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MARTIN MORK

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THE GENERATION OF SURFACE WAVES BY WIND AND THEIR PROPAGATION FROM A STORM AREA

By Martin Mork

FREMLAGT PÅ VIDENSKAPS-AKADEMIETS MØTE DEN 17. SEPTEMBER 1965 AV PALM

Summary. The response of the sea to atmospheric pressure fluctuations associated with a turbulent wind is investigated. With a sudden onset of the wind over a limited area, the wave height spectrum becomes dependent on both space and time. This dependency can simply be expressed by an "influence function".

A formal solution for the directional wave height spectrum and the mean square height is derived. It is shown that both Eckart's (1953b) and Phillips' (1957) theoretical results can be obtained from this solution.

The case of an unlimited storm has been given special attention. Guided by some observations of the pressure correlation, the author finds that the directional dependency of the spectrum is contained in the factor

$$\frac{\operatorname{sech}\alpha}{1+a^2\left(1-\dfrac{c}{u}\operatorname{sech}\alpha\right)^2}$$

where α is the angle of wave propagation relative to the wind direction, u is the wind speed, c is the phase velocity and a is a constant. The agreement between theory and data is fairly good. Finally it is shown how the present model can be modified to give exponential wave growth in accordance with Miles' (1957) theoretical results.

CONTENTS

1. Introduction. In a review of 1956 URSELL summed up the knowledge about wind-generated waves and drew attention to the unsatisfactory theoretical understanding of the actual processes behind wave-generation.

Since then the insight into the dynamics of the generating mechanism and other features of wind-generated waves has been extensively deepened, mainly by the works of ECKART (1953 b), PHILLIPS (1957) and MILES (1957).

New methods in wave observation have given valuable information about the statistical properties of wind-generated waves. One could here refer to stereophotographical methods (S.W.O.P. 1960) and the wave-recording buoy. (See LONGUET-HIGGINS, CARTWRIGHT & SMITH 1962).

In 1963 LONGUET-HIGGINS reviewed the achieved knowledge (from observations) and some of the recent ideas.

The statistical properties of water waves are such that one could expect the linear theories to be a very good first approximation.

The observed wave heights and slopes fit well with a gaussian distribution. The relationship between frequency and wave number is found to agree well with that given by the linear theory of free surface waves. Wave energy from distant storm is shown to propagate approximately with the group velocity of infinitesimally small free surface waves. All these facts are very promising in respect to the validity of linear theories. But the non-linear processes are not to be forgotten. As shown by PHILLIPS (1960), LONGUET-HIGGINS (1962) and HASSELMAN (1962 and 1963) they may lead to an appreciable transfer of energy between different wavenumbers, the tendency is to smooth out the peaks in the spectrum. The wave growth is also limited by breaking. Observations seem to verify the asymptotical spectrum found by PHILLIPS (1958) for gravity waves.

As for the generating mechanism, we are now in a position to say that it is a kind of resonant interacting mechanism even though the mutual coupling between air and water motion, as in MILES' (1957) theory, or the forced water motion as in ECKART's (1953 b) and PHILLIPS' (1957) theories, is the most effective one.

Possibly, as MILES (1960) and others have pointed out, the mechanism is a combined one:

In the first stages the wave growth is forced by the random pressure fluctuations, and the mean square height then grows linearly with time. As the waves gain height or rather steepness, the mutual coupling may become more and more important, and the growth is then exponential according to MILES' theory.

Observations made by LONGUET-HIGGINS (1962 a) are favouring MILES' theory, but a considerable amount of measurements of the mean square height indicates a linear growth with time until the sea has reached a kind of saturated state.

In this paper as well as in ECKART's (1953 b) and PHILLIPS' (1957) we are concerned with the forced water motion. The response of the water to atmospheric pressure disturbances is determined through the linearized equation for the dynamic boundary condition. Friction is neglected, the water is assumed to be incompressible, the motion irrotational and the depth infinite.

Because so little is known about the near sea atmospheric turbulence the main trouble is to define the pressure field.

ECKART (1953 b) assumed that the pressure field could be represented by a random distribution of circular pressure gusts of a given characteristic radius L and duration T moving with the wind velocity u, all parameters being taken as constants.

Thus for a simple gust centred at origo at $t=0$ we have from ECKART's work:

$$p_0 = \text{const.}\, e^{-\frac{1}{2}[((x_1-ut)^2+x_2^2)/L^2+t^2/T^2]},$$

where x_1 and x_2 are horizontal coordinates.

ECKART considered a circular storm area and the gusts were given different weights depending upon their position relative to the storm centre.

By assuming steady state conditions he calculated the wave heights outside and inside the storm area.

For the mean square height at a distance X (much larger than the storm radius R) from the centre of the storm in the wind direction ECKART obtained

$$\overline{h^2} = 10\overline{p^2}\frac{R^2}{XL}$$

where $\overline{p^2}$ is the mean square pressure given in water height. ECKART concluded that values of $\overline{p^2}$ from experiments were too small in order to explain the observed wave heights by this formula.

PHILLIPS (1957) in his model considered an unlimited storm area with a sudden onset of the wind such that the conditions were spatially statistically homogeneous. He then applied Fourier-Stieltje integrals in his calculations. PHILLIPS defined an advection velocity u for the turbulent eddies as the velocity of a reference frame in which the eddies have their greatest characteristic time of development. The advection velocity was assumed to be equal to the wind speed at a height comparable with the scale of the eddies. If the mean square wave height and the pressure correlation in time are defined respectively as

$$\overline{h^2} = \int\int \phi(\boldsymbol{k}, t)\, dk_1 dk_2\,,$$

$$\overline{pp'} = \int\int \pi(\boldsymbol{k}, t) \cos(\boldsymbol{k}\cdot\boldsymbol{u}\tau)\, dk_1 dk_2\,,$$

PHILLIPS' result for large t can be expressed as

$$\phi(\boldsymbol{k},\ ) = \omega_0^2 t \int_0^\infty \pi(\boldsymbol{k}, t) \cos[(\boldsymbol{k}\cdot\boldsymbol{u}-\omega_0)\tau]\, d\tau\,,$$

where $\boldsymbol{k}$ $(k_1\, k_2)$ is the wave number vector and $\omega_0 = \sqrt{gk}$ is the frequency of free gravity waves. With a rough approximation for the integral and an estimated value of $\overline{p^2} = 0.1\rho_a^2 U^4$, PHILLIPS obtained results which did agree well with the observed values.

In his comments PHILLIPS tried to explain what he called, "the probable reason for the failure of ECKART's theory to predict the magnitude of the wave heights generated by wind" by saying: "His (ECKART's) less precise specification of the pressure distribution has smoothed off the resonance peaks of the response of the water surface, and it is the wave numbers near this peak that can contribute largely to the wave spectrum at large durations". This must be a misunderstanding. With the value of $\overline{p^2}$ given by PHILLIPS, ECKART's results are just as good as PHILLIPS' when compared with data.

The failure or success of both theories depend mainly on the magnitude of the mean square pressure.

As it is shown in paragraph 4c, the total energy input to the waves is only slightly dependent on the pressure distribution.

Even though the forced water model fails to explain the observed magnitude of the wave heights it reveals much of processes in a wind generated sea.

Neither ECKART's (1953 b) nor PHILLIPS' (1957) model alone are suited for wave prediction in a real ocean, ECKART's model consists of a limited storm area but he claims steady state conditions. — PHILLIPS' model is time dependent but it applies to an infinite storm area. But combined, these two theories might prove useful in wave forecasting.

In the present paper a model is developed which takes into account the storm geometry, fetch length and time of development as well as the displacement of the storm.

It is also shown that MILES' (1957) theoretical result with coupling and exponential growth may easily be incorporated in this model.

In order to determine the pressure correlation, use has been made of some recent measurements by WILLMARTH and WOOLDRIDGE (1962). It is found that in the reference frame of least decay, the decay of the pressure correlation is nearly exponential in time and the characteristic spectral decay time is a function of the wave number. This result is derived from measurements in a wind tunnel at a wall beneath a thick boundary layer, but it will be assumed that qualitatively it applies to conditions in the open.

First of all the pressure correlation is derived without specifying any of the parameters involved.

Following an idea by ECKART (1953 b) it will be assumed that the pressure field associated with the turbulent eddies in the air can be represented by random distributed pressure gusts. The gusts are assumed identical, of limited duration and advected with the wind velocity.

The gusts are distributed over infinite time and space but their influence will be weighted in such a way that they are only effective within a certain limited area and in a certain time. The water surface is assumed initially undisturbed. This is adequate with a sudden onset of the wind.

2. The pressure correlation. A simple gust centered at the place $\boldsymbol{x}_i$ at the time t_j has a pressure effect at the sea surface given by

$$p_{ij} = \int A(\boldsymbol{k}, \sigma) e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}_i) - i\sigma(t-t_j)} d\boldsymbol{k} d\sigma \tag{2.1}$$

where $d\boldsymbol{k}=dk_1dk_2$, $\boldsymbol{x}$ and $\boldsymbol{k}$ are horizontal vectors with components (x_1, x_2) and (k_1, k_2). The x_1- and k_1-axes will be taken in the wind direction. The integration goes from $-\infty$ to $+\infty$. The pressure correlation will now be defined as

$$\overline{p(\boldsymbol{x},t)p(\boldsymbol{x}',t')}= \int q(\boldsymbol{x}_i, t_j)p_{ij}p_{ij}'d\boldsymbol{x}_idt_j \tag{2.2}$$

where $q(\boldsymbol{x}_i, t_j)$ is a weight function which takes account of the geometry of the storm and its duration and

$$p_{ij}p_{ij}'= \int A(\boldsymbol{k}',\sigma')A^*(\boldsymbol{k},\sigma)e^{is}d\boldsymbol{k}d\boldsymbol{k}'d\sigma d\sigma' , \tag{2.3}$$

$$s=\boldsymbol{k}'\cdot\boldsymbol{x}'-\boldsymbol{k}\cdot\boldsymbol{x}-\sigma't'+\sigma t-(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{x}_i+(\sigma'-\sigma)t_j .$$

This can be simplified by a change of the variables

$$\bar{\boldsymbol{k}}=\frac{\boldsymbol{k}'+\boldsymbol{k}}{2}, \quad \bar{\boldsymbol{x}}=\frac{\boldsymbol{x}'+\boldsymbol{x}}{2}, \quad \bar{\sigma}=\frac{\sigma'+\sigma}{2}, \quad \bar{t}=\frac{t'+t}{2},$$

$$\boldsymbol{m}=\boldsymbol{k}'-\boldsymbol{k}, \quad \boldsymbol{r}=\boldsymbol{x}'-\boldsymbol{x}, \quad \varepsilon=\sigma'-\sigma, \quad \tau=t'-t .$$

Accordingly

$$d\boldsymbol{k}'d\boldsymbol{k}d\sigma'd\sigma=d\bar{\boldsymbol{k}}d\boldsymbol{m}d\bar{\sigma}d\varepsilon ,$$

$$s=\bar{\boldsymbol{k}}\cdot\boldsymbol{r}-\bar{\sigma}\tau+\boldsymbol{m}\cdot\bar{\boldsymbol{x}}-\varepsilon\bar{t}-\boldsymbol{m}\cdot\boldsymbol{x}_i+\varepsilon t_j , \tag{2.4}$$

$$(2\pi)^3A^*(\boldsymbol{k},\sigma)A(\boldsymbol{k}',\sigma')=B(\bar{\boldsymbol{k}},\bar{\sigma},\boldsymbol{m},\varepsilon) .$$

The weight function has such a form that it can be written as a Fourier integral

$$q(\boldsymbol{x}_i,t_j)= \int Q(\boldsymbol{m},\varepsilon)e^{i(\boldsymbol{m}\cdot\boldsymbol{x}_i-\varepsilon t_j)}d\boldsymbol{m}d\varepsilon , \tag{2.5}$$

$$(2\pi)^3Q(\boldsymbol{m},\varepsilon)= \int q(\boldsymbol{x}_i,t_j)e^{-i\boldsymbol{m}\cdot\boldsymbol{x}_i+i\varepsilon t_j}d\boldsymbol{x}_idt_j .$$

Using this, the equation (2.2) takes the form

$$\overline{pp'}= \int B(\boldsymbol{k},\sigma,\boldsymbol{m},\varepsilon)Q(\boldsymbol{m},\varepsilon)e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\sigma\tau)+i(\boldsymbol{m}\cdot\boldsymbol{x}-\varepsilon t)}d\boldsymbol{k}d\sigma d\boldsymbol{m}d\varepsilon . \tag{2.6}$$

Integration over $\boldsymbol{x}_i$ and t_j has been performed with the aid of equation (2.5). Now, since the scales of the storm are much larger than the scales of the gusts, it follows that the integrand in equation (2.6) is dominated by $Q(\boldsymbol{m}, \varepsilon)$. The integrand is only significant when ε and $\boldsymbol{m}$ are small compared with the inverse values of the storm scales. It is then evident that $\boldsymbol{m}$ and ε can both be put equal to zero in B and the integration performed, giving

$$\overline{pp'}=q(\bar{\boldsymbol{x}},\bar{t}) \int B(\boldsymbol{k},\sigma)e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i\sigma\tau}d\boldsymbol{k}d\sigma . \tag{2.7}$$

In the case of a circular storm of radius R and duration $\mathcal{T}$, moving with constant velocity $\boldsymbol{v}$, a convenient form of $q(\boldsymbol{x}, t)$ is

(2.8) $$q = e^{-(x-vt)^2/2R^2 - t^2/2\mathcal{T}^2},$$

and accordingly for the transform we obtain

$$Q = (2\pi)^{-3/2} R^2 \mathcal{T} e^{-(m^2R^2/2) - (m\cdot v - \varepsilon)^2\mathcal{T}^2/2}$$

which can serve to illustrate what has just been stated. The pressure correlation can always formally be written as (2.7). But it is desirable to find an alternative and more specific expression for the spectrum in agreement with experimental results. Measurements have shown that the variation of the pressure correlation with timelag is least when the correlation is taken in a reference system moving downwind with a certain velocity u. This is defined as the advection velocity of the gusts and for most winds except very light ones it can be identified with the wind speed. Accordingly the pressure correlation may be expressed as

(2.9) $$\overline{pp'} = \int \psi(k) e^{ik\cdot(r - u\tau)} D(k, \tau) dk ,$$

where u is the vectorial advection velocity taken in the r_1 direction, $D(k, \tau)$ is called a decay function since by definition it is unity for $\tau = 0$ and decreases with increasing values of τ, $\psi(k)$ is the spatial pressure spectrum,

$$\overline{p^2} = \int \psi(k) dk .$$

For correspondence between (2.7) and (2.9) we must have;

$$B(k, \sigma) = \frac{1}{2\pi} \psi(k) \int D(k, \tau) e^{i(\sigma - k\cdot u)\tau} \, d\tau .$$

Unfortunately there are only a few measurements of the decay function and those we have are from flows in pipes and wind tunnels. Some measurements of the pressure correlation at the wall beneath a thick boundary layer can serve to illustrate the properties of the decay function. (Willmarth and Wooldridge (1962)).

A plot of the function D against $k_1 u\tau/2\pi$ becomes quite remarkable when it is transferred to a linear-logaritmic diagram. See figure 1.

The decay function can be very well approximated by a straight line through the points. Thus

(2.10) $$D = e^{-(|k_1 u\tau|/a)}, \quad a = 23.2 .$$

A consequence of (2.10) is

(2.11) $$B(k, \sigma) = \frac{1}{\pi} \psi(k) \frac{\left|\dfrac{a}{k_1 u}\right|}{1 + a^2 \dfrac{(k\cdot u - \sigma)^2}{(k_1 u)^2}} .$$

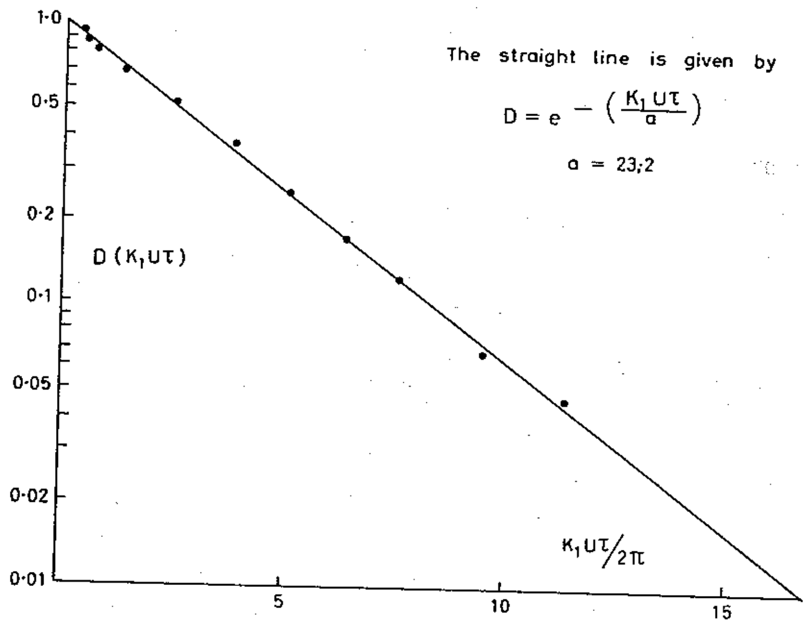

Fig. 1. The decay of the pressure correlation as observed in a reference system of least decay. The dots indicate values from observations by WILLMARTH and WOOLDRIDGE.

3. The water surface response. The water is assumed to be frictionless and incompressible, and the motion is starting from rest. There exists then a velocity potential for the motion which has to satisfy the Laplace equation. Thus

(3.1)
$$v=\nabla\phi\,,\quad \nabla^2\phi=0\,.$$

The dynamical and kinematical boundary conditions are respectively (linearized),

(3.2)
$$\frac{\partial\phi}{\partial t}+gh-\frac{\gamma}{\rho}\nabla_1^2 h=-gp \qquad (z=0)\,,$$

$$\frac{\partial h}{\partial t}=\left[\frac{\partial\phi}{\partial z}\right]_{z=0}$$

$$v\to 0 \qquad z\to-\infty\,.$$

The pressure is given in water height, h is the wave height reckoned from the plane of equilibrium, γ is the surface tension, ρ is the density of water, and

$$\nabla_1^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}\,.$$

In order to satisfy the infinite depth condition a solution of Laplace equation is

$$\phi=\phi(t)e^{i(k\cdot x+kz)}\,.$$

In order to calculate the effect of a simple gust a corresponding form of h_{ij} is

(3.3)
$$h_{ij}=\int C(k,t,t_j)e^{ik\cdot(x-x_i)}dk\,.$$

Then we have from (2.2)

$$\ddot{C}+\omega^2 C=-gk\int A(k,\sigma)e^{-i\sigma(t-t_j)}d\sigma \tag{3.4}$$

where $\omega^2=gk+(\gamma/\rho)k^3$ is the square frequency of free surface waves. With initial conditions $C=0$, $\dot{C}=0$, for $t=0$, the solution of (3.4) is

$$C=-\frac{gk}{\omega}\int\int_0^t A(k,\sigma)e^{i\sigma t_j-i\sigma\tau}\sin\omega(t-\tau)d\tau d\sigma\,. \tag{3.5}$$

The wave height correlation is defined as

$$\overline{hh'}=\int q(x_i,t_j)h_{ij}h_{ij}'dx_i dt_j\,. \tag{3.6}$$

By putting $t'=t$ and $\boldsymbol{x}'=\boldsymbol{x}$, we get the mean square height. Doing this, together with a change of variables as in the 2nd paragraph, and integrating over $\boldsymbol{x}_i$ and t_j we obtain

$$\overline{h^2}=g^2\int\frac{kk'}{\omega\omega'}B(\boldsymbol{k},\sigma,\boldsymbol{m},\varepsilon)Q(\boldsymbol{m},\varepsilon)e^{i\boldsymbol{m}\cdot\boldsymbol{x}}I\,d\boldsymbol{k}d\sigma d\boldsymbol{m}d\varepsilon\,,$$

$$I=\tfrac{1}{2}\int\int e^{-i\sigma\tau_2-i\varepsilon\tau_1}\left\{\cos\left[(\omega'-\omega)(t-\tau_1)-\frac{\omega'+\omega}{2}\tau_2\right]-\cos\left[(\omega'+\omega)(t-\tau_1)-\frac{\omega'-\omega}{2}\tau_2\right]\right\}d\tau_1 d\tau_2$$

Here the new variables τ_1 and τ_2 are:

$$\tau_1=\tfrac{1}{2}(\tau'+\tau)\,,\quad \tau_2=\tau'-\tau\,,$$

where both τ and τ' go from 0 to t.

By repeating the arguments from paragraph 2 we find that the integrand is dominated by $Q(\boldsymbol{m},\varepsilon)$ which is only significant when

$$\boldsymbol{m}\equiv\boldsymbol{k}'-\boldsymbol{k}\approx 0 \quad\text{and}\quad \varepsilon\equiv\sigma'-\sigma\approx 0\,.$$

The following approximations are then valid for small $\boldsymbol{m}$ and ε

$$B(\boldsymbol{k},\sigma,\boldsymbol{m},\varepsilon)\approx B(\boldsymbol{k},\sigma,0,0)\quad\text{called}\quad B(\boldsymbol{k},\sigma)\,,$$

$$kk'\approx k^2\,,\quad \omega\omega'\approx\omega^2\,,\quad \omega'+\omega\approx 2\omega\,,$$

$$\omega'-\omega\approx\frac{\partial\omega}{\partial k_1}m_1+\frac{\partial\omega}{\partial k_2}m_2=\frac{d\omega}{dk}\frac{\boldsymbol{m}\cdot\boldsymbol{k}}{k}\,.$$

The last term of the time integral I turns out to be negligibly small in comparison with the first and we will therefore only consider the first term. Thus,

$$\overline{h^2}=\tfrac{1}{2}g^2\int\frac{k^2}{\omega^2}B(\boldsymbol{k},\sigma)Q(\boldsymbol{m},\varepsilon)e^{i(\boldsymbol{m}\cdot\boldsymbol{x}-\varepsilon\tau_1)-i\sigma\tau_2}\cos\left[(\boldsymbol{m}\cdot\boldsymbol{G})(t-\tau_1)-\omega\tau_2\right]d\boldsymbol{k}d\sigma d\boldsymbol{m}d\varepsilon d\tau_1 d\tau_2 \tag{3.7}$$

where $\boldsymbol{G}=(d\omega/dk)\boldsymbol{k}/k$ is the group velocity of free surface waves. By comparing (3.7) with (2.5) it is seen that integration over $\boldsymbol{m}$ and ε can now be performed. Then (3.7) yields

$$\overline{h^2} = \overline{h^2_+} + \overline{h^2_-}\,,$$

(3.8) $$\overline{h^2}_{+,-} = \frac{g^2}{4} \int \frac{k^2}{\omega^2} B(\boldsymbol{k},\sigma) q(\boldsymbol{x} \pm \boldsymbol{G}(t-\tau_1),\tau_1) e^{-i(\sigma\pm\omega)\tau_2} d\boldsymbol{k}, d\sigma, d\tau_1 d\tau_2\,.$$

The integrations over τ_1 and τ_2 can best be treated by considering figure 2.

The integration has to be taken in two steps, for example over the triangles ABD and BCD:

$$\int_0^{t/2} F_1(\tau_1) d\tau_1 \int_{-2\tau_1}^{2\tau_1} F_2 d\tau_2 + \int_{t/2}^{t} F_1(\tau_1) d\tau_1 \int_{-2(t-\tau_1)}^{2(t-\tau_1)} F_2 d\tau_2 = \int_0^{t/2} [F_1(\tau_1) + F_1(t-\tau_1)] d\tau_1 \int_{-2\tau_1}^{2\tau_1} F_2 d\tau_2\,.$$

The integration over τ_2 can now be performed:

$$\int_{-2\tau}^{2\tau} e^{-i(\sigma\pm\omega)\tau_2} d\tau_2 = 2\frac{\sin 2(\sigma\pm\omega)\tau}{\sigma\pm\omega}\,.$$

Since t is assumed to be much greater than $1/\omega_c$, where ω_c is a characteristic frequency of wind generated surface waves, of order 1 sek^{-1}, the integrand will be dominated by this term for most values of τ except for the smallest ones. The contribution from integration over σ is largest when $\sigma = \mp\omega$. The function is treated as a δ-function and integration performed, σ being put equal to $\mp\omega$ in B $(\boldsymbol{k}, \sigma)$ which is a slowly varying function of σ.

Then we get

(3.9) $$\overline{h^2_{+,-}} = \frac{\pi}{2} g^2 \int \frac{k^2}{\omega^2} [B(\boldsymbol{k}, \mp\sigma)]_{\sigma=\omega} d\boldsymbol{k} \int_0^t q(\boldsymbol{x} \pm \boldsymbol{G}(t-\tau), \tau) d\tau\,.$$

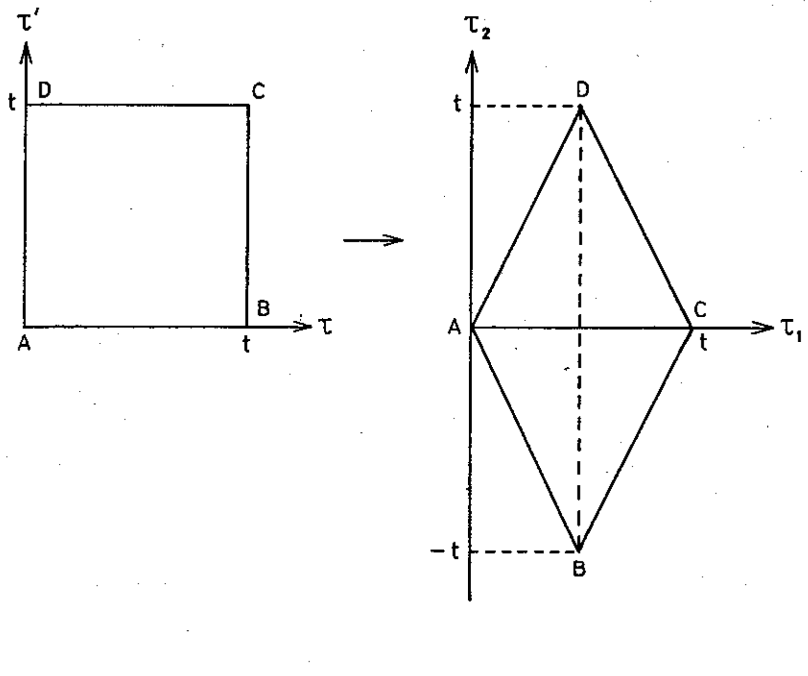

Fig. 2. The area, $ABCD$, of integration in the two coordinate systems.

Since by definition

$$B(k,\sigma)=(2\pi)^{-3}\int \overline{pp'}\cos(k\cdot r-\sigma\tau)drd\tau$$

it follows that

$$B(k,-\sigma)=B(-k,\sigma)\,,$$

$$B(k,\sigma)=B(-k,-\sigma)\,.$$

The integration over k_1 from $-\infty$ to ∞ can then be substituted by an integration from 0 to ∞. Finally we get

$$\overline{h^2}=\pi g^2\int c^{-2}B_1(k)\int_0^t q(x-(t-\tau)G,\tau)d\tau dk$$

(3.10)

$$+\pi g^2\int c^{-2}B_2(k)\int_0^t q(x+(t-\tau)G,\tau)d\tau dk\,,$$

where $B_1(k)=(B(k,\sigma))_{\sigma=\omega}$ and $B_2(k)=(B(k,-\sigma))_{\sigma=\omega}$.

The integration is over the right half k-plane. The last term gives the contribution from waves going against the wind, but this term is negligible compared with the first and can accordingly be dropped.

The time integral in (3.10) will play an important part in the following analysis. It will be denoted by the letter w and called the influence function,

$$w=\int_0^t q[x-(t-\tau)G,\tau]d\tau\,. \tag{3.11}$$

The final result can then be written in shortened form as

$$\overline{h^2}=\pi g^2\int c^{-2}B_1(k)w(x,t,k)dk\,. \tag{3.12}$$

The name "influence function" has been chosen because w determines the time and space dependency of the spectrum.

In paragraph 4 it will be clear that the integrand q can be interpreted as a wave filter since it determines the permissable range of wave numbers for a given x and t.

4. Some special examples.

a. *Moving storm.* Thus far the result is general. Now we will proceed by taking some special examples. The case of a storm moving with velocity v has earlier been mentioned. This will be treated again with storm duration $\mathcal{T}\gg t$. From (2.8) we then have

$$q(x,t)=e^{-(x-vt)^2/2R^2}\,.$$

Accordingly the influence function, (see (3.11) and (3.12)), is

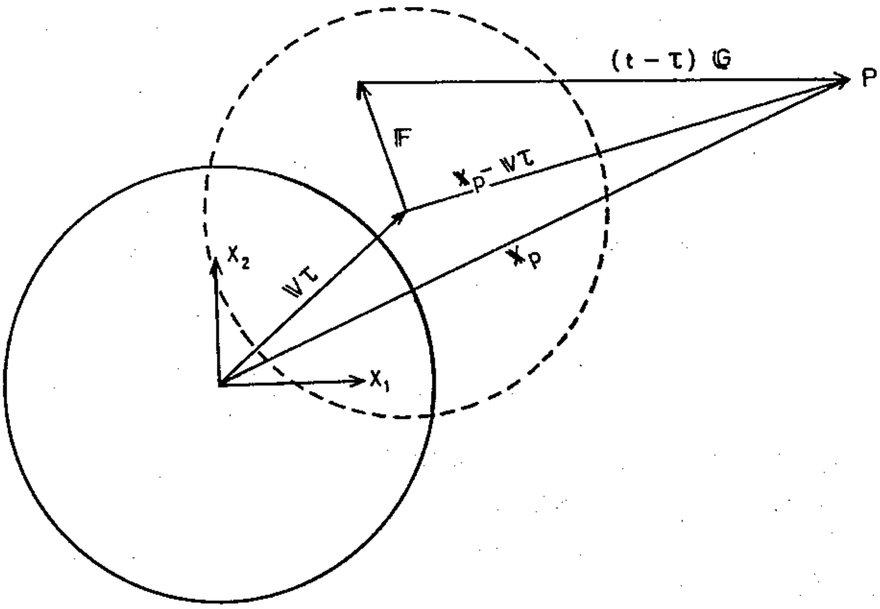

Fig. 3. The case of a moving circular storm.

$$w = \int_0^t q[x' - G(t-\tau), \tau] d\tau = \int_0^t e^{-F^2/2R^2} d\tau\,,$$

$$x' = x - v\tau\,, \quad F = \left| x' - (t-\tau) G \right|.$$

The contribution is greatest for small F, that is $F < R$. Figure 3 illustrates the case.

Only waves starting from places inside the dotted circle at time τ with appropriate direction can give appreciable contribution to $\overline{h^2}$ at the place x at a later time t, provided that the time interval $(t-\tau)$ is long enough for the waves to reach x. As the storm is moving along, waves are propagated from new areas traversed by the storm. We see that the wave energy is propagated with velocity G, the group velocity. The mean square height of waves within the storm grows proportionally with time.

b. *A stationary storm.* The case of stationary storm is of some interest. If we further let the time $t \to \infty$ as well as the storm duration, we get the case which has been treated by Eckart (1953b). Thus,

$$w = \int_0^\infty e^{-[(x-G\tau)^2/2R^2]} d\tau = e^{-[x^2 \sin^2\theta/2R^2]} \int_0^\infty e^{-[(G\tau - x\cos\theta)^2/2R^2]} d\tau\,,$$

$$\cos\theta = \frac{x \cdot G}{xG}\,.$$

For $x/R \gg 1$ this is simply

$$w \approx \frac{\sqrt{2\pi} R}{\frac{d\omega}{dk}} e^{-x^2\theta^2/2R^2}$$

since θ has to be small.

Equation (3.10) now gives, after a change to polar coordinates,

$$k_1 = k\cos\alpha\,,\quad x_1 = x\cos\phi\,,\quad \theta = \phi - \alpha\,,$$

$$k_2 = k\sin\alpha\,,\quad x_2 = x\sin\phi\,,$$

$$\overline{h^2} = \tfrac{1}{2}(2\pi)^{3/2} Rg \int_0^{\infty}\int_{-\pi/2}^{\pi/2} c^{-2}\left(\frac{d\omega}{dk}\right)^{-1} B_1 e^{-x^2\theta^2/2R^2} k\,dk\,d\alpha$$

$$\approx (2\pi)^2 \frac{R^2}{x}\sqrt{g}\int_0^{\infty} k^{5/2}(B_1(k))_{\alpha=\phi}\,dk\,.$$

Integration over α has been performed in view of the dominating exponential term, and the relation $\omega^2 = gk$ for gravity waves has been invoked.

In order to carry out the last integration, the function B_1 must be specified. In doing so, with

$$[B_1]_{\alpha=\phi} = \upsilon\overline{p^2}\, e^{-[k^2L^2 + T^2(\omega - uk\cos\phi)^2]}$$

where υ is a constant depending upon the parameters L and T which are respectively the characteristic radius and duration of the pressure gusts, Eckart obtained

$$\overline{h^2} \approx 10\,\overline{p^2}\frac{R^2}{xL} \quad \text{for} \quad \phi = 0\,.$$

From the available data of the magnitude of the mean square pressure he concluded that this equation gave wave heights which were much smaller than those obtained from observation.

c. *Unlimited storm.* This is the case which has been treated by Phillips (1957). The function q is then equal to unity, $w = t$ and (3.10) yields

$$\overline{h^2} = t\pi g^2 \int c^{-2} B_1(k)\,dk\,. \tag{4.1}$$

Remembering the way B_1 was derived, $\overline{h_2}$ can also be written

$$\overline{h^2} = g^2 t \int c^{-2}\psi(k)\,dk \int_0^{\infty} D(k,\tau)\cos((k\cdot u - \omega)\tau)\,d\tau\,. \tag{4.2}$$

Since $\omega^2 = g^2c^{-2}$ for gravity waves, this result is in full agreement with that obtained by Phillips. The pressure spectrum which he called $\pi(k, \tau)$ is here written as $\psi(k)D(k, \tau)$. It will now be shown that this integral can be estimated without specifying the parameters involved.

From the second mean-value theorem for integrals we can formally write

$$\int_0^{\infty} D(k,\tau)\cos((k\cdot u - \omega)\tau)\,d\tau = \int_0^{T(k)} \cos((k\cdot u - \omega)\tau)\,d\tau\,,$$

since $D(\boldsymbol{k}, 0)=1$ and $D(\boldsymbol{k}, \infty)=0$. Accordingly, after a change to polar coordinates, we have

(4.3) $$\overline{h^2}=\frac{gt}{u}\int_0^\infty \psi(k)kdk \int_{-\pi/2}^{\pi/2} \frac{\sin\left(\left(\cos\alpha-\frac{c}{u}\right)kuT\right)}{\cos\alpha-\frac{c}{u}}d\alpha ,$$

where $\psi(\boldsymbol{k})$ has been assumed to be an isotropic function of wavenumber.

Since

(4.4) $$\overline{p^2}=2\pi\int_0^\infty \psi(k)kdk$$

it follows from the first mean-value theorem for integrals that

(4.5) $$\overline{h^2}=\frac{gt}{u}\overline{p^2}R(k_s) ,$$

where $$R(k_s)=\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \frac{\sin\left[\left(\cos\alpha-\left(\frac{c}{u}\right)_s\right)(kuT)_s\right]}{\cos\alpha-\left(\frac{c}{u}\right)_s}d\alpha$$

and k_s is a specific value in the interval $(0, \infty)$.

The characteristic time, T, might possibly be a function of α. In the discussion which follows T is regarded as a function only of k. But since T is always positive, the final result is expected to be valid in the general case.

It is of interest to see the variation of $R(k)$ as a function of c/u and $N=kuT/2\pi$. The last parameter may be interpreted as the characteristic number of periods of development.

The maximum value of R for $N>1$ is approximately

$$R(k)_{\max}=R\left(\frac{g}{u^2}\right)\approx\frac{1}{2\pi}\int_{-\infty}^{\infty} 2\frac{\sin(\pi N\alpha^2)}{\alpha^2}d\alpha=\sqrt{2N} .$$

Having established this upper limit the question still remains: "What is the most likely value of k_s and $R(k_s)$?"

The information we have about the pressure spectrum indicates that the maximum is only slightly dependent upon the wind speed. Since the maximum of $R(k)$ occurs for $k=g/u^2$ we find that for wind speeds of order 10 m/sec or higher the main contribution to the mean square pressure comes from the range of wave numbers where $c/u<1$ and $R(k)$ is close to unity. The highest value of $R(k_s)$ is expected when the maximum of $k\psi(k)$, nearly coincides with the maximum of $R(k)$, but this is only possible for light winds. The lower limit of $R(k_s)$ is therefore taken to be unity.

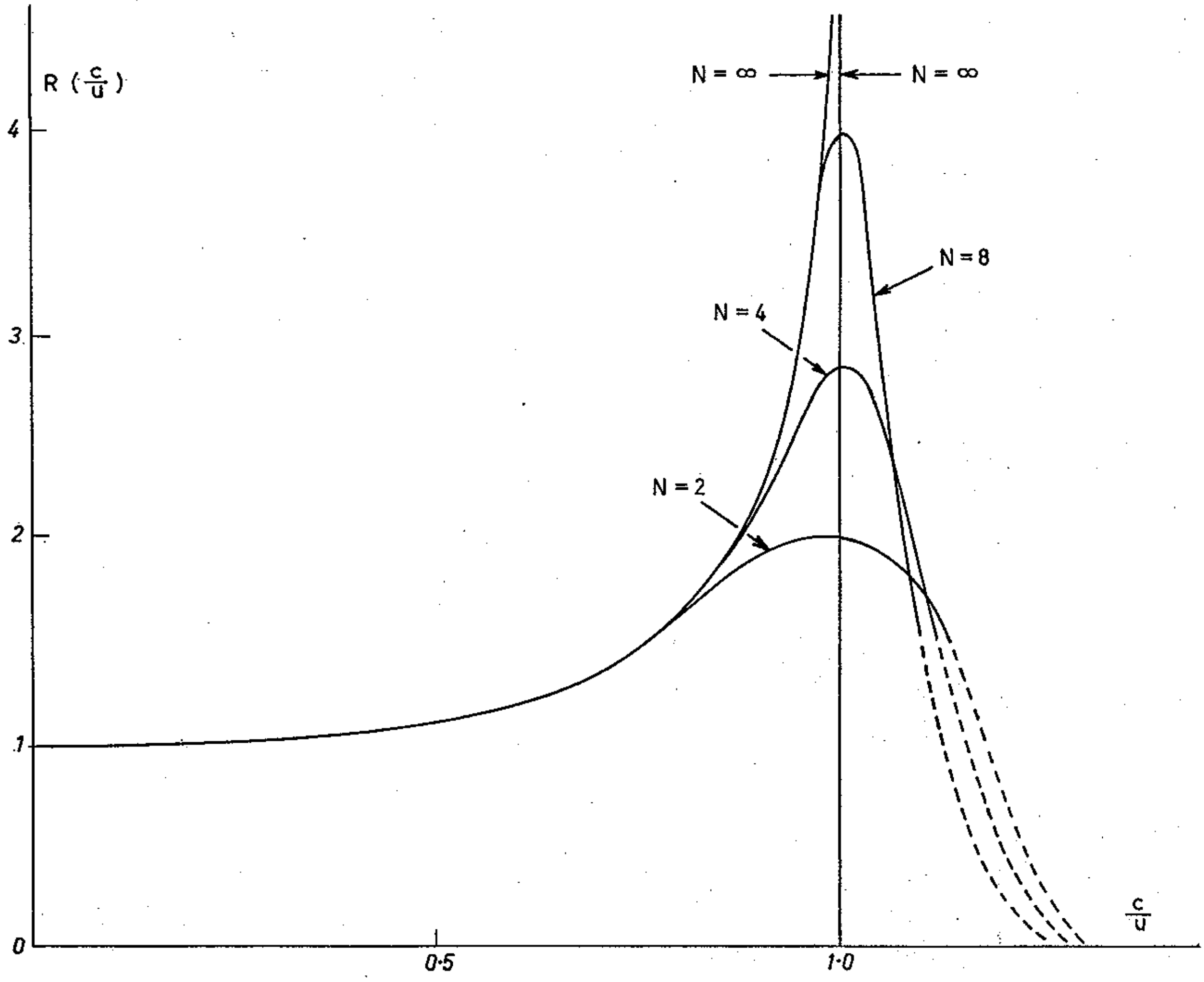

Fig. 4. The relationship between the function R and c/U for 4 different values of the parameter N.

Thus

$$1 < R(k_s) < \sqrt{2N} \quad \text{for} \quad N > 1 .$$

$R(k_s)$ is seen to be smaller than 10 even for an extremely large value of N such as 50. A more realistic upper limit seems to be 10 periods of development which indicate a value of $R(k_s)$ of order unity.

Then we get the interesting result

$$\overline{h^2} \approx \frac{gt}{u} \overline{p^2} . \tag{4.6}$$

Instead of referring to mean square height, it is oceanographic practice to use significant waveheight H defined by

$$H^2 = 8\overline{h^2} .$$

If we take $\overline{p^2}=\beta^2(\rho_a/\rho_w)^2(u^4/g^2)$ it follows from (4.6) that

$$\frac{gH}{u^2}=2\sqrt{2}\,\beta\left(\frac{\rho_a}{\rho_w}\right)\left(\frac{gt}{u}\right)^{\frac{1}{2}} \tag{4.7}$$

Comparing with data given by SVERDRUP and MUNK (1947) and others, see ROLL (1957), we must have

$$\beta \geqslant 0.15$$

in order to explain the observed amplitudes by this model. However, measurements of the mean square pressure made with the recording buoy indicate that β is much smaller. Even though the forced water motion model fails to explain the observed energy input to the waves, it might qualitatively give a good description of the wave spectrum. It has in fact been found that the width of the observed spectrum corresponds well with the theoretical resonance angle.

d. *The directional spectrum.* In this paragraph the directional dependency will be specified by use of (2.11). In polar coordinates, the mean square height is then given by

$$\overline{h^2}=\frac{gt}{u}\int_0^\infty k\psi(k)dk\int_{-\pi/2}^{\pi/2}\frac{a\,\mathrm{sech}\,\alpha d\alpha}{1+a^2\left(1-\frac{c}{u}\mathrm{sech}\,\alpha\right)^2}. \tag{4.8}$$

The function

$$F(\alpha)=\mu\frac{\mathrm{sech}\,\alpha}{1+a^2\left(1-\frac{c}{u}\mathrm{sech}\,\alpha\right)^2}$$

has been plotted in fig. 5 for 3 different values of c/u and compared with data from S.W.O.P., the STEREO WAVE OBSERVATION PROJECT (1960). The factor μ, taking different values with c/u, is chosen so that the scale of $F(\alpha)$ fits that of the data, u is taken as the wind speed at 10 m height, the constant $a=4$. The agreement is best for the longest waves, that is for $c/u\geqslant 1$. But still more observations of the directional spectrum are needed before any conclusion about the validity of the theoretical spectrum can be made.

In order to compare the directional spectrum with data obtained by LONGUET-HIGGINS (1962 a) from the recording buoy, the angular width ψ_1 of the spectrum has been calculated where ψ_1 is defined as

$$\psi_1^2=\frac{\int_0^{\pi/2}\alpha^2F(\alpha)d\alpha}{\int_0^{\pi/2}F(\alpha)d\alpha}$$

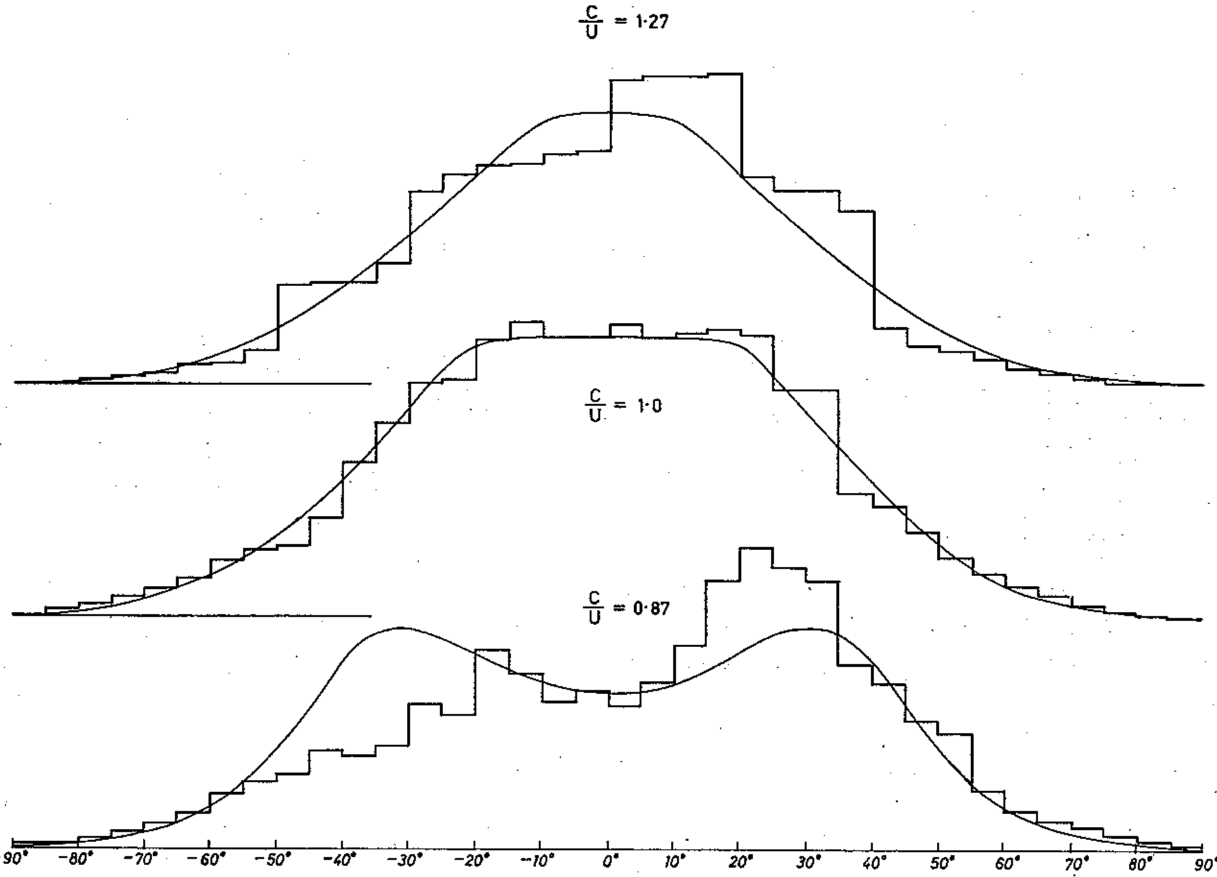

Fig. 5. The theoretical directional spectrum compared with data from the Stereo Wave Observation Project. The angular distribution is shown for three values of c/U, or adequately, for three different wavelengths.

The result is shown in Fig. 6 where ψ_1 has been plotted as a function of u_1/c instead of c/u by use of the formula

$$\frac{u}{c}=\frac{u_1}{c}\ln\frac{2\pi}{\Omega}\left(\frac{u_1}{c}\right)^{-2}, \quad \Omega=\frac{gz_0}{u_1^2}.$$

This means that u has been obtained from the logarithmic wind profile taking the wind speed at a height $2\pi/k$ which corresponds to the scale of the eddies, Ω has been chosen as $2\pi \cdot 10^{-2}$. The resonance curve is given by $\cos\psi_1=c/u$. The agreement between theory and data is fairly good except for $u_1/c<0.1$. The minimum angular width given by the theoretical formula is 29° and ψ_1 is almost constant for $u_1/c<0.1$. For large values of u_1/c, ψ_1 tends to the resonance curve.

e. *Wind blowing from a shore.* Again by using (3.10) we can easily treat the case with wind blowing from a shore in any direction, but for simplicity we will choose the wind direction normally to a straight shoreline given by $x_1=0$.

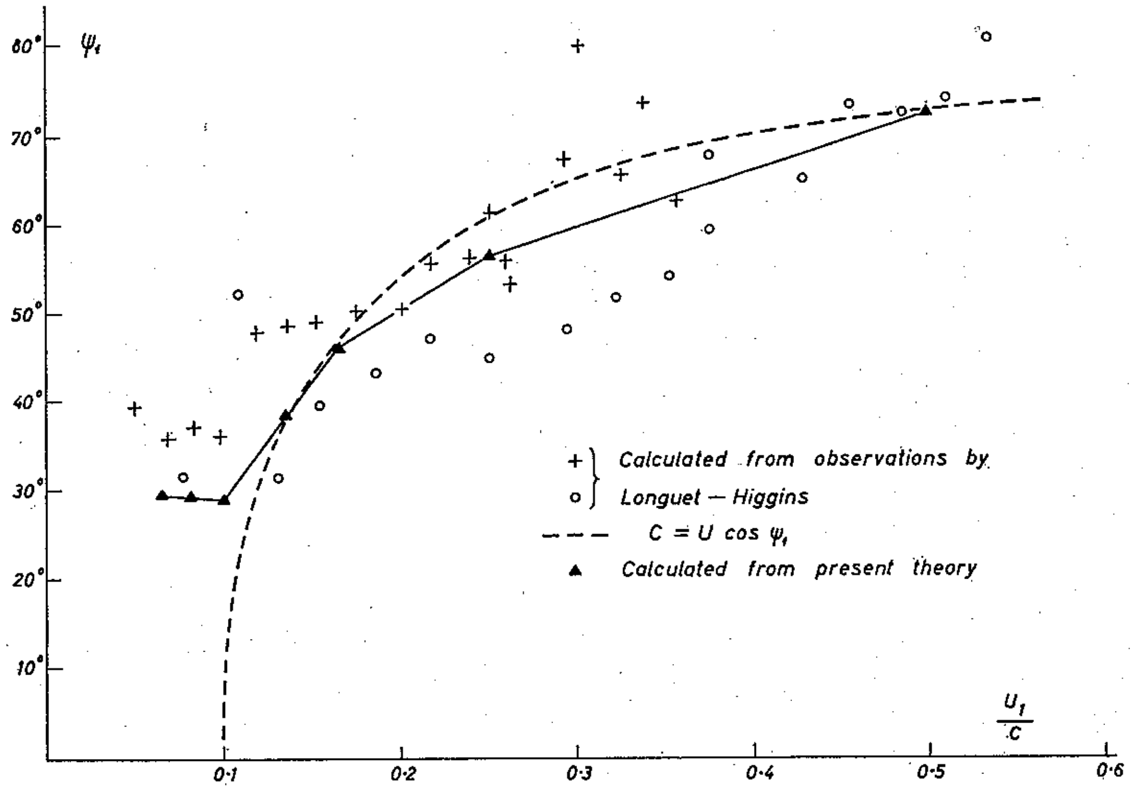

Fig. 6. The angular width ψ_1 of the spectrum. The resonance curve is also shown.

Assuming infinite duration time we must have in (2.7):

$q(\boldsymbol{x})=1$ when $x_1 \geqslant 0$, and zero otherwise. Accordingly in (3.11), by change of variable from τ' to $(t-\tau)$, we get:

$q(\boldsymbol{x}-\boldsymbol{G}\tau)=1$ when $x_1-G_1\tau \geqslant 0$ and zero otherwise. Hence,

$$0 \geqslant \tau \geqslant \frac{x_1}{G_1} = \frac{x_1}{\frac{d\omega}{dk}\cos\alpha} .$$

If the wind has blown for a sufficiently long time we obtain

$$\overline{h^2} = \pi g^2 x_1 \int_0^\infty \frac{c^{-2} k dk}{\frac{d\omega}{dk}} \int_{-\pi/2}^{\pi/2} \frac{B_1(k)}{\cos\alpha} d\alpha .$$

We see that the mean square height is directly proportional to the distance from the shore and that the waves travelling nearly normal to the wind direction are given large weight. Their contribution to the mean square height is somewhat suppressed by the form of $B_1(\boldsymbol{k})$. The greatest angle for resonance waves is

$$\alpha = \arccos\left(\frac{c_{\min}}{u}\right), \quad c_{\min} = 23\,\text{cm/sek}\,.$$

At short distances from a shore we are thus likely to observe pronounced shortcrested waves travelling nearly normal to the wind direction ($u \geqslant 3$ m/sec). This is confirmed by observations. Cfr. ROLL (1957).

f. *Swell.* The longer a gravity wave, the faster it propagates. Observing waves from a storm we should therefore expect that the spectrum is changing with time, getting broader as the shorter waves approach. This has also been confirmed by observation, but has not until recently been properly explained. In order to investigate this case with the aid of (3.11) and (3.12), we may start with a very simple configuration of a storm area. The storm is defined by $q(\boldsymbol{x})$ where $q(\boldsymbol{x})=1$ when $|\boldsymbol{x}| \leqslant R$ and zero otherwise. Consequently

$$q(\boldsymbol{x}-\boldsymbol{G}\tau)=1 \quad \text{when} \quad |\boldsymbol{x}-\boldsymbol{G}\tau| \leqslant R\,, \quad 0 \leq \tau \leqslant t\,, \quad \text{and}$$

zero otherwise.

This determines the integration time for τ and imposes restrictions on $Gt = \frac{1}{2}\sqrt{g/k}\; t$. The limits of α are also determined by the above inequality.

For a given t we then must have for a specific value of α either

(1.) $Gt \leq S_2$ that is $k \geqslant \dfrac{gt^2}{4S_2^2}$ and $w=0$,

or (2.) $S_2 \leq Gt \leq S$ that is $\dfrac{gt^2}{4S^2} \leq k \leq \dfrac{gt^2}{4S_2^2}$ and $w=t$,

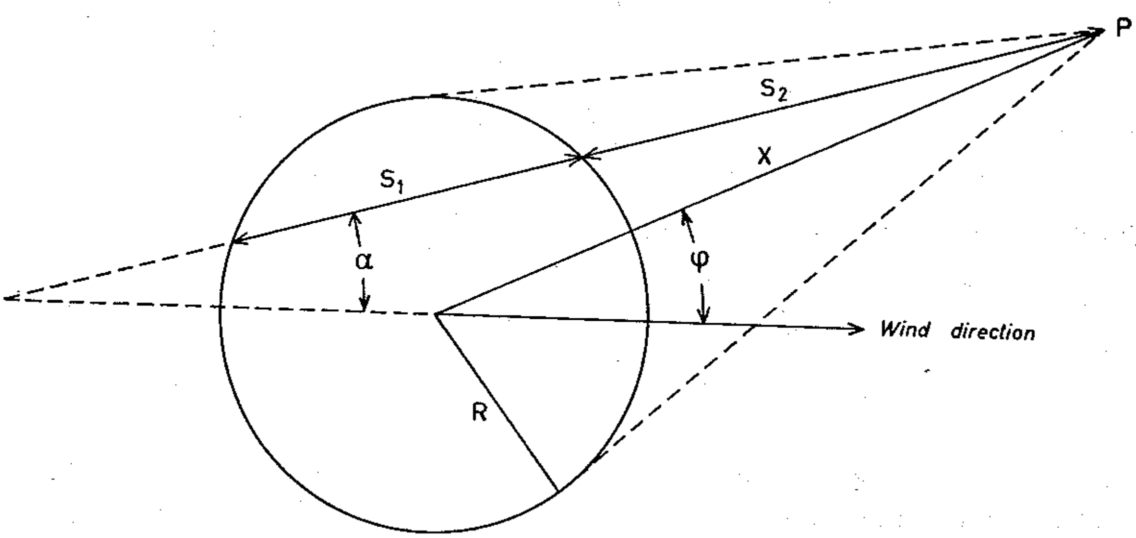

Fig. 7. Swell from a circular storm.

or (3.) $Gt \geqslant S$ that is $0<k\leq\frac{gt^2}{4S^2}$ and $w=\frac{S_1}{G}$,

where
$$S_1=2\sqrt{R^2-x^2\sin^2(\alpha-\phi)}\,,$$
$$S_2=x\cos(\phi-\alpha)-\sqrt{R^2-x^2\sin^2(\alpha-\phi)}\,,$$
$$S\;=S_1+S_2\,.$$

The mean square height at P can then be calculated from (3.10). In polar coordinates, the solution is

(4.9)
$$(\overline{h^2})_p=\pi g^2\int_{\alpha_1}^{\alpha_2}\int_0^{k_a}c^{-2}B_1(k)\frac{S_1}{G}kdkd\alpha$$
$$+\pi g^2\int_{\alpha_1}^{\alpha_2}\int_{k_a}^{k_b}c^{-2}B_1(k)tkdkd\alpha\,,$$

where
$$\alpha_1=\phi-\arcsin\frac{R}{x}\,,$$
$$\alpha_2=\phi+\arcsin\frac{R}{x}\,,$$
$$k_a=\frac{gt^2}{4S^2}\,,\quad k_b=\frac{gt^2}{4S_2^2}\,.$$

When $x/R\gg 1$ the solution may be approximated by
$$(\overline{h^2})_p=2\pi g^2\int_{-R/x}^{R/x}\int_0^{k_m}c^{-3}(B_1(k))_{\alpha=\phi}\cdot 2\sqrt{R^2-x^2(\alpha-\phi)^2}kdkd(\alpha-\phi)$$

The integration over α can then be performed and the result is

(4.10)
$$(\overline{h^2})_p=2\pi^2g^{\frac{1}{2}}\frac{R^2}{x}\int_0^{k_m}k^{5/2}[B_1(k)]_{\alpha=\phi}dk\,,$$

where
$$k_m=\frac{gt^2}{4x^2}\,,$$

k_m being the highest wave number present in a wave record at the time t. On the contrary, from the wave record we should be able to locate the origin of the waves. The result, equation (4.9), agrees also with the Cauchy-Poisson theory for propagation of surface waves.

The actual wave propagation will of course be greatly modified by effects of viscosity through dissipation and by the air resistance as well as other factors. The energy loss is greatest for the shortest waves. Since the maxima in $B_1(\boldsymbol{k})$ is decreased and shifted towards higher k-values as α is increased, it is obvious that swell can only be registered at small angles with the wind direction (within 30°).

g. *Model with exponential growth.* As mentioned in the introduction, MILES' (1957) theoretical result for the wave growth (i.e. the exponential increase caused by coupling) may easily be incorporated in the present model.

The actual change of (3.10) is then

$$\overline{h^2}=\pi g^2 \int c^{-2}B_1(\boldsymbol{k})d\boldsymbol{k} \int_0^t e^{M\tau}q(\boldsymbol{x}-(t-\tau)\boldsymbol{G},\tau)d\tau$$

when waves going against the wind have been neglected.

The only difference between the original equation and this equation is the exponential factor $e^{M\tau}$ in the time integral, M is a function of $\boldsymbol{k}$, the two dimensional wave number, and has to be calculated for the actual wind profile above the sea.

In the case of waves from a distant circular storm of radius R, the domain of τ and t is given by

$$|\boldsymbol{x}-(t-\tau)\boldsymbol{G}|\leq R\,,\quad 0\leq\tau\leq t\,.$$

The solution in polar coordinates can formally be written as

$$(\overline{h^2})_p=\pi g^2 \int_{\alpha_1}^{\alpha_2}\int_0^{k_a} c^{-2}B_1(\boldsymbol{k})\frac{e^{MS_1/G}-1}{M}kdkd\alpha \tag{4.11}$$

$$+\pi g^2 \int_{\alpha_1}^{\alpha_2}\int_{k_a}^{k_b} c^{-2}B_1(\boldsymbol{k})\frac{e^{Mt}-1}{M}kdkd\alpha$$

where α_1, α_2, S_1, k_a and k_b are as defined in paragraph 4f.

In the model with exponential growth, the storm area has to be limited by the condition $q(\boldsymbol{x})\equiv 0$ for $\boldsymbol{x}$ values outside the storm. Otherwise we should have to impose restrictions on the wind profile which would demand a much more complicated model.

The similarity between (4.9) and (4.11) should be noted. In fact when $MS_1/G\rightarrow 0$ and $Mt\rightarrow 0$ they become identical. As t is increased, $k_a\rightarrow k_b$ and only the first term is important.

5. Conclusion. It has been shown that both ECKART's (1953 b) and PHILLIPS' (1957) theoretical results can be obtained from the present general model, eventually modified to take account of the exponential wave growth as given by MILES' (1957) theory. This model can also be made suitable for wave forecasting. In the same way as for the circular storm, it is possible to obtain a formal solution for the case of a moving storm of any geometrical shape.

If $B_1(k)$ is known, the mean square height can be calculated. The main issue is to derive this function. If the directional dependency given by (2.11) is acceptable, $B_1(k)$ is determined as soon as $\psi(k)$ is given. The pressure spectrum is known to be a smooth function of wave number tending to zero as $k \to 0$ or $k \to \infty$. The water response is not critically dependent upon the actual form of $\psi(k)$. This suggests that a simple functional form may be chosen, for example:

$$\psi \sim k^{-n} e^{-k_0/k}, \quad n > 2 .$$

The theory has to be tested by comparison with observations. Unfortunately, the bulk of available observations comes from fully developed seas. In order to isolate the non-linear phenomena, observations from the primary stage of wave generation are highly desirable.

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