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LUMINOSITY CURVES OF HIGH AURORAE

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Abstract: From photographs obtained by the late Prof. Carl Størmer the luminosity variation with height has been studied in 23 aurorae with their maximum luminosity at a height between 200 and 400 km. The luminosity curves and absolute intensities of high aurorae are discussed briefly. It is concluded that the primary electrons are dispersed in energies and/or the angles which their paths make with the magnetic field lines and that the number density in the electron bundles causing high rays is of the order 10^2 to 10^4 cm^{-3}

1. Introduction. The analysis of the luminosity curve of different auroral forms and their theoretical explanation based on various models of excitation processes has been the subject of study in a number of papers. The first quantitative estimates of the mean luminosity curves were given by VEGARD and KROGNESS (1920), who estimated the vertical extension of the auroral forms from photographs. It was evident that the type effect is very predominant in the luminosity distribution of different auroral forms, the vertical extension of rays being more than ten times the vertical extension of quiet homogeneous arcs.

A photometric study of auroral forms, principally arcs and draperies, has previously been made (HARANG 1946). The luminosity curves were characterized by the quantities l_l^p and l_u^p, which were the differences between the height of maximum intensity and the two heights (lower and upper) at which the intensity had fallen off to p times this maximum intensity. The l^p's were given for $p = 2/3$, $1/2$ and $1/3$. The measurements included 54 aurorae, and the results are included in Fig. 2 of this paper. Numerical values are given in the previous paper. These values were obtained from arcs with their maximum intensity in the height interval 100—130 km and from draperies in the height interval 130—160 km. Besides these, two examples of sunlit rays with their maximum intensity at 270 km were discussed.

It would be of interest to extend these studies of the vertical extension of the auroral luminosity to aurorae lying at very great heights. These auroral forms are usually faint and look less spectacular than the brilliant low aurorae and are therefore often neglected during auroral photography.

2. Data obtained from Størmer's photographs. The late professor CARL STØRMER measured the heights of a great number of *high* aurorae, some of which were sunlit. His great stock of auroral photos were inspected, and 23 plates showing auroral rays of a simple character and with little fog or diffuse auroral background luminosity were selected. The plates were not furnished with any photometric intensity scale, but a photographic intensity scale made through a Zeiss Stufenfilter on similar photographic plates was used as a reference.

By means of a recording photometer a number of records were taken at directions approximately at right angles to the direction of the ray. Fig. 1 shows four examples of rays with their luminosity distribution represented through isophotes. Drawings of the rays with STØRMER's height measurements are given to the left. By combining STØRMER's height measurements and the isophotic representation the vertical lumi-

Table 1.

No.	Date	MET	h_{max} km	$l_l^{1/2}$ km	$l_u^{1/2}$ km	Remarks
1	18—19. Sept. 1930	21 18 04	291	48	62	sunlit, isolated ray
2	20—21. April 1936	00 15 56	328	58	79	ray in glow
3		23 39	276	41	55	ray in glow
4		39 30	311	38	39	sunlit(?) ray in glow
5		40 45	325	51	68	sunlit ray in glow
6		49 16	346	54	56	ray in glow
7		49 43	303	39	54	sunlit ray in glow
8		01 08 25	305	47	60	sunlit, isolated ray
9	16—17. Oct. 1936	05 01 46	375	64	95	group of rays
			370	56	91	
10	3— 4. Nov. 1936	21 14 00	205	42	86	isolated ray
11	21—22. March 1938	04 23 21	342	68	90	isolated ray
12		24 15	335	57	105	isolated ray
13		25 42	286	75	104	isolated ray
14		26 20	305	51	95	isolated ray
15		27 14	280	43	100	extinction not negligible
16		30 39	312	113	110	sunlit, reversed
17	14—15. Sept. 1938	03 58 58	225	31	94	irregular form
18	15—16. Sept. 1938	21 32 05	360	44	40	sunlit, reversed
19		34 04	325	66	49	sunlit, reversed
20	30.9— 1. Oct. 1938	00 40 09	248	55	74	faint, isolated ray
21	2—3. April 1940	03 40 50	293	27	94	sunlit, ray in glow
22		46 49	409	135	159	two faint rays
23		47 30	355	114	132	sunlit, ray in glow

nosity curve of the ray could be constructed. On each figure the distances to the upper and lower half-intensity points, $l_l^{1/2}$ and $l_u^{1/2}$, are given.

In Table 1 the numerical values of $l_l^{1/2}$ and $l_u^{1/2}$ are given together with the height of maximum luminosity, h_m, for each of the 23 cases. In Fig. 2 $l_l^{1/2}$ and $l_u^{1/2}$ are plotted versus h_m, and on this figure the old data (HARANG 1946, Table 3) are included. The curves drawn on this figure are estimated by eye, neglecting the points from sunlit aurorae. The results indicate that for non-sunlit aurorae the vertical extension depends strongly on h_m below 200 km, whereas above 200 km the extension increases only slightly with height. The points from sunlit aurorae are more scattered, but the vertical extension of these aurorae is of the same order of magnitude as that of ordinary aurorae.

The fact that STØRMER's plates were not furnished with any intensity calibration scale, introduces an additional source of error in the new data, and this error is even difficult to estimate very accurately. It turns out that a certain percentage error in the assumed value of γ, the contrast of the plate, yields approximately the same percentage error in $l_l^{1/2}$ and $l_u^{1/2}$. Altogether we feel confident that the lack of calibration curves on each plate should not give rise to larger errors than 20 per cent, and we believe the scatter of the points on Fig. 2 to be significant.

It should also be born in mind that the earlier work was done in Tromsø in the auroral zone, while Størmer's plates were obtained in southern Norway, about 7 degrees south of the auroral zone.

3. The luminosity distribution function. In the previous paper (HARANG 1946) it was assumed that the light emission per unit air mass along the beam of incident electrons is expontially decreasing, i.e. the emission is given by a function

$$L(\xi) = L_o \exp(-\mu\,\xi) \tag{1}$$

where ξ is the penetrated air mass per unit area. Taking the air density distribution to be given by the standard exponential formula with a constant scale height H, the formula for the emission per unit height interval in the atmosphere, $L(h)$ will be the same as that for electron production by monochromatic light in the ionosphere, i.e. one obtains the Chapman function

$$L(h) = L_m \exp\{1 - z - \exp(-z)\}, \tag{2}$$

where z is the height relative to the height h_m of maximum emission L_m and with the scale height H as unit:

$$z = (h - h_m)/H \tag{3}$$

The maximum emission occurs where

$$\mu\varrho H = 1, \tag{4}$$

where ϱ is the air density. $L(h) = {}^1/_2 L_m$ for $z = z_u = +\,1.46$ and $z = z_l = -\,0.99$; these two values correspond to the upper and lower half intensity points given in the

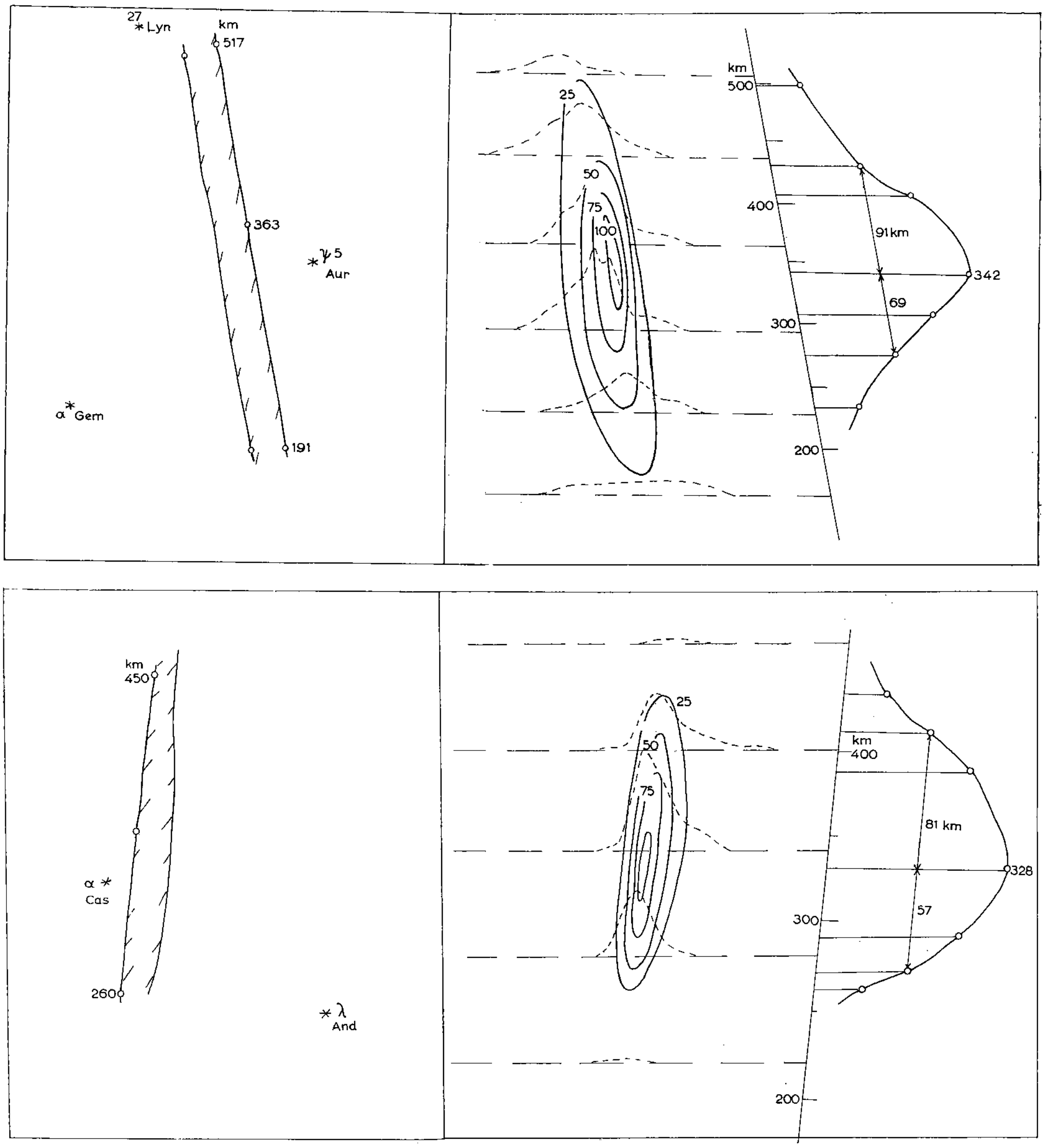
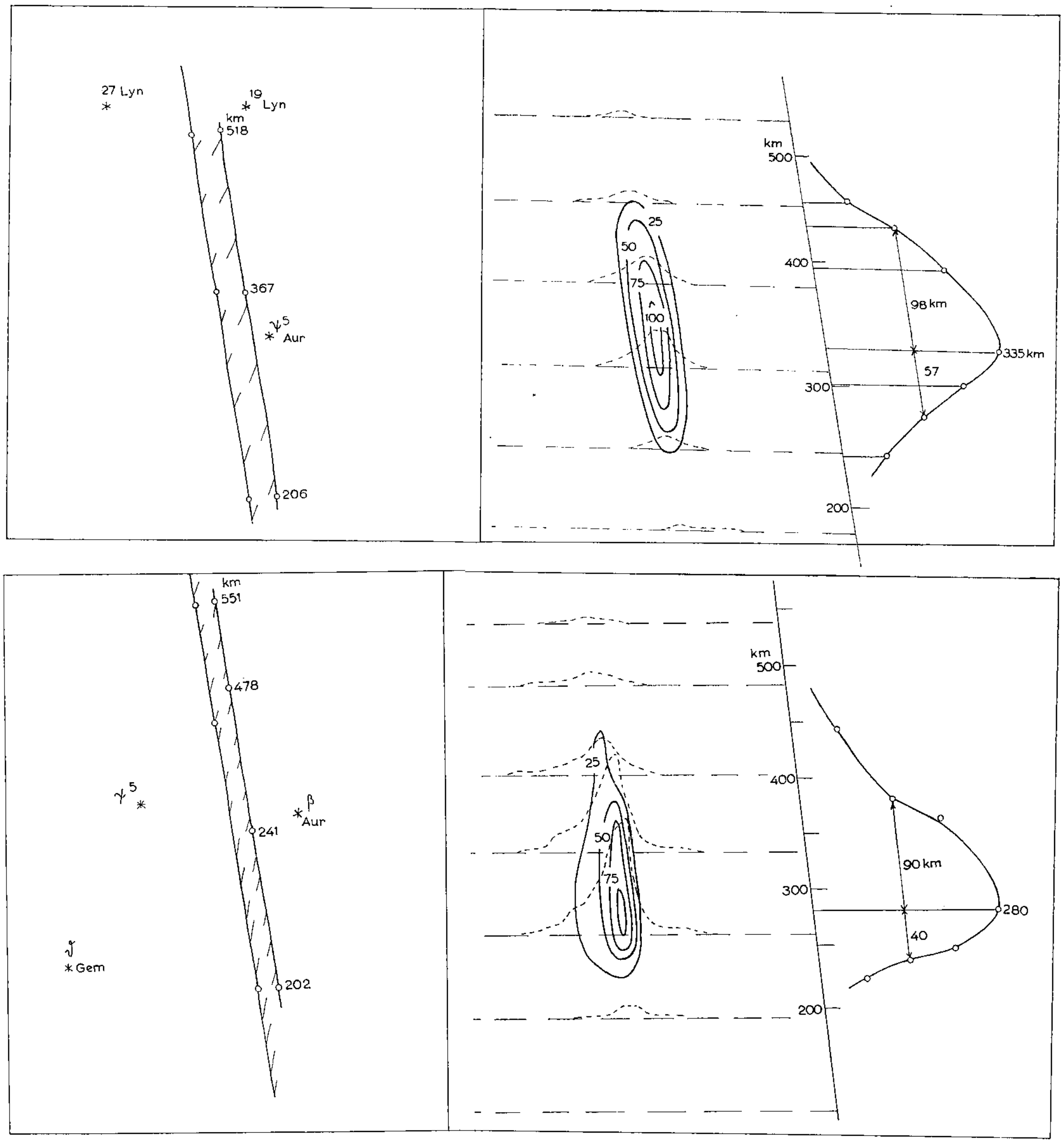


Fig. 1. Examples of the luminosity distribution in auroral rays. Height measurements to the left, isophotes and vertical luminosity distribution to the right.

preceeding section, and we may derive the scale height H from l_u and l_l, by the equations

$$z_u = (h_u - h_m)/H = l_u/H = 1.46 \qquad (5a)$$

$$z_l = (h_l - h_m)/H = -\, l_l/H = -\, 0.99 \qquad (5b)$$



Using the observed values of l_u and l_l we obtain two values of the scale height which we shall designate respectively H_u and H_l.

In Table 2 are given H_u and H_l computed for various values of h_m, taking l_u and l_l from the curve drawn on Fig. 2. In Table 2, column 4 is further given the average of H_u and H_l, in column 5 and 6 the scale height H and the atmospheric density ϱ taken from KALLMANN (1959), and in column 7 μ computed from eq. (4).

BATES and GRIFFING (1953) have critizised this method of analysing the auroral

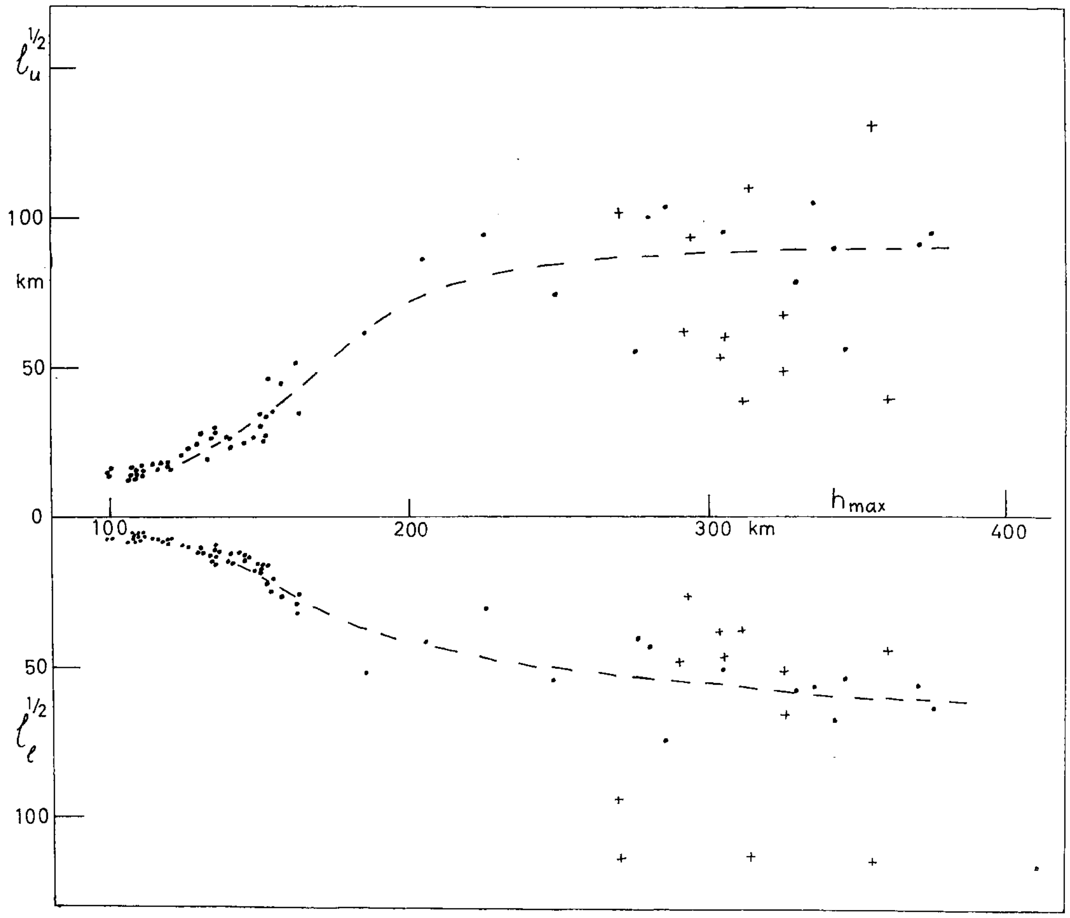


Fig. 2 $l_l^{1/2}$ and $l_l^{1/2}$ versus h_m. • aurorae in the dark, + sunlit aurorae. All points to the left of $h_m = 200$ km and the two at $h_m = 270$ km are from the earlier measurements (Harang 1946).

luminosity curves. Their main objection is that the basic assumption in the analysis, that the light emission per unit air mass is an exponential decreasing function, is invalid. In the first analysis it was assumed that the intensity of the incident electron beam is exponentially decreasing, and that the light emission is proportional to the absorption of electrons from the beam. However, the exponential law is not concerned with the absorption of energy in the air, but with the loss of particles from a monoenergetic, parallel beam of electrons, when any electron suffering a change in either the direction or energy through a collision is regarded as lost from the beam. In the auroral case the electrons will still follow the main beam, even if it is subject to a large scatter angle, since the magnetic field lines prevent a linear spreading out from the beam. The light emission curve from a monoenergetic electron beam is therefore not

Table 2.

h_m km	H_u km	H_l km	H_{av} km	H (Kallmann) km	ϱ (Kallmann) g/cm^3	μ cm^2/g
350	62	60	61	67.8	1.9×10^{-14}	8.3×10^{6}
300	61	58	59	58.1	4.8×10^{-14}	3.5×10^{6}
250	58	50	54	48.3	1.5×10^{-13}	1.2×10^{6}
200	49	42	46	37.8	5.9×10^{-13}	3.7×10^{5}
150	21	18	20	20.2	6.1×10^{-12}	7.8×10^{5}
120	12.6	8.9	10.8	12.9	6.3×10^{-11}	1.4×10^{4}
100	9.2	6.9	8.1	8.1	6.9×10^{-10}	1.8×10^{3}

an exponential decreasing function, it rather raises to a sharp maximum close to the end of the beam.

It is therefore not possible to take our computations seriously as a derivation of the scale height H in the atmosphere. But the very fact that the derived scale heights are in reasonable agreement with those obtained from rocket and satellite data indicates that the light emission per unit air mass really is approximately described by the exponentially decreasing function given in eq. (1). We shall return to this question later, but first we want to consider a little more in detail the emission function we obtain from the observations.

It is certainly possible to take a single height luminosity curve $L(h)$, choose a model atmosphere, and from this compute the emission function $L(\xi)$, since

$$L(\xi) = L(h)\frac{dh}{d\xi} \tag{6}$$

Since the density distribution is not very accurately known and the luminosity curves show large variations, this procedure would hardly give any informations of more significance. What we want to point out is that eq. (1) seems to give a representative description of the light emission in many aurorae. Attempts to study the curves in more detail confirm this view, but it should be strongly emphasized that this description is not a very accurate one. Our conclusion is therefore only that the emission per unit air mass along the beam of incident particles is a steadily decreasing function, and that it can be roughly described by the exponential function given in eq. (1). The exponential factor μ is thus purely a descriptive parameter, and is in no way directly related to any absorption coefficient.

Recent rocket measurements indicate that the density of the arctic atmosphere above 100 km may differ considerably from that at lower latitude. Thus the work by LaGow, Horowitz and Ainsworth (1959) indicates that at 60° N the scale height at a height of 200 km is close to 100 km in the summer time. It appears that the structure of the auroral zone atmosphere at night is still rather uncertain, also local heating by the aurora itself may affect the density distribution. If it should be proved that the scale height appropriate to our case is higher than the values given in Table 2, then our model of the emission function has to be changed. The emission function

$$L(\xi) = C\,\xi^{n-1}\exp\left(-\mu\xi^{n}\right) \tag{7}$$

yields the luminosity height distribution

$$L(z) = L_m \exp\left(1 - nz - \exp(-nz)\right), \tag{8}$$

which is of the same form as eq. (2) and which seems to describe the luminosity curves of high aurorae reasonably well when n is properly chosen.[1] In this case nz and nl/H replace z and l/H in eq.s (5a, b), and the scale height H would be $H = nH'$, where H' now is the scale height derived by eqs. (5a, b) and given in Table 2 (for $n = 1$). The maximum luminosity ($z = 0$) occurs now where $\mu\ (\varrho H)^n = 1$. For n less than 2 we still obtain an emission function $L(\xi)$ which decreases with increasing ξ, for all values of ξ.

That H_u in Table 4 is greater than H_l is in agreement with the fact that the scale height increases with increasing altitude. The fact that we in our analysis consider H to be constant for any one aurora does not introduce any error of importance.

4. The composition of the primary electron beam. Adopting the view that ordinary aurorae are caused neither by protons nor by local discharges in the ionosphere (cf. OMHOLT 1959a), we shall assume that the aurorae we have been considering were caused by electrons entering the ionosphere along the magnetic lines of force. Further, it seems most natural to assume that the luminosity produced by these electrons is proportional to the rate of ionization, as was originally done by BATES and GRIFFING (1953) and later by other workers when excitation of aurora by protons was considered (see e.g. BATES 1960). The number of ions produced by such an electron beam, per unit penetrated air mass, may be readily derived from data given in the LANDOLT-BØRNSTEIN (1952) tables. Fig. 3 shows the number of ion pairs produced per penetrated cm air N.T.P., per electron, as a function of residual range $\xi'(= \xi_m - \xi$, where ξ_m is the residual range when they enter the atmosphere and ξ penetrated air distance along the path). For some selected electron energies the starting point is indicated, i.e. where the residual range $\xi' = \xi_m$. The electrons then move towards smaller residual range ξ', to the left on the figure, and the curve between $\xi' = \xi_m$ and $\xi' = 0$ gives the appropriate function $L(\xi)$ for these electrons.

Electrons with initial energy about 150 eV penetrate only to a height of about 220 km even if they go straight down along the magnetic field lines, thus aurorae with their maximum luminosity below 350 km (lower limit below about 220 km) must at least partly be caused by electrons with higher energies. It is readily seen that for electrons with initial energies above 150 eV the curve in Fig. 3 can not be reconciled with

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[1] Low aurorae (h_m at 100—130 km) seem to be even better described by the formula $L(z) = L_m (1 + n'z)\ \exp.\ (-n'z)$.

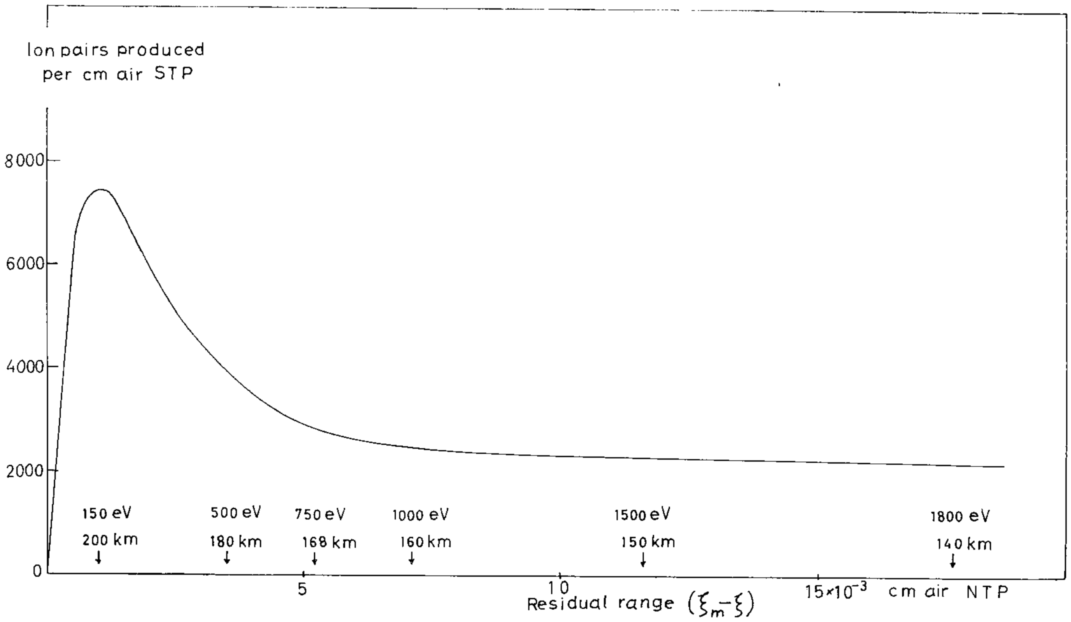

Fig. 3. Ionization per penetrated air mass versus residual range of the electrons ($\xi - \xi$). Arrows indicate the residual range for certain given energies, for which also the altitudes to which they may penetrate into the atmosphere are given.

the observations, which yield an $L(\xi)$ of the form given by eq. (1) or perhaps by eq. (7) with n most likely between 1 and 2. For the curve in Fig. 3 then goes through a maximum while that given by eq. (1) is a steadily decreasing function. Therefore we conclude that most of the aurorae analysed in this paper can not be caused by monoenergetic electrons penetrating along the magnetic field lines. It is certainly no major difficulties in constructing a composite bundle of electrons, dispersed in energy as well as in the angle which their paths make with the magnetic field lines, so that the ionization (and emission) function of the bundle would resemble that obtained from the observations. Similar work has already been done on protons (see e.g. CHAMBERLAIN 1958 or OMHOLT 1959a). We shall make no such attempt here, in as much one also should consider the effect of scattering of the low energetic electrons.

For sunlit aurorae the interpretation is still more difficult, because much of the light emitted from these is due to resonance scattering of sunlight by N_2^+ molecules (cf. BATES 1960).

5. Structure and absolute luminosity. The very high auroral forms are almost exclusively rays or ray bundles, sometimes forming draperies. From the photographic material available it seems that the cross section of rays may be as low as one km^2, but the more diffuse rays or ray bundles may well have a thickness of about 10 km.

On the other hand Gartlein has measured individual rays which were less than 100 m across (cf. BOOKER, GARTLEIN and NICHOLLS 1955).

Since the emission of light is also accompanied by ionization processes, the rays are associated with narrow trails of ionization. The rate of ionization is related to the brightness, so that measurements of the light emission from the rays also give an idea of the rate of ionization. The surface of rays, when viewed at approximately right angles to its length, shows a brightness corresponding to international brightness coefficient I to II, sometimes a little higher. Photoelectric measurements (OMHOLT 1959b) yield an emission of up to about 3×10^{10} and 1.5×10^{9} photons $\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$ of the green [OI] line λ 5577 and the N_2^+ band λ 4709 respectively. Even if we assume that the thickness of the brightest rays is about 10 km the rates of photon emission are as high as 3×10^{4} and 1.5×10^{3} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$ for the λ 5577 line and the λ 4709 band respectively. The available data suggest that the rate of ionization is about 500 times the rate of emission of the λ 4709 band and about 25 times that of the λ 5577 line (OMHOLT 1959a, BATES 1960), so that it is fairly certain that the rate of ionization in an auroral ray may be of the order 10^{6} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$. It may perhaps occasionally be as high as 10^{7} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$, but 10^{5} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$ is probably a more common value.

How weak an auroral ray may be is not yet settled. If the rays are due to an electrodynamical constriction phenomenon the structure may depend on the intensity and flux of primary particles which yields the current. Thus a minimum value for the flux of primary particles may be necessary before a ray is formed. The presence of sunlit auroral rays, which in certain cases exhibit almost only the First Negative N_2^+ bands as fluorescent scattering of sunlight (STØRMER 1955), indicates that the rays as a primary phenomenon may be extremely feeble, the radiation excited by the primary and secondary electrons being perhaps below the threshold of visibility. In this case the rate ionization may be 10^{4} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$ or less. An ordinary auroral arc of brightness coefficient I, which is already difficult to detect with the eye, corresponds to the same rate of ionization (with optical thickness 20 km). Our present knowledge does not contradict the presence of extremely weak rays or glow, invisible to the eye, throughout most of the *E*- and *F*-region at disturbed nights, yielding a rate of ionization of about 10^{3} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$ or so.

The presence of aurorae up to very great heights has been regarded as evidence for the excitence of an atmospheric density of importance up to nearly 1000 km. The rate of ionization in an aurora is of course given by

$$q = n_0 \varphi \sigma, \tag{9}$$

where n_0 is the atmospheric density, φ the flux of ionizing particles and σ the effective ionization cross section. If the very high rays are caused by primary electrons, these have an energy of a few hundred eV, and the ionization cross section is about 2×10^{-16} cm^{2}. Thus, with a rate of ionization of 2×10^{4} $\mathrm{cm}^{-3}\,\mathrm{sec}^{-1}$ (brightness coefficient I, optical thickness 10 km) we obtain

$$n_0 \varphi = 10^{20}\ \mathrm{cm}^{-5}\,\mathrm{sec}^{-1} \tag{10}$$

This should be about the minimum value of $n_o\varphi$ required to produce visible aurora between 200 and 400 km. At 300 km the density is about 10^9 cm^{-3}, so that the minimum value of φ should be 10^{11} cm^{-2} sec^{-1}. It seems likely that values of 10^{12} to 10^{13} cm^{-2} sec^{-1} also occur. With an average velocity of 10^9 cm sec^{-1} this yields an electron density in the beam of 10^2, 10^3 and 10^4 cm^{-3} respectively.

Because of the convergence of the magnetic lines of force running from the equatorial plane to the poles, the density of the primary electrons hitting the atmosphere is about 100 times less close to the equatorial plane. On the other hand, a large fraction of the electrons being present in the neighbourhood of the equatorial plane may be reflected magnetically on their way towards the poles. If we follow the magnetic lines of force out to the equatorial plane one should therefore expect that the density of electrons active in producing aurora is somewhere between 1 and 10^3 cm^{-3}, a not unreasonable value. These estimates of the electron density is based on the assumption that the dipole approximation to the earth's magnetic field is valid in these regions of space, also during magnetic storms, which of course may not be true. The main point in our discussion is, however, to ascertain that the required electron densities are not undue.

6. Conclusions. The observational results show that the emission per penetrated distance air NTP is roughly of the form given by eq. (1) or by eq. (7) with n between 1 or 2. It is concluded that these aurorae must be caused by bundles of electrons with a dispersion in energies and/or the angles which their paths make with the magnetic lines of force. The number density in the electron bundles causing high rays is of the order 10^2 to 10^4 cm^{-3} when they hit the atmosphere. It seem likely that the individual aurorae show large variations from the "average" so that the data obtained here are accurate enough to compare the observations with the theories in their present stages.

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