{2-2} is the transition probability of the (2—2) Atmospheric band (originating from the O_2 $(b^1\Sigma)$ state), A_{21} that for the red [OI] doublet, $I(2—2)$ and $I(6300)$ the intensities of the same emissions, then we have that for $\beta' n(O) \gg A'$:

$$\frac{I\,(2-2)}{I\,(6300)} \approx \frac{A_{2-2}\,\beta\, n\,(O_2)}{A_{21}\,\beta'\, n\,(O)} \cdot \frac{63}{77}\,. \tag{3.5}$$

We have heie neglected other modes of excitation of the $b^1\Sigma$ state. The (2—2) band carries about 2/3 of the transitions from the $v' = 2$ level (NICHOLLS 1956) and the electronic transition probability for these bands is about 0.12/sec. (cf. SEATON 1958), and we get

$$\frac{I\,(2-2)}{I\,(6300)} \approx 5\, \frac{n(O_2)}{n\,(O)}\,. \tag{3.6}$$

With the densities adopted by SEATON we obtain the result that the intensity of the (2—2) band is about half of that of the red doublet. From the observations it appears that the intensity of the (2—2) band is actually only a few per cent of that of the red doublet (cf. CHAMBERLAIN, FAN and MEINEL 1954 compared with OMHOLT 1957a). The observational results thus indicate that, contrary to SEATON's assumption, $A' \gg \beta' n(O)$ in most parts of the aurorae, in which case

$$d_2 = \beta\, n\,(O_2) + \beta_M\, n\,(M). \tag{3.4a}$$

If process (3.3) shall be effective at all, β cannot be much smaller than the value 2×10^{-11} cm³/sec assumed by SEATON (1958).

In an earlier paper (OMHOLT 1957a) it was concluded from relative intensities that if only the Atmospheric bands are emitted by the $O_2(b^1\Sigma)$-molecules and if redistribution of the pure vibrational energy ($v' = 2 \rightarrow v' = 1$ or 0) through collisions is effective (which would increase A'), it is doubtful, but not impossible that the $O(^1D)$ atoms are deactivated through process (3.3). Deactivation of the vibrational energy might increase A' sufficiently to make equation (3.4a) valid, but the probability of such collisions is generally very low (cf. MASSEY and BURHOP 1952) so that it is doubtful whether such deactivation can take place at a sufficient rate. Other kinds of collisional deactivation of the $O^2(b'\Sigma)$ molecules might occur, the $v' = 0$ and 1 levels being excited by secondary electrons rather than by deactivation from the $v' = 2$ level. In this case we would expect an altitude distribution of the Atmospheric $(1 - v'')$ and $(0 - v'')$ bands above the $O - O_2$ transition region similar to the red doublet. This seems again not to be the case (OMHOLT 1957a) but the fluorescent scattering of the emission (cf. CHAMBERLAIN 1954) and the uncertainties in the densities and rate coefficients involved makes the interpretation uncertain.

The most simple solution is that either is the transition probability for

$$O_2\,(b^1\Sigma) \rightarrow O_2\,(a^1\Delta) + h\nu \tag{3.7}$$

(OMHOLT 1957a) much higher than assumed by SEATON (about 1/sec) or is the probability of deactivation of the $O(^1D)$ atoms by other collision processes than (3.3) sufficiently high. The (2—2) band of the transition (3.7) (which apparently would be the most intense one) lies close to 20 000 Å, a region in the auroral spectrum is not yet well explored.

KVIFTE and VEGARD (1947) did not determine the probability of deactivation of O (^1D) atoms, but state that in pure oxygen gas the intensity ratio $I(6300)/I(5577)$ is independent of the pressure and the tube size. Following their own interpretation of the experiments, this indicates that also for the $O(^1D)$ atoms it was chiefly collisions which determined the lifetime, as was presumably the case for the $O(^1S)$ atoms. From this follows only that the probability of collisional deactivation for the $O(^1D)$ atoms in the experiments must have been much higher than the probability of spontaneous emissions, whereas SEATON (1954, 1958) seems to conclude that it was the same as for

$O(^1S)$. KVIFTE and VEGARD's work does not contradict a much higher effective probability of deactivation for $O(^1D)$ atoms by collisions with O_2.

SEATON (1958) concludes from the available observations that either must the relative rates of excitation of the ^1D and ^1S terms vary suitably with height or must the deactivation probability for the $O(^1D)$ atoms decrease less with height than does the density of the main atmosphere. Equation (3.4a), with constant β's, leads to a deactivation probability that decreases faster with height than does the density, but we may not exclude that β increases with the temperature, and thus with the height in the atmosphere.

4. Rapid fluctuations in the intensity of the [OI] lines. Aurorae may frequently vary very rapidly in intensity. As demonstrated earlier (OMHOLT and HARANG 1955, OMHOLT 1956) this may be a powerful tool for studying the effective lifetime of the oxygen atoms in the ^1D and ^1S terms, and the excitation mechanism. Such measurements may, as also earlier proposed by SWINGS (1948), give information about the possible collisional deactivation of the $O(^1D)$ and $O(^1S)$ atoms in the upper atmosphere.

If N_n is the number of atoms in the term in question (^1D or ^1S), then

$$\frac{dN_n}{dt} = Q_n - (\sum_m A_{nm} + d_n)\, N_n = Q_n - \frac{1}{\tau_n} N_n, \tag{4.1}$$

where

$$\frac{1}{\tau_n} = \sum_m A_{nm} + d_n \tag{4.2}$$

Here the A_{nm}'s are the spontaneous transition probabilities to lower levels, d_n the probability that an excited atom suffers deactivation by collisions, τ_n the mean lifetime of an excited atom, and Q_n the rate of excitation, including cascade from higher levels. Since the intensity of a particular line with frequency ν_{nj} is

$$I_{nj} = h\, \nu_{nj}\, A_{nj}\, N_n, \tag{4.3}$$

we get from eq. (4.1)

$$I_{nj} + \tau_n \frac{dI_{nj}}{dt} = h\nu_{nj}\, \tau_n\, A_{nj}\, Q_n. \tag{4.4}$$

The simplest assumption we can make regarding Q_n is that it is proportional to the rate of excitation of the First Negative N_2^+ bands with $v' = 0$, Q_N. Since these bands arise from permitted transitions, the intensity I_{Ni} of a particular band is obviously equal to $k_i\, Q_N$, where k_i is a constant, and we obtain

$$I_O + \tau \frac{dI_O}{dt} = K\, I_N, \tag{4.5}$$

where K is a constant given by

$$K = h\, \nu_{nj}\, A_{nj}\, \tau_n\, Q_n\, / k_i\, Q_N, \tag{4.6}$$

and the indices nj and Ni are replaced by O and N.

It is evident that we, under the given assumptions, can compute τ from eq. (4.5), provided dI_O/dt is large enough compared to I_O and I_N.

The assumption that K is a constant seems not unreasonable if the excitation of the [OI] lines is due to secondary electrons and we consider a very small volume element in the atmosphere (cf. sec. 2a). In the case of local discharges in the atmosphere it is rather difficult to predict the variation of Q_n/Q_N with time, and if some of the mechanisms discussed in sec. 2c are operative there will almost certainly be an appreciable time delay between Q_N and Q_n.

Complications arise also because we are not able to isolate the emission from a small volume element in the atmosphere. A photometer necessarily records the light from a narrow, infinitely long cone, the center of which lies along the optical axis of the photometer. τ as well as Q_n/Q_N may vary with the height in the atmosphere. It is likely that particularly for the red doublet there are rather large variations in K (and τ) with the height (cf. HARANG 1958).

Thus the values of τ derived by applying eq. (4.5) on the records may be influenced by the time delay between the two excitation rates Q_N and Q_n as well as other factors. During the analysis we shall regard eq. (4.5) as defining two quantities τ and K, namely those which makes eq. (4.5) to fit the record as good as possible. We shall return to the interpretation of τ in sec.s 8 and 9.

5. Observations. The new observations from Tromsö reported in this paper were all made with the photometer described earlier (OMHOLT 1959a). Two series of measurements were made in Tromsö. The first series was made in October and November 1957. This consisted of measurements of rapidly fluctuating aurorae, and the intensities of the [OI] line λ5577 and the First Negative N_2^+ band λ4278 (0—1) were recorded while the photometer was kept in a fixed position. Ordinary metallic interference filters were used, having half-widths about 100 Å. This seems rather much, but for these strong emissions no appreciable error is likely to be introduced by neighbouring bands or lines. No attempt was made to measure the intensities on an absolute scale.

The second series of measurements was made in March 1958, in order to establish the correct value for the intensity ratio between the [OI] line λ5577 and the First Negative N_2^+ bands. This series consisted of measurements of the green line and the λ4709 (0—2) band. Quiet auroral forms were measured by scanning through these with the photometer; moderately fluctuating aurorae were measured by keeping the photometer in a fixed direction and recording the intensity variations. In this series improved, multilayer filters were used. The filter for the λ4709 band is described earlier

(OMHOLT 1959a); the half width was about 60 Å. The filter for the [OI] line had a half width of about 90 Å, and its maximum transmission was at 5560 Å.

The absolute intensities were computed from measurements of a standard lamp as well as the star Capella (*α Aur*) (cf. OMHOLT 1959a). The two sources gave very nearly the same result. Capella gave 13 % higher value for the intensity ratio $I(5577)/I(4709)$ and 25 % lower value for the absolute intensity of the λ4709 band than did the standard lamp. The values derived from the star measurements were adopted, as the star may serve as a standard for other workers as well. In no cases were auroral observations made closer than 10° to the horizon.

In this paper we shall also include some observations of flashing aurorae made at Yerkes Observatory of the University of Chicago, Wisconsin, in March 1957. The photometer has been briefly described elsewhere (OMHOLT 1957b) and was in principle similar to that used in Tromsö. The intensities of the green [OI] line and the λ3914 band of N_2^+ were recorded, and the filters had half-widths of about 100 Å. A Brush two-pen recorder was used, this responds to frequencies up to 100 c/sec.

6. The equilibrium value of $I(5577)/I(4709)$. We shall first deal with the second series of measurements in Tromsö, namely that of the green [OI] line and the λ4709 N_2^+ band, since the result of these observations are important for the interpretation of the observation of the rapid intensity fluctuations.

In the cases of slowly varying aurorae the intensities were measured where the green line had its maximum, i.e. dI_o/dt was zero. In the case of scans through an aurora, the intensities were measured at the maximum points. In the cases of vertical scans through arcs and bands, the intensities were also measured in the upper part of the aurora, approximately where the intensities were half of the maximum value. Rays were scanned horizontally at various points, but usually the time permitted only one or two scans of each ray. In all cases the intensity of the background was subtracted, so that the given values are those for aurorae only, not including the night sky emission. Inhomogeneities in the night sky emission may cause some small, random errors.

It was found convenient to divide the observations into three groups according to the type of aurora:

a. Diffuse and patchy aurorae, glow and pulsating aurorae.
b. Upper and middle part of rays.
c. All other aurorae, including arcs, bands and bottom of rays.

The results are displayed in Fig. 2, where the intensity ratio $I(5577)/I(4709)$ is plotted against $I(4709)$. The unit for $I(4709)$ is 10^9 λ4709-quanta emitted per cm² (column) of the aurora per sec, or 1 KR., whereas the ratio $I(5577)/I(4709)$ is the energy ratio.

It will be seen that within group (c) there is a slight, systematic variation in the intensity ratio with $I(4709)$. Within the two other groups the points are somewhat more scattered. It does not seem to be any systematic variation in the intensity ratio, but the ratio is definitely lower than that for aurorae of the same intensity in group (c).

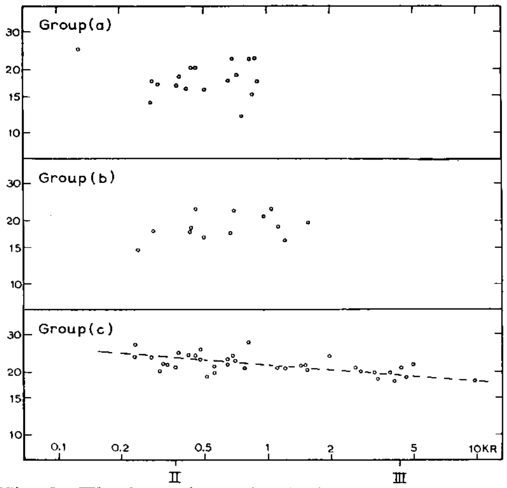

Fig. 2. The intensity ratio $I(5577)/I(4709)$ against $I(4709)$ in KR. $I(4709)$ for aurorae with international brightness coefficients II and III is indicated on the figure. Group (a), (b) and (c), cf. the text. The point at $I(4709) = 10$ KR is for type B aurora.

We shall postpone further discussion of these results to sec. 9, except for noting the fact that $I(5577)/I(4709)$ in almost all cases lies within $\pm$ 25 % from 20. This is somewhat encouraging for the application of eq. (4.5), on the records of rapidly fluctuating aurorae, as it indicates that K in this equation does not vary excessively.

From the first series of observations, which were made in October—November 1957, K_c, i.e. the intensity ratio $I(5577)/I(4278)$ relative to a fixed scale factor, was computed and plotted against $I(4278)$. This material consists mainly of aurorae belonging to group (c) above, and showed no significant variation of K_c with $I(4278)$. The scatter of the measured points is considerable, but 63 % of the 135 values lie within $\pm$ 15 % from the average value. A fairly large number of corrections for variations in the sensitivety of the photometer had to be made, and the extinction correction is considerably greater for the λ4278-band than for the λ4709-band. The results are therefore much less reliable than those obtained during the second series of measurements. These second series was made particularly to avoid much of the difficulties encountered during the analysis of the first series. As we shall see later, the scale value of K_c does not affect the computation of the lifetime τ.

7. Methods for analysing records of rapidly varying aurorae.

a. Basic equations. Eq. (4.5) is rather simple, and is based on relatively simple assumptions about the excitation processes. Our first step in examining the records shall be to see if this equation is applicable to the actual records. If the equation, taking K and τ to be constants, gave an exact relation between the two intensities, the analysis would indeed be very easy. However, we cannot expect that K and τ are true constants, they may vary somewhat during one record. Also random errors in the measurements introduce errors. Because of the statistical nature of the photocurrent the recorded intensities contain a noise component. It would therefore be impossible to measure the intensities, and particularly the derivative, at any particular moment without smoothing the curve. Because of this and of the considerable work involved in the nummerical analysis, the best approach was found to be to integrate the intensities and the derivative over certain small time intervals. Thus the total time of the record to be analysed was divided into a finite number of shorter intervals, the integration being performed over each interval.

Considering eq. (4.5) this leaves us with a finite number of equations

$$\mathcal{J}_{Op} + \tau\, \mathcal{J}'_{Op} = K_c\, \mathcal{J}_{Np} \tag{7.1p}$$

where $K_c = Kc_O/c_N$, $\mathcal{J}_{Op}$, $\mathcal{J}'_{Op}$ and $\mathcal{J}_{Np}$ are the integrals of $c_O I_O$, $c_O dI_O/dt$, and $c_N\, I_N$ over the time interval t_p to t_{p+1}. c_O and c_N are the scale factors for I_O and I_N on the records (cf. also OMHOLT and HARANG 1955). The greatest random errors in the measurements are now undoubtedly in $\mathcal{J}'_O$, and the intensity curves were slightly smooted before $\mathcal{J}'_{Op}$ was measured. $\mathcal{J}'_{Op}$ is of course the difference between $c_O I_O(t_{p+1})$ and $c_O I_O(t_p)$.

We are now faced with the problem of solving a number of equations of the form (7.1) to find the two unknown K_c and τ. Since the number of equations usually exceeds two, we have to choose a solution which satisfies all the equations as good as possible. How good this fit is gives us also an indication about how well eq. (4.5) represents the actual case.

b. The graphic method. The method used earlier (OMHOLT and HARANG 1955, OMHOLT 1956) was a very simple graphic method. In an orthogonal K_c, τ coordinate system eq. (7.1) represents a straight line. For each time interval we obtain a set of values $\mathcal{J}_{Op}$, $\mathcal{J}'_{Op}$ and $\mathcal{J}_{Np}$ and a straight line in the K_c, τ coordinate system. If eqs. (7.1p) were exact, all the lines should cross in one point, giving the true values of K_c and τ. Actually all the lines never crossed in one point, but the "best" values of K_c and τ were selected by subjective judgement. Also the uncertainty in τ was estimated from the graph.

This method is simple and fast, and one gets a certain impression of the accuracy of the results. However, the results may to a certain degree depend on the person who undertakes the analysis and his unconscious wishes. The estimated uncertainties in τ and K_c are subject to this criticism to an even higher degree.

c. A numeric, linear method. If we consider the sum

$$\sum_p \mathcal{J}_{Op} + \tau \sum_p \mathcal{J}'_{Op} = K_c \sum_p \mathcal{J}_{Np}, \tag{7.2}$$

we usually find that $\sum_p \mathcal{J}'_{Op}$ is very small, as we usually analyse records where the values of I_O at the beginning and end are very close. We may therefore with negligible error adopt a preliminary value of τ, called τ', and compute K_c from eq. (7.2). If we adopt this value of K_c we may compute a set of τ_p's from the individual eq.s (7.1*p*), or rewritten:

$$\tau_p = (K_c\, \mathcal{J}_{Np} - \mathcal{J}_{Op}) / \mathcal{J}'_{Op}. \tag{7.3p}$$

It seems now reasonable to take $|\mathcal{J}'_{Op}|$ as a weight factor in computing the average value of τ, as the accuracy obviously depends on the numeric values of the numerator and denominator in eq. (7.3). Since τ_p is always positive unless great errors or deviations occur, we can usually put

$$\tau = \frac{\Sigma\, \tau |\mathcal{J}'_{Op}|}{\Sigma\, |\mathcal{J}'_{Op}|} = \frac{\Sigma\, |K_c\, \mathcal{J}_{Np} - \mathcal{J}_{Op}|}{\Sigma\, |\mathcal{J}'_{Op}|} \tag{7.4}$$

If τ deviates so much from τ' that it effects the value of K_c derived from equation (7.2) a new approximation has to be made, but this happens very seldom.

The results obtained by this method are very close to those obtained by adding all eq.s (7.1p) with positive values of J'_{Op} and all with negative values of J'_{Op}. This gives us two equations from which K_c and τ can be found. The method described here is slightly faster when we also want to evaluate the individual τ_p's to study the variations during one record. It is of course also possible to study the apparent variations in K_c by adopting a fixed value of τ and to to compute individual K_{cp}'s from eq. (7.1).

d. The method of least squares. The method most commonly applied when experimental errors or random variations occur, is the method of least squares. This method may be applied to our problem in the following way: If τ_0 and K_0 are the sought values of τ and K_c, then

$$J_{Op} + \tau_0 J'_{Op} - K_0 J_{Np} = q_{Op} T_p, \tag{7.5p}$$

where $T_p = t_{p+1} - t_p$ and q_{Op} is a small value, being the average (during the time interval t_p to t_{p+1}) of the difference between J_{Op} and the value of J_{Op} derived from eq. (7.1p) when J_{Np} and J'_{Op} are inserted. We may now seek the minimum value of

$$S = \sum_p q^2_{Op} T_p, \tag{7.6}$$

or the values of τ_o and K_o which give

$$\frac{\partial S}{\partial K_0} = O \text{ and } \frac{\partial S}{\partial \tau_0} = O. \tag{7.7 a, b}$$

The resulting equations are

$$K_0 \sum_p J^2_{Np}/T_p - \tau_0 \sum_p J_{Np} J'_{Op}/T_p - \sum_p J_{Np} J_{Op}/T_p = O, \tag{7.8}$$

and

$$K_0 \sum_p J_{Np} J'_{Op}/T_p - \tau_0 \sum_p J'^2_{Op}/T_p - \sum_p J_{Op} J'_{Op}/T_p = O, \tag{7.9}$$

from which K_0 and τ_0 can be found.

There is no particular reason for using the sum of $q^2_{Op} T_p$ in eq.s (7.7 a, b). We might as well have replaced q_{Op} by q_{Np}, the average, during the time interval T_p, of the difference between J_{Np} and the value of J_{Np} derived from eq. (7.1p) when J_{Op} and J'_{Op} are inserted. q_{Np} is given by

$$\frac{1}{K_0} J_{Op} + \frac{\tau_0}{K_0} J'_{Op} - J_{Np} = q_{Np} T_p, \tag{7.10}$$

and we obtain eq. (7.9) together with

$$K_0 \sum_p J_{Op} J_{Op}/T_p - \tau_0 \sum_p J'_{Op} J_{Op}/T_p - \sum_p J^2_{Op}/T_p = O. \tag{7.11}$$

An equation of the same form as (7.9) is obtained from equations (7.3p) by taking $\mathcal{J}'^2_{Op}/T_p$ as weight factor rather than $|\mathcal{J}'_{Op}|$ in computing the average value of the τ_p's computed from eq.s (7.3p).

In the work reported in this paper we have arbitrarily used eq.s (7.8) and (7.9). A few examples showed that the values of K_0 and τ_0 obtained from these equations did not differ significantly from those obtained from equations (7.9) and (7.11).

A more general discussion of the application of the method of least squares is given in the appendix.

e. The delay method. When a record consists of a single "pulse" of light, such as may frequently be the case when pulsating and flaming aurorae occur, a fourth method may be used. We let I_O and I_N in equation (4.5) represent the intensities above the constant "background". For an accepted pulse I_O is zero at the beginning and end of the record. From eq. (4.5) follows that in this case

$$\int_1^2 I_O \, dt = K \int_1^2 I_N \, dt, \tag{7.13}$$

and

$$\int_1^2 t\, I_O dt = K \int_1^2 t\, I_N \, dt - \tau \int_1^2 t \frac{dI_O}{dt} dt = K \int_1^2 t\, I_N \, dt + \tau \int_1^2 I_O \, dt, \tag{7.14}$$

since $I_{O2} = I_{O1} = O$.

From equations (7.13) and (7.14) it is not difficult to show that

$$\tau = \frac{\int_1^2 t\, I_O \, dt}{\int_1^2 I_O \, dt} - \frac{\int_1^2 t\, I_N \, dt}{\int_1^2 I_N \, dt}. \tag{7.15}$$

If I_O is not exactly zero at the beginning and end of the record, but very small, correction terms must be introduced. In the cases were I_O is zero at the beginning and equal to I_f at the end, one obtains, in the first approximation, a correction factor to τ, as given by eq. (7.15), which is

$$f = \left\{ 1 + (T \div t_N)\, I_f \Big/ \int_1^2 I_O \, dt \right\}, \tag{7.18}$$

where

$$t_N = \int_1^2 t\, I_N \, dt \Big/ \int_1^2 I_N \, dt,$$

T is the duration of the record and t is reckoned from the beginning of the record.

8. Observations of rapid fluctuations in I (5577) and I (N_2^+).

a. Re-examination of the earlier records. In an earlier paper (OMHOLT 1958) the result obtained from 62 records of the green [OI] line and the N_2^+ band λ4278 was given. The records were analysed by the graphic method described in sec. *7b* and 71 values of τ and K_c were derived. The distribution of the measured values of τ was asymmetric and showed a peak close to 0.75 sec. The mean value of τ was 0.64 sec. The peak in the distribution agrees very well with the theoretical value of 0.73 sec computed by GARSTANG (1951) for the life time of the undisturbed, excited atom. The distribution suggests that the theoretical lifetime is fairly correct and that collisional deactivation of the excited atoms frequently occurs.

It is, however, somewhat doubtful whether the distribution itself is trustworthy. The variation of K_c (equal to K_1/τ multiplied by τ, cf. Fig. 3, OMHOLT 1956) with τ is very small. Since K_c is proportional to τ this requires Q_n/Q_N (cf. equation (4.6)) to vary in inverse proportion to τ. This may be so by accident, but it may be tempting to suspect the variation in τ to be due to random errors in the records or random deviations from the theoretical equation (4.5). In fact, if we at will change only 8 of the measured values of τ (2 at 0.70 sec and 6 at 0.75 sec) we may obtain a fairly symmetric distribution around a new mean value of 0.62 sec. The average change in τ is only 0.16 sec for the 8 values. When in addition one considers the fact that the theoretical value of 0.73 sec was known during the analysis, the final stage of which is rather subjective, one must admit that the former conclusion is somewhat suspect.

Although the records in this series are less accurate than those which have been obtained later and which will be described in the following section, it was thought worth while to recompute τ by the linear numeric method described in sec. *7c*. The resulting distribution is shown in Fig. 3. It is seen that the distribution of the new values is more symmetric around the mean value, which is 0.63 sec. The distribution is fairly broad, the mean square deviation ($\sigma = (\Sigma \triangle \tau^2/n)^{1/2}$) being 0.21 sec. The correlation coefficient between the new and the old values of τ's is 0.64. The records are not good enough to exclude that the distribution is due to random or systematic errors, and it was not found worth while to make further computations on the basis of these records. It is noteworthy, however, that one very good record, the one that is shown in the paper by OMHOLT and HARANG (1955), exhibits a very pure afterglow phenomenon at the end, and that this part of the record fits eq. (4.5) very well, giving $\tau = 0.75$ sec. This value agrees very well with those derived by the other methods described in sec.s *7b*, *c* and *d* (0.75, 0.76 and 0.79 sec. respectively). A few of the records giving shorter life-times are quite good, and certainly the lifetime, as defined by eq. (4.5), do vary to some extent.

b. Flashing and flaming aurorae. A great number of records of flashing and flaming aurorae were obtained at Yerkes Observatory, but only a few were made with sufficient speed to permit an accurate and careful analysis (1″/sec). The records which are analysed in detail are all from a flashing and flaming aurora which occured during the

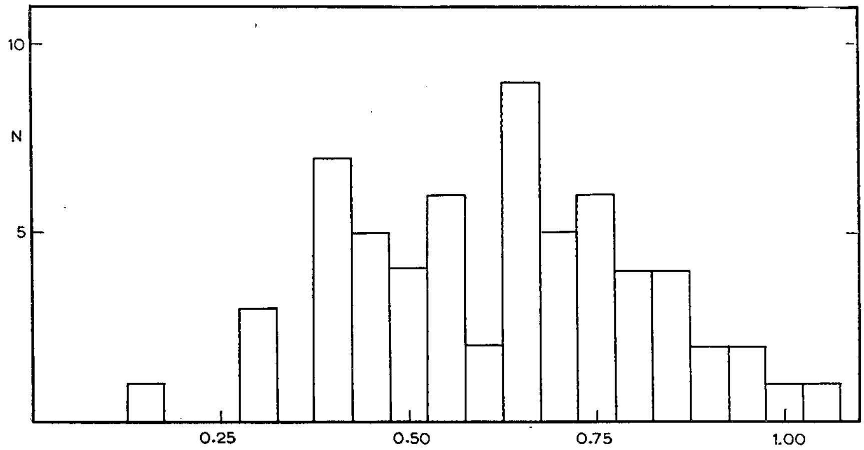

Fig. 3. The distribution of the recomputed values of τ from the old Tromsø records.

night of March 28/29 1957. The aurora almost filled the northern half of the sky and consisted of small, diffuse, raylike elements lasting for a second or less only. The appearance of the individual elements seemed quite incoherent and the sky gave an impression similar to that of boiling water. Occasionally waves of flaming aurora occured. A series of pulses recorded with relatively low recorder speed (0.2″/sec.) is shown in Fig. 4. Incidentally, it seems to the author that this type of aurora is relatively much more common far away from the auroral zone than close to it.

16 high-speed records were found suitable for analysis, and 8 were also suitable for the delay method described in sec. *7e*. These 8 records were analysed by all the methods described in sec. *7*, and the other ones by the linear numeric method and the

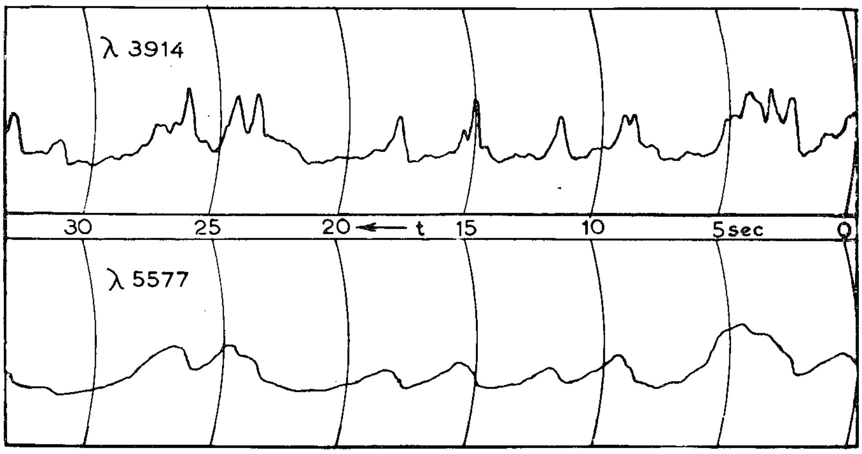

Fig. 4. Record of flashing aurora, Yerkes Observatory.

method of least squares (secs. *7c* and *7d*). The records were divided into intervals of about 0.5 sec. The measured values of τ are given in Table 1, together with K_c computed by the method of least squares. There is very good agreement between the various methods. The correlation coefficient between the τ's derived by the linear method and by the method of least squares is 0.95.

Table 1. *Flashing aurorae, Yerkes Observatory*

τ computed by the graphic method (τ_G), the linear numeric method (τ_1), the delay method (τ_D) and the method of least squares (τ_2). K_c is from the method of least squares. The mean square deviation is $\sigma = (\Sigma\, \Delta\tau^2/n)^{1/2}$. The unit for τ is 10^{-2} sec.

Rec. No.	τ_G	τ_1	τ_D	τ_2	Average	K_c
1	55	52	48	46	50	0.75
2	63	65	65	64	64	0.75
3	75	78	76	76	76	0.94
4	60	58	60	55	58	0.95
5	58	56	54	55	56	0.84
6	59	59	57	58	58	0.83
7	69	68	67	72	69	1.14
8	65	64	58	66	63	0.96
Average 1–8	63	63	60	62	62	
9		60		58		0.91
10		53		54		0.73
11		54		55		0.88
12		48		32		0.64
13		69		66		0.77
14		57		54		1.21
15		68		56		1.00
16		65		59		0.87
Average 1–16		61		58		
σ		10		06		

Although this high correlation coefficient at a first glance may suggest that the basic equation describes the phenomenon reasonably well and that there is a real variation in τ, a detailed study of the curves shows that they are not quite in harmony with eq. (4.5). In Fig. 5 is shown a typical one of the 16 records. This is a single pulse that gives a value of about 0.76 sec for τ (No 3, Table 1). By means of eq. (4.5) two curves for I_N were constructed from the curve for I_O, taking τ to be 0.7 and 0.8 sec respectively. It will be seen that both of the constructed curves for I_N show a peculiar deviation from the observed curve. They show a slight delay at the beginning and a higher and sharper maximum. This seems to be typical, as it is found to be so in most

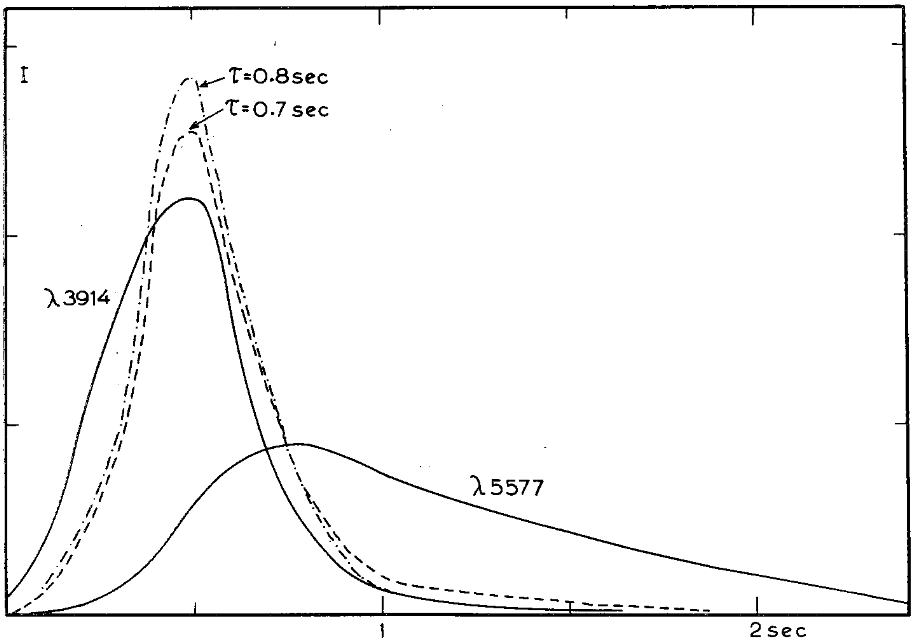

Fig. 5. High speed record of a "pulse" of flashing aurora. The intensities are normalised so that $K_c = 1$. The dashed curves for $I(3914)$ are constructed from the curve for $I(5577)$ by eq. (4.5), taking τ to be 0.7 and 0.8 sec. The computed values for τ_2 and τ_1 are 0.76 and 0.78 sec respectively.

of the cases analysed. If we keep K_c constant and compute τ along the curve, we find a corresponding systematic variation of τ.

Examination of the linear numeric method shows that in all cases of flashing aurorae τ is mainly determined by the part of the record where dI_O/dt is negative, because here I_N is very low. Therefore inaccuracies in K_c are not very important. If I_N was zero during this period, we should have a pure decay phenomenon from which τ could be determined by eq. (7.2).

It is difficult to see how the described effect can be due to systematic errors in the measurements. If the effect is real it seems as if the excitation mechanism is somewhat different from that on which eq. (4.5) is based. But any attempt to interpret this effect is bound to be quesswork.

Although there is quite good correlation between the various sets of τ's this does not necessarily imply that the individual τ's are correct. The value computed by the various methods are based on the same set of basic figures derived from the intensity curves. A change in any of these figures will tend to alter the computed value of τ in the same direction, regardless of the method used. The correlation therefore does not prove that the scatter in τ is significant.

As mentioned earlier K_c should be proportional to τ if the ratio between the excitation rates is constant (cf. eq. (4.6)). There is no significant correlation between τ and K_c for these records, but because of the scatter of the points in a τ, K_c diagram such a variation cannot be excluded.

Also a number of records made with lower speed (0.2″/sec) have been analysed. Although these are less accurate, the results may have a certain interest. They have been analysed by the linear numeric method only, as this seems adequate in this case. The average value of τ for 48 records is 0.67 sec. The distribution has no significant asymmetry and the mean square deviation of τ is 0.12 sec. There are no significant correlation between τ and K_c.

c. Pulsating aurorae. Among the observations in the first series from Tromsö are some quite good records of pulsating aurorae. A typical example of such a record is given in Fig. 6. The curves are redrawn from the photographic records, some of the high-frequency noise being smoothed. Altogether 27 pulses were found suitable for analysis, and these were analysed by the linear numeric method and by the method of least squares. 19 of the cases, group (a), are successive pulsations of the same multiple weak arc, about 45° to the south of zenith. The other 8 cases, group (b), are successive pulsations of patches with diffuse ray structure, about 30° to the south of zenith. In all cases the constant "background" of light was subtracted from the total intensity. The records were divided in intervals of one seconds duration $(t_{p+1} \div t_p)$. τ_2, computed by the method of least squares, is plotted against K_c in Fig. 7. The recorded intensities seem to follow equation (4.5) quite well, and there is good correlation between the τ's computed by the two methods.

The pulsations usually last some seconds and exhibit a flat top, from which K_c can be computed separately $((dI_0/dt) = 0)$. The values of K_c computed by the

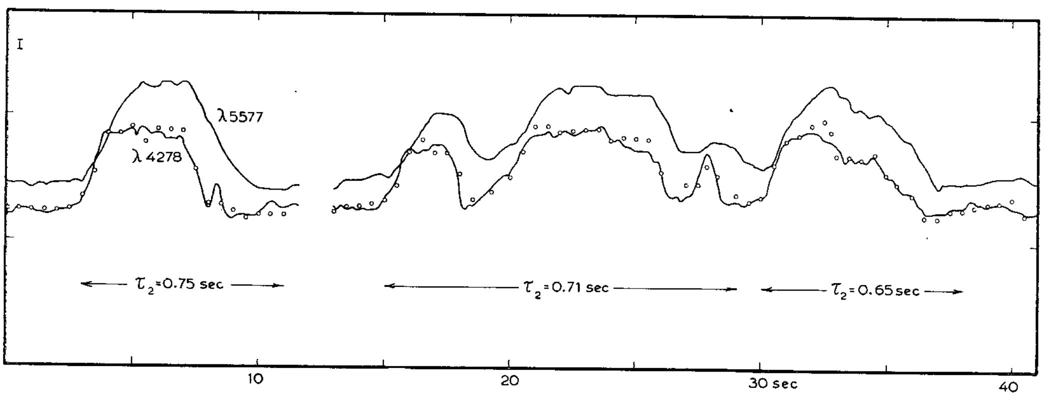

Fig. 6. Record of pulsating aurora, group (a), Tromsø. The intervals indicated by ⟷ were treated as individual records from which the given values of τ_2 were computed. ooo: Curve for $I(4278)$ constructed from the curve for $I(5577)$ by eq. (4.5), taking $\tau = 0.75$ sec.

various methods agree within two or three per cent. Since the middle part of a pulse plays an important part in determining K_c, but not τ, it may be of interest to compute τ separately from the parts of the curve where dI_0/dt is positive and negative, excluding the center part of the record where dI_0/dt is zero or small. We denote these values by τ_+ and τ_-. These have been computed and found to correlate well with τ computed by the other methods. Some values of significance have been collected in Table 2.

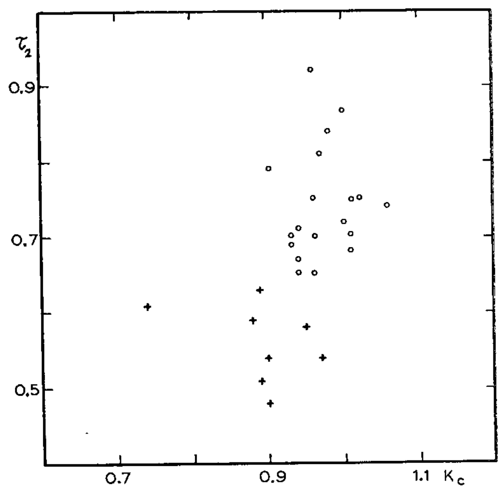

Fig. 7. τ_2 against K_c for pulsating aurorae, Tromsø. o: group (a), +: group (b).

As pointed out in the previous section the correlation between the various sets of τ's does not necessarily imply that the variation in τ is real. Fig. 6 shows a record belonging to group (a). The points show the λ4278-curve constructed from the smooted λ5577-curve with $\tau = 0.75$ sec, the average value of τ for this group. The values of τ computed by the method of least squares are given on the figure, below the curve. It does not seem likely that the differences between the constructed and the measured λ4278-curve for the last of the three pulses is significant. This means that a variation in τ of o.1 sec. is not significant. It is therefore not likely that the variation in τ within the individual groups (a) and (b) is significant, particularly because all the measurements within one group are of the same pulsating aurora. But it is felt that the difference between the two groups is real. As is seen from Fig. 7 K_c varies with τ as expected, but it is of course also very probable that the ratio between the excitation rates is different in the two cases.

Table 2. *Pulsating aurorae, Tromsö.*

Average values of τ and σ, and the correlation coefficients ϱ. τ_1 is computed by the linear numeric method and τ_2 by the method of least square. For τ_+ and τ_- see the text. The unit for τ is 10^{-2} sec.

	τ_1 / σ_1	τ_2 / σ_2	τ^+ / σ^+	τ_- / σ_-	Correlation coefficients		
					$\varrho(\tau_2, \tau_1)$	$\varrho(\tau_2, \tau_+)$	$\varrho(\tau_2, \tau_-)$
Group (a)	76	74	74	76	0.76	0.65	0.80
19 cases	13	07	10	14			
Group (b)	59	56	58	56	0.84	0.47	0.37
8 cases	05	05	07	07			
(a) (b)	71	69	69	70			
27 cases	11	10	11	15	0.91	0.76	0.78

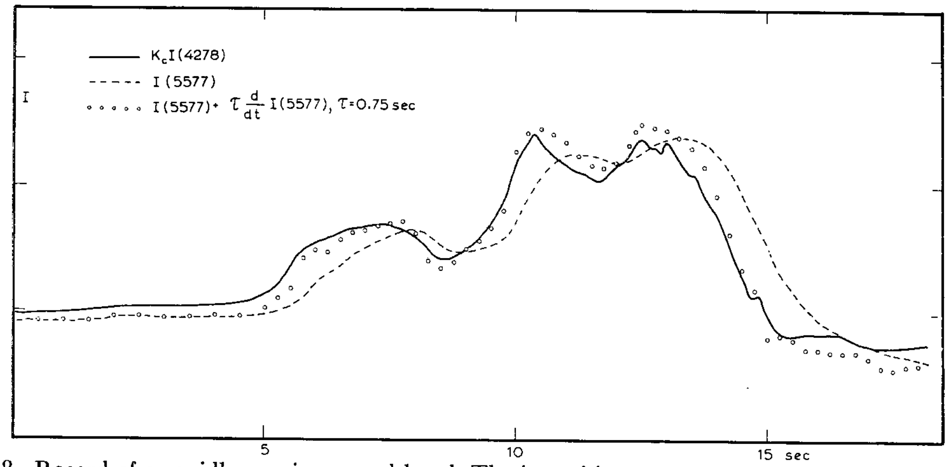

Fig. 8. Record of a rapidly varying, rayed band. The intensities are normalised so that $K_c = 1$. ooo: curve for $I(4278)$ constructed from the curve for $I(5577)$ by eq. (4.5), taking τ to be 0.75 sec. The computed values for τ_2 and τ_1 are 0.70 and 0.81 sec respectively.

d. Rayed bands. Although rapidly moving auroral bands and other rapidly varying auroral forms are frequently seen, the number of good records of such forms is not high. This is because it is difficult to predict the movements and variations of such forms. From the records only 15 cases were found good enough for a detailed and precise

Table 3. *Rayed bands, Tromsö.*

τ computed by linear method (τ_1), the method of least squares (τ_2) and from the increasing and decreasing parts of curves (τ_+ and τ_-, cf. the text). K_m/K_c, cf. the text. The unit for τ is 10^{-2} sec. σ is the mean square deviation.

Record No	τ_1	τ_2	τ_+	τ_-	$\frac{K_m}{K_c}$	Correlation coefficients
1	56	53	47	52	1.03	
2	72	80		81		
3	76	72	57	80	1.10	$\varrho(\tau_2, \tau_1) = 0.88$
4	83	78	50	87	1.08	
5	81	70	57	75	1.06	$\varrho(\tau_2, \tau_+) = 0.66$
6	97	83	71	81	1.07	
7	57	56	43	57	1.01	$\varrho(\tau_2, \tau_-) = 0.88$
8	66	60	44	66	1.04	
9	69	60		73	0.98	
10	53	49	42	55	1.08	
11	74	74	81	80	1.00	
12	79	71	64	80	1.07	
13	77	64	73	78	1.03	
14	59	55	45	59	1.01	
15	60	61	61	62		
Average	71	66	57	71	1.04	
σ	12	10	12	11		

analysis. These were all records of rayed bands that moved rapidly and passed the sight-line of the photometer in about 10 sec. or so. A typical record is shown in Fig. 8.

The records have been analysed by the linear numeric method and the method of least squares. The results are given in Table 3. As in the preceding section τ_+ and τ_- have been computed from the steepest parts of the curves. It is seen that the agreement between the various methods is less satisfactory than in the cases of pulsating aurorae. τ_2, computed by the method of least squares, is usually lower than τ_1, computed by the linear method. τ_- agrees reasonably well with τ_1, whereas τ_+ is usually much lower than both τ_1 and τ_2. Values of K_c derived by the different methods agree well.

The curves exhibit also another unsatisfactory feature. Fig. 8 shows the record No 3 in Table 3. The curves have been normalised so that K_c is one. It will be seen that if we compute K_c from the points where dI_O/dt is zero, we obtain values larger than one in the maximum points and lower than one in the minimum points. This is the case for most for the records. In Table 3 is also given K_m/K_c, K_m being computed from the maximum point, and K_c by the method of least squares. K_m is usually a few per cent greater than K_c. This effect is similar so that found in the records from Yerkes Observatory, cf. sec. *8b*. The scale values were not accurate enough to correlate K_c and τ.

In this analysis no corrections have been made for the background of night sky emission and diffuse auroral emissions. A proper correction for the diffuse auroral background is difficult to make, as this also seems to vary considerably. The night sky emission is negligible in these cases. One procedure has been attempted, namely to study the variation of K_c computed from the extremal points as a function of I_N. In a few cases a sufficient systematic variation of K_c has been found, and K_c could be expressed as

$$K_c = K_{co} I_N^{p-1} \tag{8.1}$$

where p is larger than one.

What this suggests is simply that the rate of excitation of the oxygen line is proportional to I_N^p rather than to I_N (since it is supposed to be proportional to $K_c I_N$), and that I_N^p has to replace I_N in our basic equation (4.5). This procedure also corrects automatically for the background, as eq. (8.1) was found to be valid for the background as well. In five of the fifteen cases eq. (8.1) was found to describe the variation of K_c satisfactory, p varied between 1.10 and 1.36. In the other cases the number of extremal points were too few (two or three) to get a satisfactory determination of p.

The introduction of I_N^p gave consistently higher values of τ. τ_1 and τ_2 increased on the average 0.25 and 0.19 sec respectively. That the adoption of $p > 1$ increases τ is reasonable from a physical point of view. The relative variation of the assumed rate of excitation is then increased, and the "damping constant" τ must be increased to explain the observed variation of I_O.

Since K_{cp} is greatest at the maximum points and smallest at the minimum points, the curves may also fit the equation

$$K_c I_N = I_O + \tau \frac{dI_O}{dt} + \alpha \frac{d^2 I_O}{dt^2} \tag{8.2}$$

with K_c constant, better than the original eq. (4.5). To be in agreement with the observations α has to be positive and of the order of 0.1 sec^2. The introduction of this term has a negligible influence on τ and K_c. Using the method of least squares, and with (d^2I_O/dt) as well as I_O equal at the beginning and end of the record, we obtain in the first approximation:

$$\text{(8.3)} \qquad \frac{\triangle\tau}{\tau} = \frac{\triangle K}{K} = \alpha \left(\sum_p \mathcal{J}_{Np} \mathcal{J}''_{Op} / T_p\right) \Big/ \left(\sum_p \mathcal{J}_{Np} \mathcal{J}_{Op} / T_p\right)$$

where $\mathcal{J}''_{Op}$ is the integral of d^2I_O/dt^2 from t_p to t_{p+1}. Since $\mathcal{J}''_{Op}$ is an oscillating quantity $\Sigma\ \mathcal{J}_{Np}\mathcal{J}''_{Op}/T_p$ will be much smaller than $\Sigma\ \mathcal{J}_{Np}\mathcal{J}_{Op}/T_p$ and $\triangle\tau/\tau$ is of the order 0.01. The introduction of the term $\alpha d^2I_O/dt^2$ is usually less satisfactory than the introduction of I_N^p. At the flat parts of the records before and after the "pulse" the inconsistency is almost unchanged, which means that this method does not take care of the background.

It is difficult to judge whether or not a correction to τ should be introduced. The fact that K_c given by eq. (8.1) varies in the opposite direction of $I(5577)/I(4709)$ in Fig. 2, group (c), casts some doubt on the validity of eq. (8.1) as an expression for the ratio between the excitation rates.

e. Uncertainties in τ. In the preceding sections we have made no attempt to estimate the uncertainty or mean square deviation in τ for the individual cases. In the case of the linear numeric method (cf. sec. 7c) one might think that it should be possible to compute a mean square deviation σ for each record from the τ_p's.

It is, however, rather doubtful whether σ would have any significance. The number of intervals t_p to t_{p+1} is an arbitrarily chosen quantity and the τ_p's are certainly not independently of each other. The errors in $\mathcal{J}'_{Op}$ are undoubtedly the largest ones of those due to errors in the reading of the curves. But errors in the $\mathcal{J}'_{Op}$'s are not independent, as $\mathcal{J}'_{Op}$ is the difference between sucessive $I_O(t_p)$'s.

Apart from this, the variations in τ_p are mostly not due to errors in the measurements of the curves, at least not in the cases of strong aurorae, but "errors", or rather deviations, in the curves themselves. We do not presently know why τ_p or K_c varies during a record. The variations may be real, i.e. due to physical events preceding the light emission. τ and K may for example be functions of the height within an aurora. The height of the aurorae may vary at random, but nevertheless the height itself is probably a continuous function of time, and we do not know its autocorrelation function. The computed value of τ is certainly also influenced by the excitation mechanism.

It is possible that part of the deviations in the curves, particularly for the N_2^+ bands, is due to the scatter of light in the atmosphere. Light from a strong aurora close to the area in the sky which is covered by the photometer is scattered into the photometer and causes a relative increase in intensity which is larger for the N_2^+ band than for the [OI] line. Also this effect would yield correlation between neighbouring τ_p's.

Wind in the upper atmosphere is probably an important cause for errors in τ and the scatter of the τ_p's. It is known that winds of velocities 50—100 m/sec exist in the upper atmosphere (BEDINGÉR, GOSH and MANRING 1957, HARANG and PEDERSEN 1956). Velocities of the order of 500 m/sec for the auroral scatter elements have also been measured (cf. KAISER 1958), but these velocities probably concern the ionization maxima or the ions and electrons only. When directed towards the zenith the photometer covers an area with a diameter of about 1.5 km at an altitude of 110 km. With a wind velocity of 50 m/sec the entire air volume from which the emission is observed by the photometer drifts out of the field in about 30 sec. For a pulsating aurora this effect may be less important, because as far as can be judged the entire patch or arc pulsates in phase, and any area of the aurora is probably representative for the whole aurora. For moving bands this is not the case, the excited oxygen atoms move out of the field and are replaced by fewer or more excited atoms moving into the field from other directions. This process depends on the geometry of the aurora and it is at present not possible to treat this effect quantitively.

All the arguments presented in this section show that the computation of standard errors for the individual τ's has no meaning, and that much of the variation in the computed values of τ may be due to the causes mentioned above.

9. Conclusions.

a. The intensity ratio I(5577)/I(4709). The observations reported in sec. 6 show that the intensity ratio $I(5577)/I(4709)$ does not vary very much in the various types of ordinary, quiet aurorae. Observations by other workers, notably by HUNTEN (1955), are partly contradictory to this result, but it seems likely that much of the discreapancy may be due to the observation technique. The scanning from the N_2^+ band λ3914, which was used by HUNTEN in these comparisons, to the λ5577 line takes some time, in which the intensity may vary. Also, the correction for the atmospheric absorption is difficult, and, as pointed out by HUNTEN, the measured intensity ratio $I(3914)/I(4278)$ varies also with more than a factor 2 although it should be constant.

The results presented in this paper show that $I(5577)/I(4709)$ is constant within $\pm$ 25 % in all common, quiet auroral forms, showing a distinct structure that can be easily distinguished from the background of nightglow or auroral glow. It is likely that the systematic variation in this intensity ratio for group (c), as shown in Fig. 2, is real. The other set of observations reported in sec. 6 does not support this view, but these observations are neither accurate enough to contradict it.

Within any one record of rapidly varying, rayed bands, we found that K_c usually increases slightly with the absolute intensity (cf. sec. 8d), i.e. the variation in K_c with intensity is opposite to that in Fig. 2, group (c). Since these records are difficult to interpret, the variation in K_c should not be taken too literally as an expression for the variation in the relative excitation rates of the two emissions.

HARANG's (1958) measurements do show a systematic variation in $I(5577)/I(4278)$ with height, this ratio decreases with increasing height, particularly in the upper part of draperies. (He puts $I(5577)/I(4278)$ arbitrarily equal to one at the maximum point.) This is partly in agreement with our result, that in the middle and upper part of rays (group b) $I(5577)/I(4278)$ is less than in arcs, bands and the lower parts of rays (group c). HARANG's variations are, however, greater than would be expected from our result. Again it must be pointed out that the nightglow background precludes high accuracy in the weak, upper part of aurorae, particularly in HARANG's measurements, were scans are not made across the rays, but rather along them. Also, our measurements do not include the weakest, uppermost part of auroral arcs.

Our result suggests that the energy distribution of the particles active in the excitation of the green [OI] line and the First Negative N_2^+ bands is roughly the same everywhere and in most types of aurorae. This does thus not support the theories of local discharges as a main cause for some auroral forms, although it cannot be excluded that discharges across the magnetic lines of force contribute to minor parts of the auroral emission.[1]

Finally it should be emphasized that our observations do not, except in one case, include type B aurorae (red lower border) or red surfaces, whereas they do include red tops of rays.

b. Excitation and lifetime of the $O(^1S)$ atoms. It seems fairly safe to conclude, from the measurements reported in sec. 8, that the lifetime of the undisturbed $O(^1S)$ atoms is about 0.7 sec, with an uncertainty of about $\pm$ 0.1 sec. This is very close to the theoretical result of 0.73 sec obtained by GARSTANG (1952), and thus an experimental confirmation of his result.

When we come to the detailed interpretation of the records and the apparent variation in τ, the situation is not quite satisfactory. The questions we want to answer are: (a) which processes do contribute significantly to the excitation, and (b) is collisional deacitvation of the $O(^1S)$ atoms of any importance.

It is obvious that τ, as defined by the mathematical equation (4.5) with the measured curves for I_O and I_N inserted, really do vary. But this does not exclude that variations in the physical quantity K_c and other conditions (cf. sec. 8e) may vary such as to give an apparent variation in τ.

It is concluded that the processes (2.8), (2.9) and (2.10) are of minor importance in the excitation of the ^1S term, because of the long additional time delay they otherwise would cause. Of course, if this time delay were long enough, the radiation excited by these processes would escape our observation altogether, but the average value of K_c for a "pulse" or a selected record is not significantly different from that for quiet aurorae. If some of the radiation excited by the "pulse" escaped our observation, K_c would be lower. It is of particular interest to note that the recombination process (2.10) is of minor importance. The processes (2.12), (2.13) and (2.14) are neither likely to be of importance, except perhaps in the very lowest parts of aurorae. K_c varies apparently

[1] The low latitude aurora recently observed by MANRING and PETTIT (1959), where I(6300)/I(5577) was of the order 10^3, could perhaps be due to discharge processes.

moderately with the height, whereas the relative concentration of O_2 increases strongly with decreasing height in the lower part of the auroral region.

For the other processes discussed in sec. 2c the observations do not furnish further arguments. It is interesting to note that the intensity ratio $I(5577)/I(4709)$, and also the fact that it is fairly constant, is consistent with the view that secondary electrons play a dominating role in the excitation and ionization processes. As demonstrated in sec. 2a the time delay between the ionization process and the excitation of the oxygen atoms is negligible at the heights where it is most likely that the observed aurorae occured.

The vibrational intensity distribution in the First Negative N_2^+ bands is not quite in harmony with the assumption of electron excitation (cf. BATES 1949) but this question deserves further study. For example, the concept that the electron excitation probabilities are proportional to the corresponding overlap integral for the wavefunction (cf. HERZBERG 1950) is, experimentally, based on work in the low energy range only (20—200 eV) whereas the auroral ionization is partly performed directly by primary electrons with energies of several KeV and perhaps some primary protons.

The answer to the question about whether or not collisional deactivation occurs depends critically on how reliable one considers the theoretical values of the transition probabilities to be. GARSTANG (1956) considers the values 0.73 sec for the lifetime of the undisturbed $O(^1S)$ atoms to be rather too small than too high. If this is the case it seems most likely that the correct value is about 0.75—0.80, in the upper part of the range within which our measured τ's are distributed. The shorter lifetimes measured may be due to collisional deactivation. At least it is difficult at present to find a more plausible cause for the smaller values of τ. The probability of collisional deactivation, d_3, should then be in the range 0—1/sec. If we tentatively put d_3 equal to 0.5/sec at a height of 100 km, we get a probability of deactivation per gaskinetic collision of the order 10^{-4}. This may not be an undue probability for process (3.2) (SEATON 1958).

If, on the other hand, it should be proven that the life-time towards radiation is about 0.65 sec or less it would be more reasonable to consider the variations in τ to be due to variations in the excitation mechanism. Particularly if the value is 0.60 sec. or less, it seems more reasonable to regard the longer lifetimes as due to a minor, but not ignorable, contribution to the excitation from processes which adds to the delay time, such as the recombination process (2.10). Finally, we must not exclude the possibility that such processes are operative simultaneously with deactivating collisions.

It is evident that if collisional deactivation occurs, the probability d_3 varies considerably with height, and thus varies considerably within one aurora. This may be an additional cause for the deviations from the idealized model (eq. 4.5), such as observed in some types of aurora. The correction introduced in sec. 8d (variable K_c) increases τ to 0.8 or 0.9 sec., but again it is doubtful whether this correction is justified. The values of τ computed from the records of pulsating aurorae in Tromsö are probaly the best ones. To try to interpret the finer details in the available records has at present little or no meaning. Our knowledge about the various processes involved is quite insufficient for a more detailed analysis of the records.

c. The red [OI] doublet. There is not much to add to what is already said about the red doublet in sec.s 2 and 3, but it may be worth while to point out a few things that must be born in mind during further work on these lines.

From the model computations on excitation by secondary electrons, illustrated in Fig. 1, the ratio S_{12}/S_{13} is found to be about 14, a value in reasonable agreement with the estimates by SEATON (1958). Although this ratio is fairly independent of the average electron energy, it is obvious that the introduction of the processes mentioned in sec. 2c and of other processes competing for the energy of the secondary electrons may cause S_{12}/S_{13} to vary considerably with height. For example, electron excitation of the O_2 molecules to the states $a^1\Delta$ or $b^1\Sigma$ at low heights may compete with the excitation of O for the electron energy and reduce the rate of excitation to the ^1D term as well as S_{12}/S_{13}.[1] On the other hand, excitation by processes such as (2.10), (2.12) and (2.14) may tend to increase the ratio S_{12}/S_{13} at low heights.

VEGARD (cf. VEGARD *et al.* 1958) find a correlation between the intensity of the Hα line and the red doublet, and concludes that protons are particularly effective in exciting the ^1D term, without spesifying the exact mechanism.

KRASSOVSKY (1958) suggests that charge exchange between protons and oxygen atoms may produce $O^+(^2D)$ ions, which again are transferred to $O(^1D)$ atoms through, for example,

$$O^+(^2D) + N(^4S) \rightarrow O(^1D) + N^+(^3P) \tag{9.1}$$

He does not give any reason why this mechanism should be so much more effective when the O^+ ions are produced through charge exchange rather than by ordinary ionization by protons or electrons. Also, it is unlikely that the rate of this process is high enough to make it dominating anywhere in the aurora. Protons can hardly be responsible for more than about 10 % of the total ionization in aurorae, and therefore the rate of ionization by charge exchange with atomic oxygen can only be of the order 2 % or less of the total rate of ionization in aurorae, and this is less than the rate of excitation of the green [OI] line (cf. OMHOLT 1959a).

The correlation between Hα and the red doublet at high latitudes is easy to explain if the Hα emission is mainly associated with high aurorae (cf. OMHOLT 1959a), in which the probability of collisional deactivation of the $O(^1D)$ atoms is low.

The height distribution curves for $I(6300)$ compared to $I(5577)$, on which SEATON's (1958) analysis is based, are not satisfactory, but it is doubtful if an improvement cf these curves can provide further information until we know more about the possible excitation and deactivation processes. It may, however, be useful to investigate whether the ratio $I(6300)/I(5577)$ depends on the absolute height in the atmosphere only, or

[1] The recently detected anomalous high cross section for inelastic collisions between electrons and N_2 molecules (HAAS 1957), with a sharp peak around 2 eV, may also be important, HAAS attributes this to formation of unstable N_2 molecules, and if the absolute cross section is sufficient, this would lead to an important rate of capture of "active" electrons with energies just above 2 eV, and release of "inactive" electrons with energies around 1.5 eV. This would particularly lead to a decrease in S_{12}.

if this ratio at a particular height (or rather for a particular atmospheric constitution and density) differ from one aurora to another and is correlated with the intensities of other emissions, such as the hydrogen lines. Such measurements would be a somewhat difficult task, but they may lead to a separation of pure atmospheric effects, such as collisional deactivation and indirect excitation processes, and effects owing to the nature and energy distribution of the primary particles.

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APPENDIX

On the method of least squares for determining τ and K from photoelectrical records. Denoting I_N in eq. (4.5) by X and I_O by Y, the basic equation becomes

$$Y + \tau \frac{dY}{dt} = K X, \tag{A.1}$$

X and Y being functions of time. We assume that this equation is valid for the true intensities, and that τ and K are constants, and want to find the best approximation to τ, K, X and Y. If we suppose that the corresponding measured intensities x and y deviate in a random manner from the true intensities, then it may be appropriate to assume that the best approximation is found when

$$I = \int_1^2 \{w\, K^2\, (X - x)^2 + (Y - y)^2\}\, dt \tag{A.2}$$

is as small as possible. The factor K^2 is introduced because this brings X and x on the same level as Y and y, since K is the equilibrium value of the ratio Y/X. If one curve is less accurate than the other we may allow for the difference in accuracy by introducing a weight factor w different from one. Eqs.s (A.1) and (A.2) yield

(A.3) $$I = \int_1^2 \left\{ w \left(Y + \tau \frac{dY}{dt} - Kx\right)^2 + (Y - y)^2 \right\} dt,$$

The minimum value of this integral is obtained when

(A.4) $$\left(\frac{\partial}{\partial Y} - \frac{d}{dt}\frac{\partial}{\partial\,(dY/dt)}\right) \left\{ w \left(Y + \tau \frac{dY}{dt} - Kx\right)^2 + (Y - y)^2 \right\} = 0.$$

From this equation follows that

(A.5) $$(1 + w)\; Y - w\,\tau^2 \frac{d^2 Y}{dt^2} = w\,Kx + y - w\,K\tau \frac{dx}{dt} = f\,(t).$$

The general solution of eq. (4.5) is well known (cf. MARGENAU and MURPHY 1943, p. 53). The complete solution includes two constants, C_1 and C_2 (or Y_1 and Y_2) in addition to τ and K. The integral I should also be minimized with these four constants as variables, and this gives four equations which have to be satisfied, in addition to eq. (A.5). The complete solution seems only possible by rather elaborate numeric computations.

If, on the other hand, the deviations from the ideal equation (A.1) are chiefly due to random variations in τ and K, i.e. if we assume the measured values x and y to be correct, then it may be reasonable to assume that the best values of τ and K are found when I, given by eq. (A.3) but with y replacing Y and $w = 1$, is as small as possible. I corresponds now to S given by eq. (7.6), and from the conditions $\partial I/\partial K = 0$ and $\partial I/\partial\tau = 0$ we obtain the two equations

(A.6) $$K \int_1^2 x^2 dt - \tau \int_1^2 x \frac{dy}{dt}\, dt - \int_1^2 xy dt = 0,$$

and

(A.7) $$K \int_1^2 x \frac{dy}{dt}\, dt - \tau \int_1^2 \left(\frac{dy}{dt}\right)^2 dt - \int_1^2 y \frac{dy}{dt}\, dt = 0,$$

which are analogous to eq.s (7.8) and (7.9). Similarly we may derive an equation analogous to eq. (7.11) by minimizing $I_1 = I/K^2$ rather than I.

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