Ex 1

DET NORSKE VIDENSKAPS-AKADEMI I OSLO

GEOFYSISKE PUBLIKASJONER
GEOPHYSICA NORVEGICA

Vol. XXVII. No. 7 October 1969

A method of initialization for dynamical weather forecasting, and a balanced model

By Kaare Pedersen and Knut Erik Grønskei

OSLO 1969
UNIVERSITETSFORLAGET

GEOFYSISKE PUBLIKASJONER
GEOPHYSICA NORVEGICA
VOL. XXVII NO. 7

A METHOD OF INITIALIZATION FOR DYNAMICAL WEATHER FORECASTING, AND A BALANCED MODEL

BY KAARE PEDERSEN AND KNUT ERIK GRØNSKEI
FREMLAGT I VIDENSKAPS-AKADEMIETS MØTE DEN 23DE MAI 1969 AV ELIASSEN

Summary. By assuming the divergence to be locally constant we have developed a closed system of equations in the four variables: the velocity potential, the stream function, the time derivative of the stream function, and the vertical velocity. This system of equations is solved by relaxation procedure. The method has been tested on an actual situation and the convergence proved to be fairly rapid. The solutions of the vertical velocity and the stream function obtained are compared with solutions of the vertical velocity and the stream function obtained from the usual ω-equation and the simplified "balance equation" called the ψ-equation. It is found that for this case the vertical velocity computed by the present method is significantly different from the solution of the ω-equation. The maximum value of this difference reaches a value of 30% of the maximum value of the solution of the ω-equation. The difference between the two stream functions are found to be rather small.

1. **The set of equations.** We shall use the following definitions for the horizontal velocity vector

$$\vec{v} = \vec{k} \times \nabla\psi + \nabla\chi = \vec{v}_\chi + \vec{v}_\psi \tag{1.1}$$

$\vec{k}$ is the vertical unit vector, ψ the stream function and χ the velocity potential.

For an adiabatic and frictionless atmosphere the prognostic equations are:

i) The thermodynamic energy equation

$$g\frac{\partial}{\partial t}\frac{\partial z}{\partial p} + g\vec{v}\cdot\nabla\frac{\partial z}{\partial p} + \sigma\omega = 0 \quad \text{(T.E.)} \tag{1.2}$$

where g is the acceleration of gravity, z the geopotential height, p the pressure, $\omega = \frac{Dp}{dt}$ the vertical motion and

$$\sigma = g\left(\frac{\partial^2 z}{\partial p^2} + \frac{1 - R/Cp}{p}\frac{\partial z}{\partial p}\right).$$

R is specific gas constant and Cp is the specific heat at constant pressure.

ii) The vorticity equation

$$(1.3)\qquad \frac{\partial}{\partial t}\nabla^2\psi+\vec{v}\cdot\nabla(\nabla^2\psi+f)+\omega\frac{\partial}{\partial p}\nabla^2\psi+(\nabla^2\psi+f)\nabla\cdot\vec{v}+\vec{k}\cdot\nabla\omega\times\frac{\partial\vec{v}}{\partial p}=0\quad \text{(V.E.)}$$

where f is the coriolis parameter.

iii) The divergence equation

$$(1.4)\qquad \frac{\partial}{\partial t}\nabla^2\chi+\nabla\cdot(\vec{v}\cdot\nabla\vec{v})+\nabla\cdot\left(\omega\frac{\partial\vec{v}}{\partial p}\right)-\mathcal{J}(f,\chi)-\nabla\cdot f\nabla\psi+g\nabla^2 z=0.\quad \text{(D.E.)}$$

where $\mathcal{J}$ denotes the Jacobian.

iv) The continuity equation

$$(1.5)\qquad \nabla^2\chi+\frac{\partial\omega}{\partial p}=0.\quad \text{(C.E.)}$$

In order to obtain a balanced initial velocity field, we ignore the first and second local derivative of the divergence

$$(1.6)\qquad \frac{\partial}{\partial t}\nabla^2\chi=0\quad\text{and}\quad\frac{\partial^2}{\partial t^2}\nabla^2\chi=0\,.$$

We then obtain a "balance equation"

$$(1.7)\qquad \nabla\cdot(\vec{v}\cdot\nabla\vec{v})+\nabla\cdot\left(\omega\frac{\partial\vec{v}}{\partial p}\right)-\mathcal{J}(f,\chi)-\nabla\cdot f\nabla\psi+g\nabla^2 z=0\quad \text{(B.E.)}$$

and a time derivative of this equation which we will differentiate with respect to pressure

$$(1.8)\qquad \frac{\partial}{\partial t}\frac{\partial}{\partial p}(\nabla\cdot(\vec{v}\cdot\nabla\vec{v}))+\frac{\partial}{\partial p}\left(\nabla\omega\cdot\frac{\partial}{\partial t}\frac{\partial\vec{v}_\psi}{\partial p}\right)-f\frac{\partial}{\partial p}\frac{\partial}{\partial t}\nabla^2\psi$$

$$-\frac{\partial}{\partial t}\frac{\partial}{\partial p}\nabla f\cdot\nabla\psi\qquad+g\nabla^2\frac{\partial}{\partial t}\frac{\partial z}{\partial p}=0.\quad\left[\frac{\partial}{\partial t}\frac{\partial}{\partial p}\qquad\text{(B.E.)}\right]$$

With suitable given values of z and appropriate boundary conditions the five equations (1.2), (1.3), (1.5), (1.7) and (1.8) form a closed system in the dependent variables

ψ, χ, ω, $\frac{\partial}{\partial t}\psi$ and $\frac{\partial}{\partial t}\frac{\partial z}{\partial p}$.

The variable $\frac{\partial}{\partial t}\frac{\partial}{\partial p}z$ appears only in two terms and is easily eliminated. It is convenient to do this elimination in the following way

$$(1.9)\qquad \nabla^2(\text{T.E.})-f\frac{\partial}{\partial p}(\text{V.E.})-\frac{\partial}{\partial t}\frac{\partial}{\partial p}(\text{B.E.})=0$$

We will try to find a convergent relaxation procedure to obtain solutions of the variables χ, ψ, $\frac{\partial}{\partial t}\psi$ and ω from the equations (1.5), (1.7), (1.3) and (1.9). In order to do this we shall rewrite the equations separating the terms (LORENZ 1960). The balance equation takes the form

$$\nabla \cdot (\vec{v}_\psi \cdot \nabla \vec{v}_\psi) - \nabla f \cdot \nabla \psi - f\nabla^2\psi + g\nabla^2 z$$
$$+\nabla \cdot (\vec{v}_\psi \cdot \nabla \vec{v}_\chi) + \nabla \cdot (\vec{v}_\chi \cdot \nabla \vec{v}_\psi) + \nabla\omega \cdot \frac{\partial \vec{v}_\psi}{\partial p} - J(f,\chi)$$
$$+\omega\frac{\partial}{\partial p}\nabla^2\chi + \nabla\omega \cdot \frac{\partial \vec{v}_\chi}{\partial p} + \nabla \cdot (\vec{v}_\chi \cdot \nabla \vec{v}_\chi) = 0 . \tag{1.10}$$

The vorticity equation may be written

$$\frac{\partial}{\partial t}\nabla^2\psi + \vec{v}_\psi \cdot \nabla(\nabla^2\psi + f) + \vec{v}_\chi \cdot \nabla f + f\nabla^2\chi$$
$$+\vec{v}_\chi \cdot \nabla(\nabla^2\psi) + \nabla^2\psi\nabla^2\chi + \omega\frac{\partial}{\partial p}\nabla^2\psi + \vec{k} \cdot \nabla\omega \times \frac{\partial \vec{v}_\psi}{\partial p}$$
$$+\vec{k} \cdot \nabla\omega \times \frac{\partial \vec{v}_\chi}{\partial p} = 0 . \tag{1.11}$$

Finally we get the equation (1.9) in the form

$$\nabla^2(\sigma\omega) + g\nabla^2\left(\vec{v}_\psi \cdot \nabla\frac{\partial z}{\partial p}\right) + g\nabla^2\left(\vec{v}_\chi \cdot \nabla\frac{\partial z}{\partial p}\right)$$
$$-f\frac{\partial}{\partial p}(\vec{v}_\psi \cdot \nabla(\nabla^2\psi + f)) - f\frac{\partial}{\partial p}(\vec{v}_\chi \cdot \nabla f) + f^2\frac{\partial^2\omega}{\partial p^2}$$
$$-f\frac{\partial}{\partial p}(\vec{v}_\chi \cdot \nabla(\nabla^2\psi)) + f\nabla^2\psi\frac{\partial^2\omega}{\partial p^2} - f\omega\frac{\partial^2}{\partial p^2}\nabla^2\psi$$
$$-f\frac{\partial}{\partial p}\left(\vec{k} \cdot \nabla\omega \times \frac{\partial \vec{v}_\psi}{\partial p}\right) - f\frac{\partial}{\partial p}\left(\vec{k} \cdot \nabla\omega \times \frac{\partial \vec{v}_\chi}{\partial p}\right)$$
$$-\frac{\partial}{\partial t}\frac{\partial}{\partial p}(\nabla \cdot (\vec{v}_\psi \cdot \nabla \vec{v}_\psi)) + \frac{\partial}{\partial t}\frac{\partial}{\partial p}(\nabla f \cdot \nabla\psi) \tag{1.12}$$

$$-\frac{\partial}{\partial p}\left(\nabla\cdot\frac{\partial}{\partial t}\vec{v}_\psi\cdot\nabla\vec{v}_\chi\right)-\frac{\partial}{\partial p}\nabla\cdot\left(\vec{v}_\chi\cdot\nabla\frac{\partial}{\partial t}\vec{v}_\psi\right)$$

$$-\frac{\partial}{\partial p}\left(\nabla\omega\cdot\frac{\partial}{\partial t}\frac{\partial}{\partial p}\vec{v}_\psi\right)=0\,.$$

The dominant terms in this equation are those giving the well known ω-equation

$$\nabla^2(\sigma\omega)+f^2\frac{\partial^2\omega}{\partial p^2}=f\frac{\partial}{\partial p}(\vec{v}_\psi\cdot\nabla(\nabla^2\psi+f))-g\nabla^2\left(\vec{v}_\psi\cdot\nabla\frac{\partial z}{\partial p}\right).\tag{1.13}$$

Luckily the most important terms containing ω have coefficients independent of the other variables, and the dominant terms in ψ are only slightly dependent on ω; this we may utilize in our relaxation procedure.

As a first estimate we take $\omega=\omega^{(0)}=0$ and from equations (1.5), (1.10) and (1.11) we obtain estimates of $\chi^{(0)}$, $\psi^{(0)}$ and $\frac{\partial}{\partial t}\psi^{(0)}$.

We now use equation (1.12) as an equation in ω where the differential operator has the form

$$L(\omega)\equiv\nabla^2(\sigma\omega)+f^2\frac{\partial^2\omega}{\partial p^2}\tag{1.14}$$

and where all the other terms are approximated by the above estimates. With the new value $\omega^{(1)}$ of ω we repeat the procedure.

Since the solution of the balance equation (1.10) is rather time-consuming, we have applied an alternative procedure in the computations described in a later section.

2. **The relaxation procedure.** The dominant terms in the balance equation (1.10) are those containing the stream function. A simplified form of this equation is then

$$\nabla\cdot(\vec{v}_\psi\cdot\nabla\vec{v}_\psi)-\nabla f\cdot\nabla\psi-f\nabla^2\psi+g\nabla^2z=0\tag{2.1}$$

We will now run through the procedure outlined above, retaining the solution of the stream function $\psi^{(0)}$ obtained from equation (2.1) until solutions for ω, χ and $\frac{\partial}{\partial t}\psi^{(0)}$ are obtained.

As a point of consistency in our simplification which reduced (1.10) to (2.1), we must also omit the last three terms of equation (1.12). The values of ω and χ thus obtained are then introduced in the balance equation (1.10) to obtain a new value of the stream function, but in order to have a two-dimensional equation in $\psi^{(i)}$ we approximate the term

(2.2) $$\nabla\omega \cdot \frac{\partial \vec{v}_\psi^{(i)}}{\partial p} \quad \text{by} \quad \nabla\omega \cdot \frac{\partial \vec{v}_\psi^{(i-1)}}{\partial p}.$$

The procedure is then repeated retaining all terms in equation (1.12).

As a reference we compared our ω-values with those obtained as solutions of the "ordinary ω-equation" (1.13), which was used in the first scan; therefore the $\frac{\partial}{\partial t}\frac{\partial}{\partial p}[\nabla \cdot (\vec{v}_\psi \cdot \nabla \vec{v}_\psi) - \nabla f \cdot \nabla \psi]$ terms in equation (1.12) were omitted temporarily.

3. **The numerical model and the boundary conditions.** The geopotential heights, z, are given at the pressure levels 1000, 850, 700, 500 and 300 mb. The values are given in a quadratic grid with mesh size equal to five degrees of latitude at the equator on a Mercator projection. The area used in the computations extends from 5 °N to 74.68 °N and from 125 °W to 50 °E. The initial values of z are adjusted so that they satisfy Rellich's condition (see e.g. COURANT HILBERT 1962) for a solution of equation (2.1)

(3.1) $$\frac{f}{2} + \frac{g}{f}\nabla^2 z > \frac{1}{f}\nabla f \cdot \nabla \psi = -\frac{\partial f}{\partial y}\frac{u_\psi}{f}$$

where u_ψ is the zonal velocity, which we estimate by its geostrophic value.

When Rellich's condition is applied to equation (1.10), we find the following relationship:

(3.2) $$\frac{f}{2} + \frac{g}{f}\nabla^2 z + \frac{1}{2f}(\nabla^2\chi)^2 > \frac{1}{f}\nabla f \cdot \nabla\psi + \frac{1}{f}J(f,\chi) - \frac{1}{f}\nabla\omega \cdot \frac{\partial \vec{v}}{\partial p} - \frac{1}{f}\vec{v} \cdot \nabla\nabla^2\chi - \frac{1}{f}\omega\frac{\partial}{\partial p}\nabla^2\chi$$

which form the constraints on the initial values of the field variables. If the balance condition $\frac{D}{dt}\nabla^2\chi = 0$ is used, instead of $\frac{\partial}{\partial t}\nabla^2\chi = 0$, the last two terms in the relationship (3.2) should be omitted. But if the condition $\frac{D}{dt}\nabla^2\chi = 0$ is chosen one needs an estimate of $\frac{\partial}{\partial t}\nabla^2\chi$ in equation (1.12).

In our case study we found that the maximum adjustment of z occurred at the 300 mb level and had a magnitude of 13 meters.

The upper and lower boundary conditions for the vertical velocity, ω, were taken to be

(3.3) $$\omega = 0 \quad \text{at} \quad p = 1000\text{ mb} \quad \text{and at} \quad p = 150\text{ mb}$$

At the vertical boundaries we put

(3.4) $$\omega = 0 \quad \text{and} \quad \chi = 0,$$

and

$$\frac{\partial \psi}{\partial s} = \frac{g}{f}\frac{\partial z}{\partial s} - \frac{\oint \frac{1}{f}\frac{\partial z}{\partial s}\delta s}{\oint \delta s} \tag{3.5}$$

where δs is a line-element of the boundary. The boundary condition for ψ is the same as that used by BOLIN (1955). This combination of boundary conditions for the velocity potential and the stream function is not the usual one. Since there is a flow through our area, no boundary condition of the usual type are given; i.e., v_s or $v_n = 0$, where v_s and v_n are the respective tangential and normal velocity components.

4. **Results of computations.** As a test case we have chosen the date used by PEDERSEN (1963), 28 November 1958, 00 GMT. The geopotential heights of the 1000, 500 and 300 mb levels are shown in Figs. 1—3.

The solution, $\psi^{(0)}$ of equation (2.1) at the previously noted levels are shown in Figs. 4—6. On these Figures we have also shown the values of $|\vec{v}_{\psi(0)}| - |\vec{v}_g|$ where $\vec{v}_g$ is the geostrophic wind. The units for ψ are 9.8×10^8 cm²/s and for $(|\vec{v}_{\psi(0)}| - |\vec{v}_g|)$m/s This variation appears to be what one would expect considering the difference between gradient wind and geostrophic wind.

The solution of equation (1.13), denoted $\omega^{(1)}$, is shown for the levels 925, 600 and 400 mb in Fig. 7—9. (Unit baryes/s). The areas of negative values of $\omega^{(1)}$ are in good agreement with the positions of fronts, precipitation, and cloud areas. The convergence of the relaxation procedure was fairly rapid, so that after five scans the changes in ω were less than 0.025 barye/s, i.e. $\frac{|\omega^{(5)} - \omega^{(4)}|}{|\omega^{(1)}|} > 0.01$. These values of the vertical velocity, $\omega^{(5)}$, are therefore considered to be the solution of the vertical velocity with the balance equation (2.1).

The difference $\omega^{(5)} - \omega^{(1)}$ at the levels 925, 600 and 400 mb is shown in Figs. 10—12. The difference is largest at 400 mb where the divergence field is strong, see Figs. 13—15, and where the difference between the geostrophic vortciity variation and the true vorticity variation is relatively strong. The maximum value reaches one barye/s. Generally this difference appears to be about 30% of the $\omega^{(1)}$ values. It is interesting to note that the $\omega^{(5)}$ values are somewhat smoother than the $\omega^{(1)}$ values. About 80% of this difference was reached already after scan two, that means that $\omega^{(2)}$ is a fair approximation to $\omega^{(5)}$. The velocity potential $\chi^{(5)}$ at the levels 1000, 500 and 300 mb is shown in Figs. 13—15. The divergent velocities, $\vec{v}_\chi$, are, as expected, strongest at the upper and lower levels. At 300 mb $\vec{v}_\chi$ reaches a maximum of 5.3 m/s, and at 1000 mb the maximum value of $\vec{v}_\chi$ is 3.8 m/s. At 500 mb the divergence field is rather weak with a maximum velocity of 1.6 m/s.

The stream function, $\psi^{(1)}$, obtained from equation (1.10), with $\omega^{(5)}$ and $\chi^{(5)}$ as estimates for the vertical velocity and the velocity potential, is not much different from the stream function $\psi^{(0)}$. The differences $\psi^{(1)}-\psi^{(0)}$ for the levels 500 and 300 mb are given in Fig. 16 and Fig. 17 respectively. At the lower levels the velocities of this difference field are weaker. At 500 mb the velocity field $\vec{v}(_{\psi}{}^{(1)}{}_{-\psi}{}^{(0)})$ is strongest and reaches a maximum value of 3.5 m/s, that is 8% of the maximum value of $\vec{v}_{\psi}{}^{(0)}$ at this level. It appears that at this level the term $\nabla\omega\cdot\frac{\partial\vec{v}_{\psi}}{\partial p}$ is about as large as the sum of the other terms in equation (1.10) containing ω or χ.

At 300 mb the velocity field $\vec{v}(_{\psi}{}^{(1)}{}_{-\psi}{}^{(0)})$ reaches a maximum value of 2.5 m/s, that is 4% of $\vec{v}\psi^{(0)}$ at this level. It appears that condition $\frac{\partial}{\partial t}\nabla^2\chi=0$ gives a somewhat different result for $\psi^{(1)}$ than the condition $\frac{D}{dt}\nabla^2\chi=0$, especially at 300 mb.

With values of $\omega^{(5)}$, $\chi^{(5)}$ and $\psi^{(1)}$ we obtain from equation (1.12) a new value of the vertical velocity which we denote $\omega^{(1.1)}$. The values of $\omega^{(1.1)}$ show increasing amplitude relative to $\omega^{(0)}$. The difference $|\omega^{(1.1)}-\omega^{(0)}|$ reaches a maximum value of 0.4 b/s.

After nine scans of the set of equations (1.5), (1.10), (1.11) and (1.12) the maximum residue in ω becomes less than 0.02 b/s, and the values of $\psi^{(9)}$, $\omega^{(9.1)}$ and $\chi^{(9.1)}$ are considered to be the solutions of ψ, ω and χ. The values of $\omega^{(9.1)}$ at the 600 mb level are shown in Fig. 18. At this level the difference $(\omega^{(9.1)}-\omega^{(0)})$ is largest. We find that there is an amplitude increase in ω at this level of 30% relative to $\omega^{(0)}$. The velocity field $\vec{v}(\psi^{(9)}-\psi^{(0)})$ reaches a maximum value of 5 m/s.

5. **A balanced prognostic model.** With the assumption that $\nabla^2\chi$ is locally contants we may apply equation (1.10), after differentiation with respect to time, as a prognostic equation for the geopotential height. This we have done at one pressure level. The prognostic values of the geopotential heights at the other pressure levels are more quickly computed from the thermodynamic energy equation (1.2). These computations are currently in progress, and will be published at a later date.

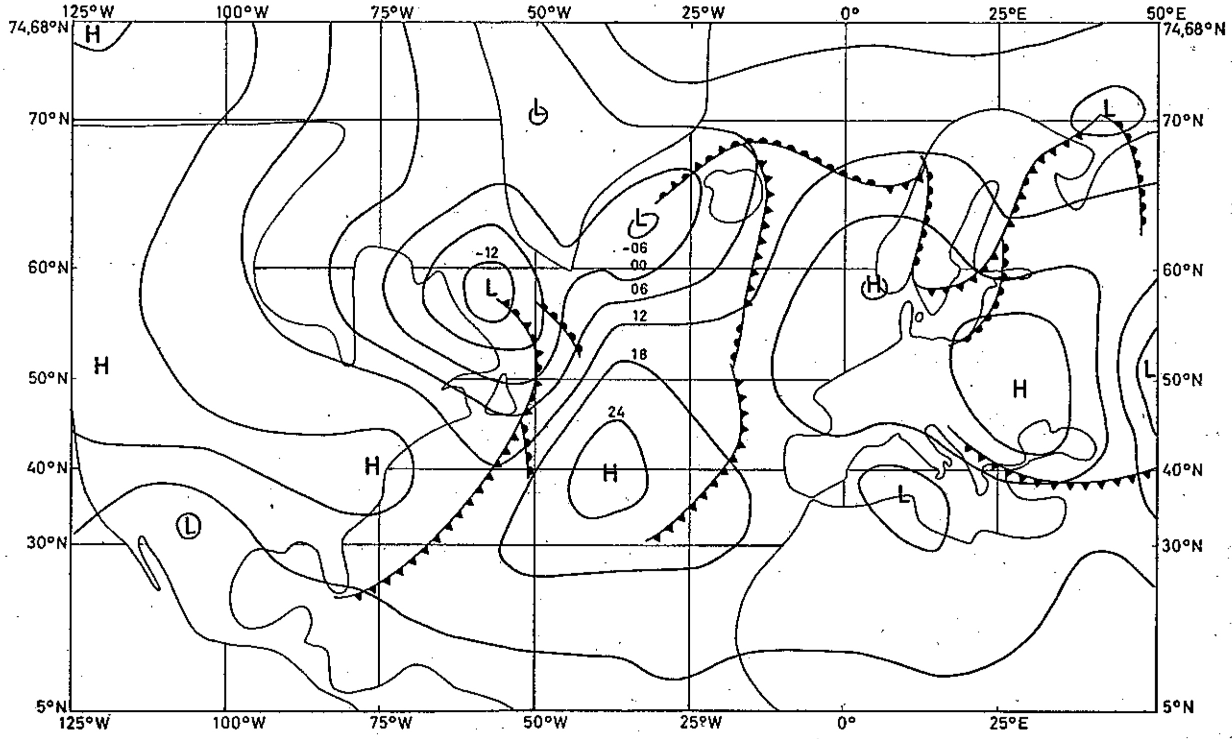
REFERENCES

BOLIN, B., 1955: Numerical forecasting with a barotropic model. *Tellus* 7, 27–49.

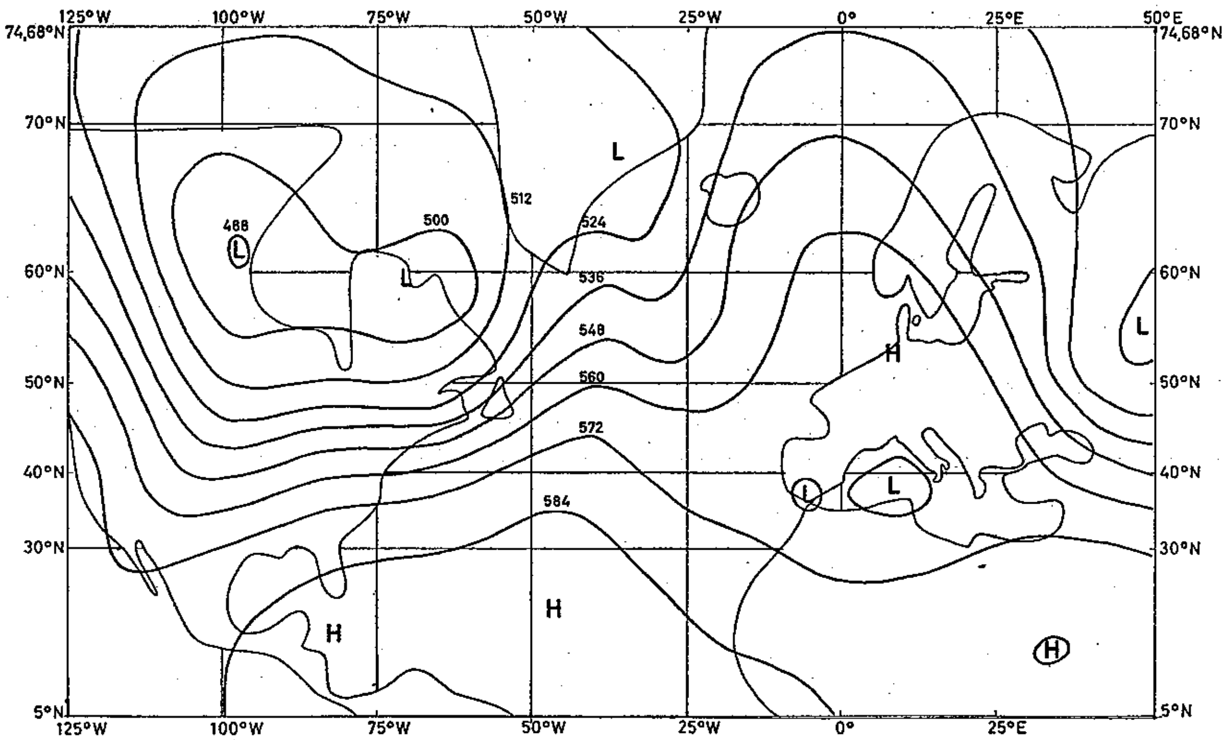
COURANT, R., and HILBERT, D., 1962: *Methods of Mathematical Physics.* Interscience Publishers, New York, p. 324.

LORENZ, E. N., 1960: Energy and numerical weather prediction. *Tellus* 12, 364–373.

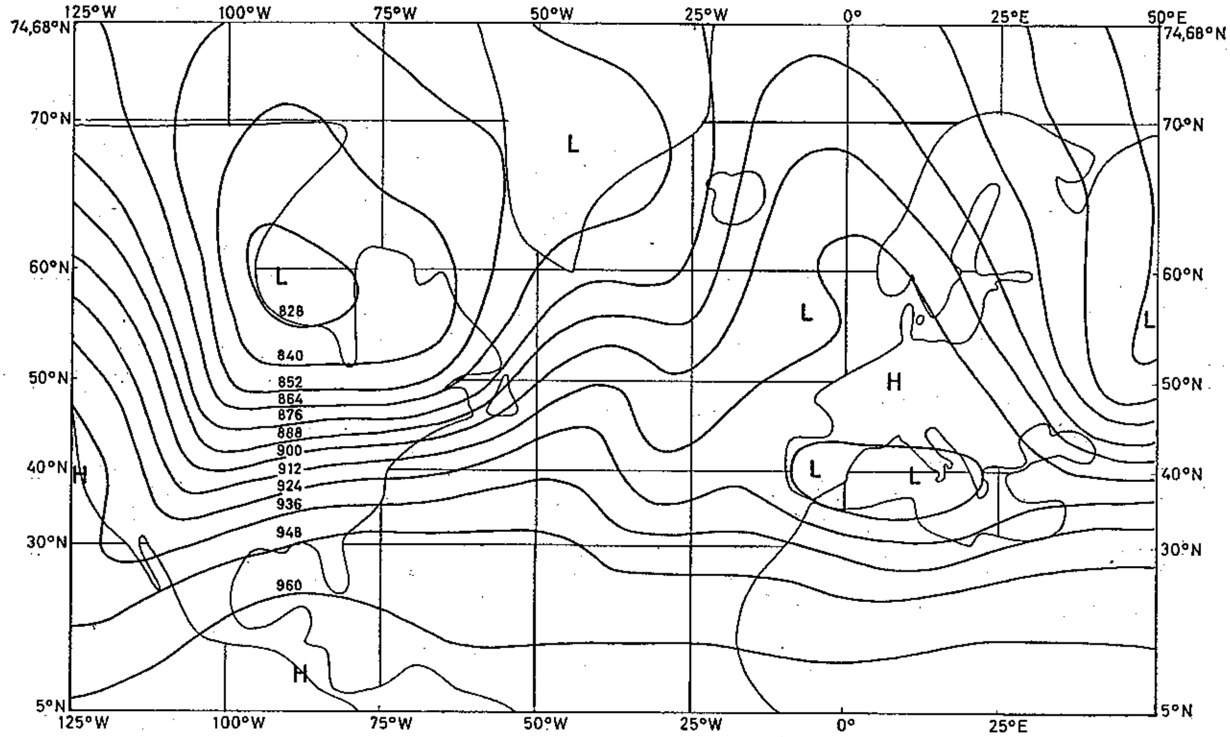
PEDERSEN, K., 1963: On quantitative precipitation forecasting with a quasi-geostrophic model.



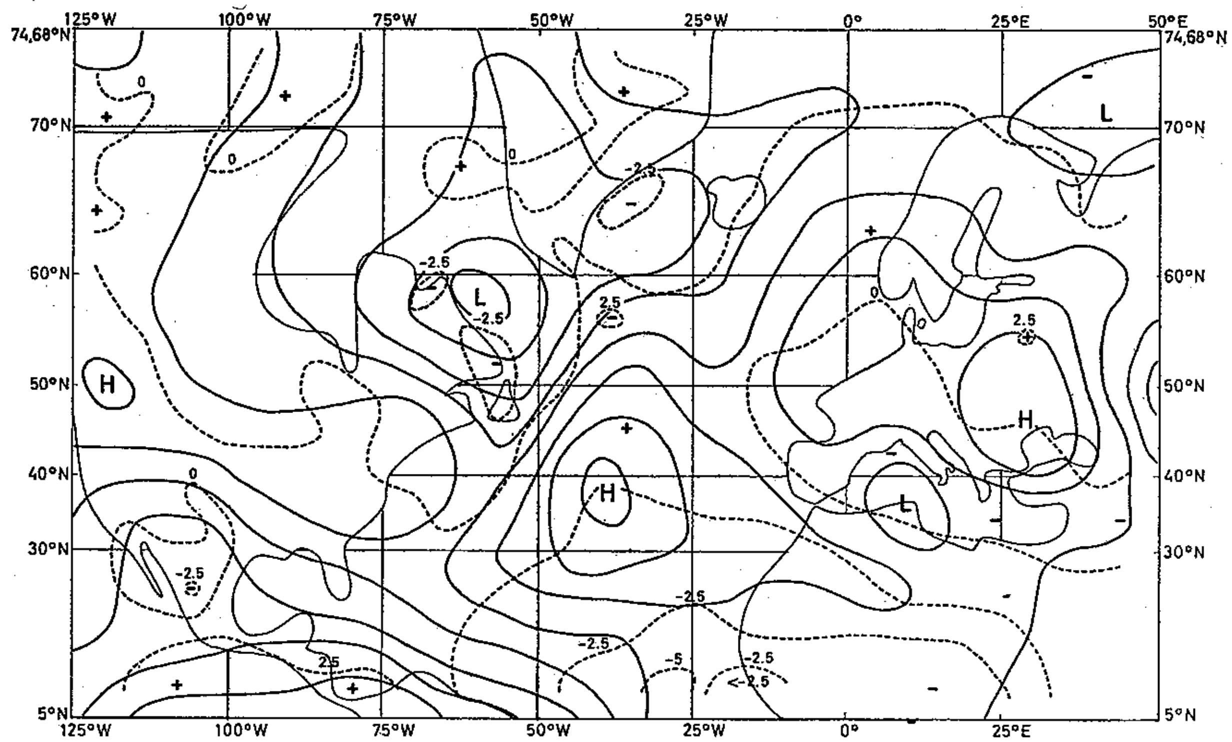
Figur 1. z 1000 mb. Unit: 10 m.



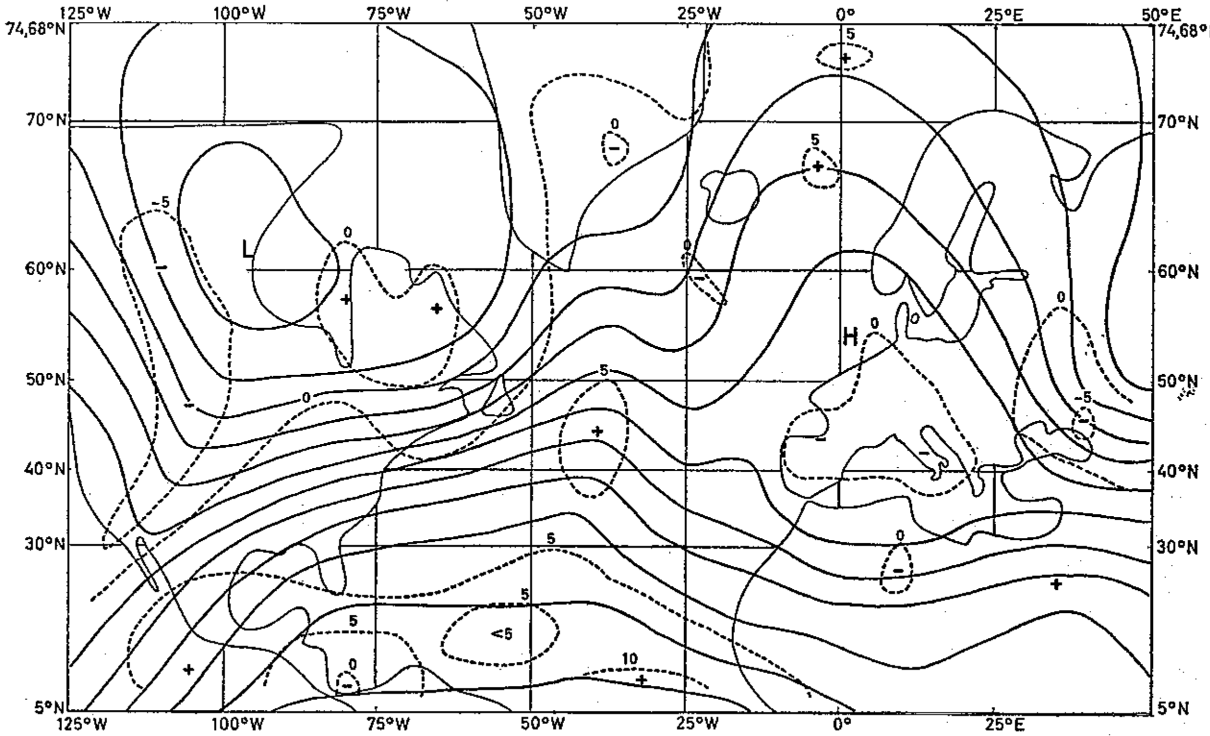
Figur 2. z 500 mb. Unit: 10 m.



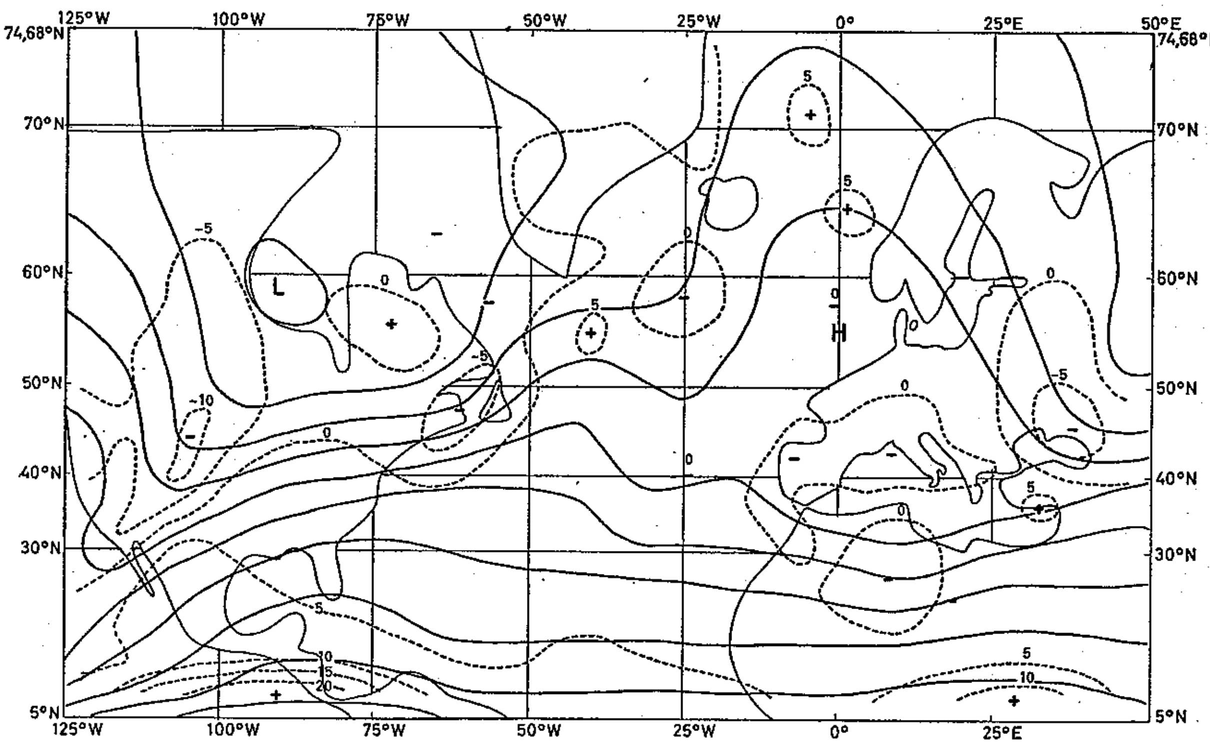
Figur 3. *z* 300 mb. Unit: 10 m.



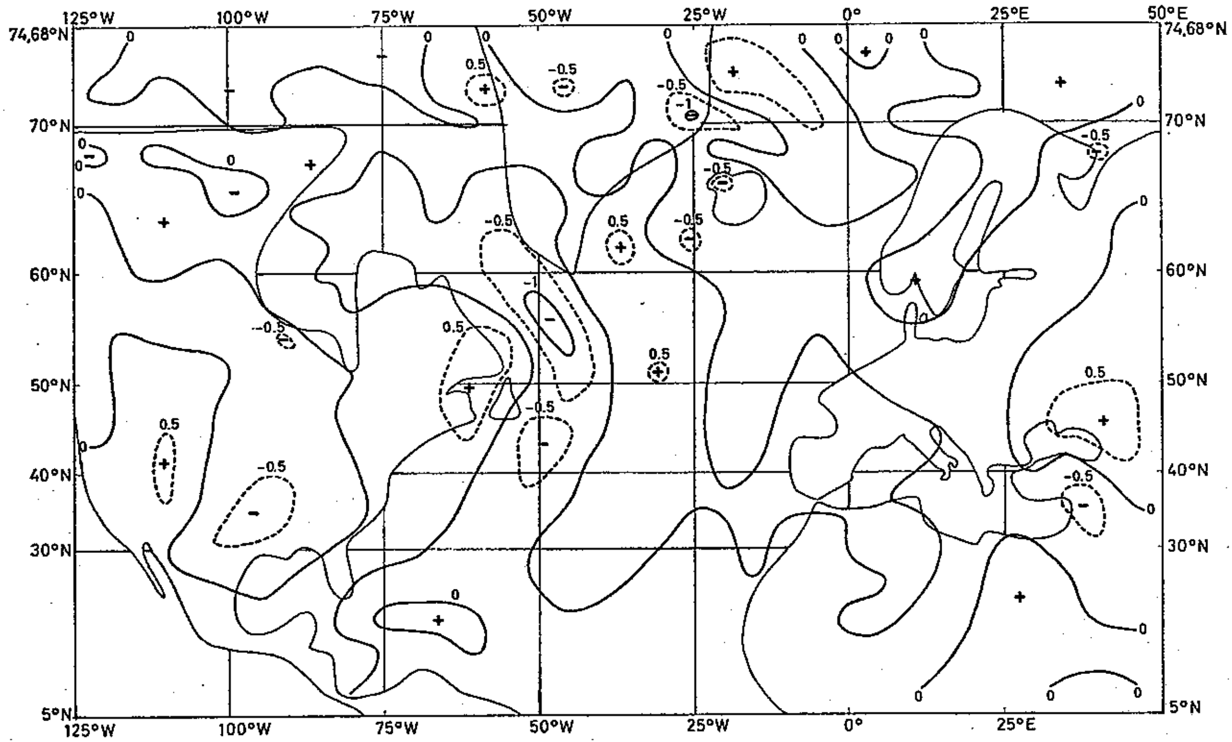
Figur 4. $\psi^{(0)}$ 1000 mb. Full lines are drawn for each $9.8 \cdot 10^{10}$ cm²/s. Broken lines show $|\vec{v}_{\psi}{}^{(0)} - \vec{v}_g|$. Unit: m/s.



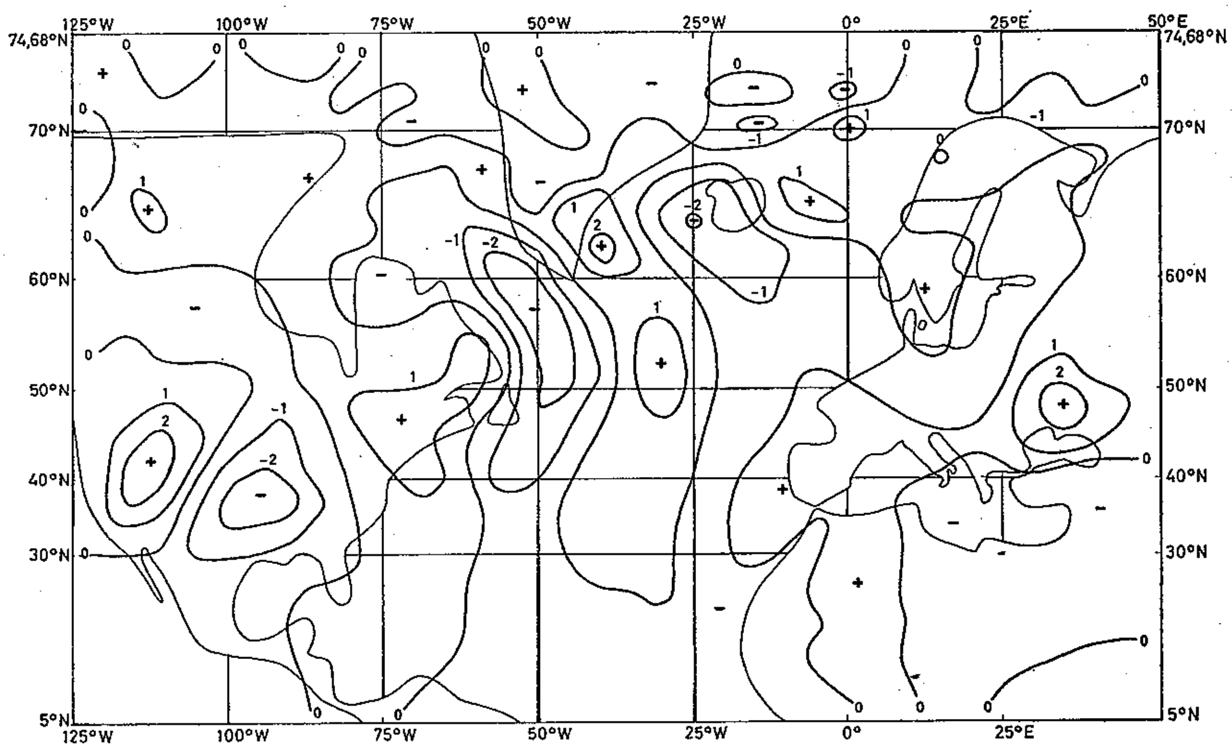
Figur 5. $\psi^{(0)}$ 500 mb. Full lines are drawn for each $9.8 \cdot 10^{10}$ cm^2/s. Broken lines show $|\vec{v}_\psi^{(0)}| - |\vec{v}_g|$. Unit: m/s.



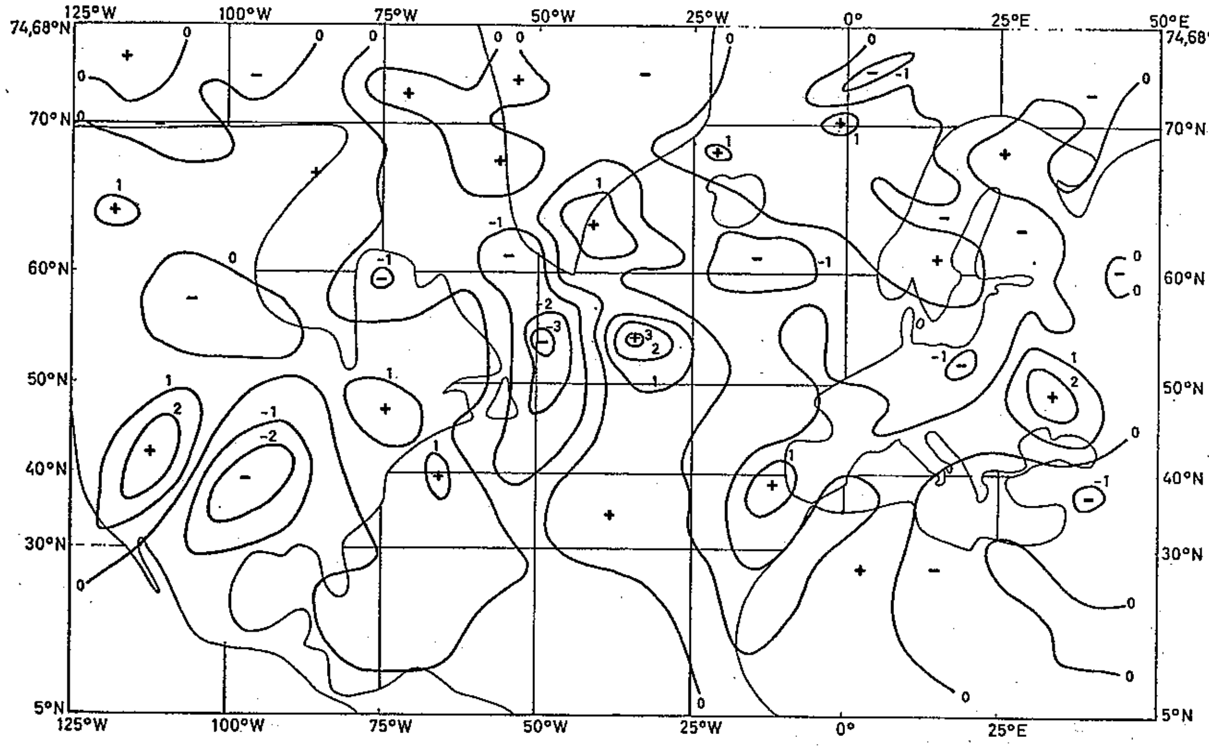
Figur 6. $\psi^{(0)}$ 300 mb. Full lines are drawn for each $2 \cdot 9.8 \cdot 10^{10}$ cm^2/s. Broken lines show $|\vec{v}_\psi^{(0)}| - |\vec{v}_g|$. Units: m/s.



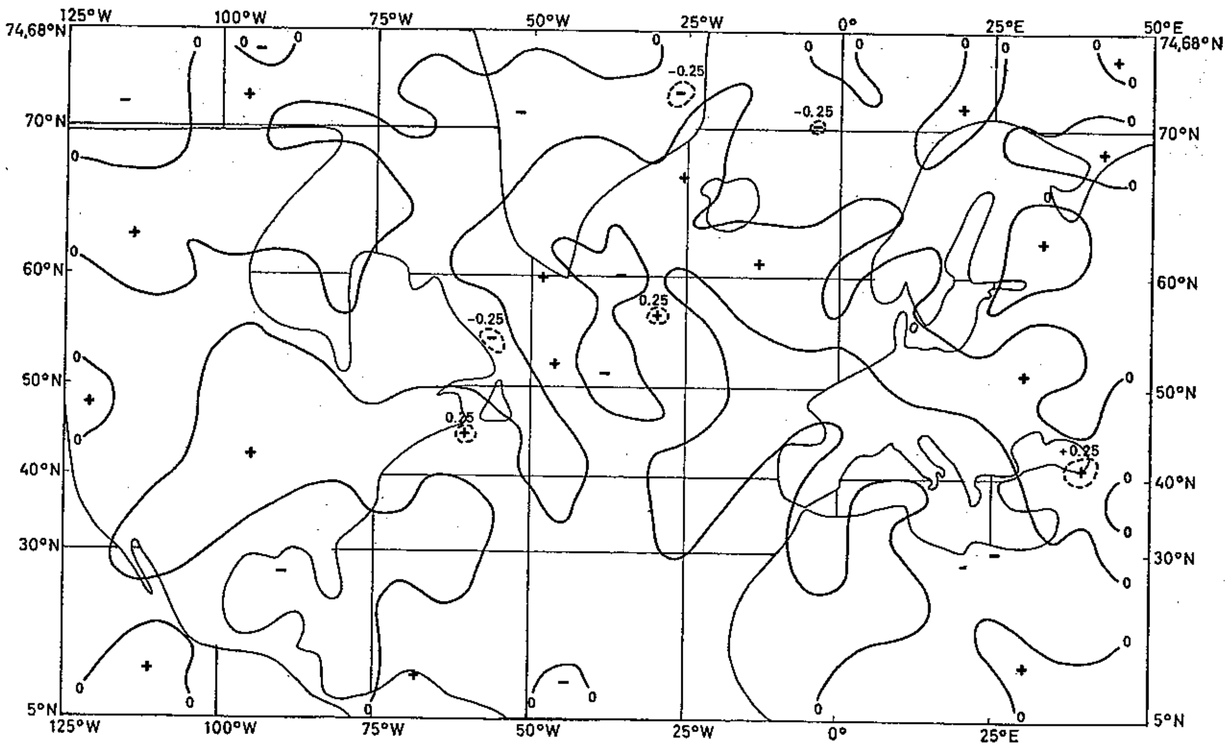
Figur 7. $\omega^{(1)}$ 925 mb. Full lines are drawn for each 1.0 barye/s.



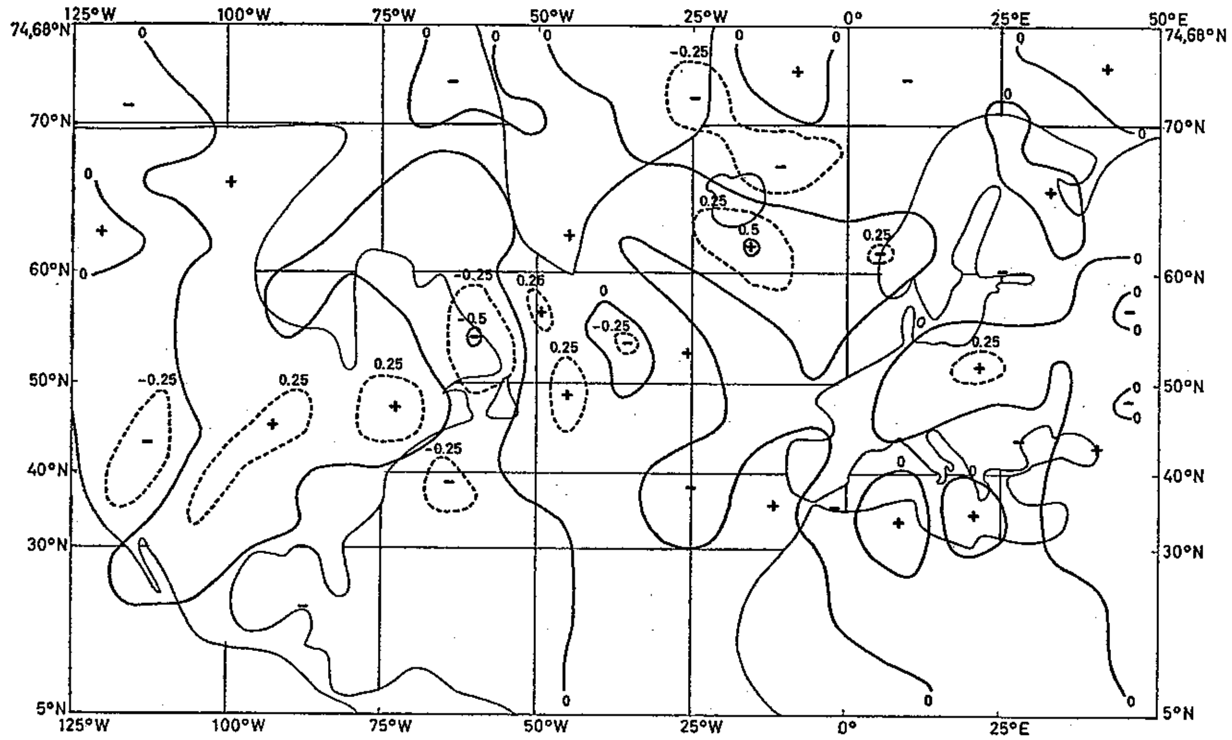
Figur 8. $\omega^{(1)}$ 600 mb. Full lines are drawn for each 1.0 barye/s.



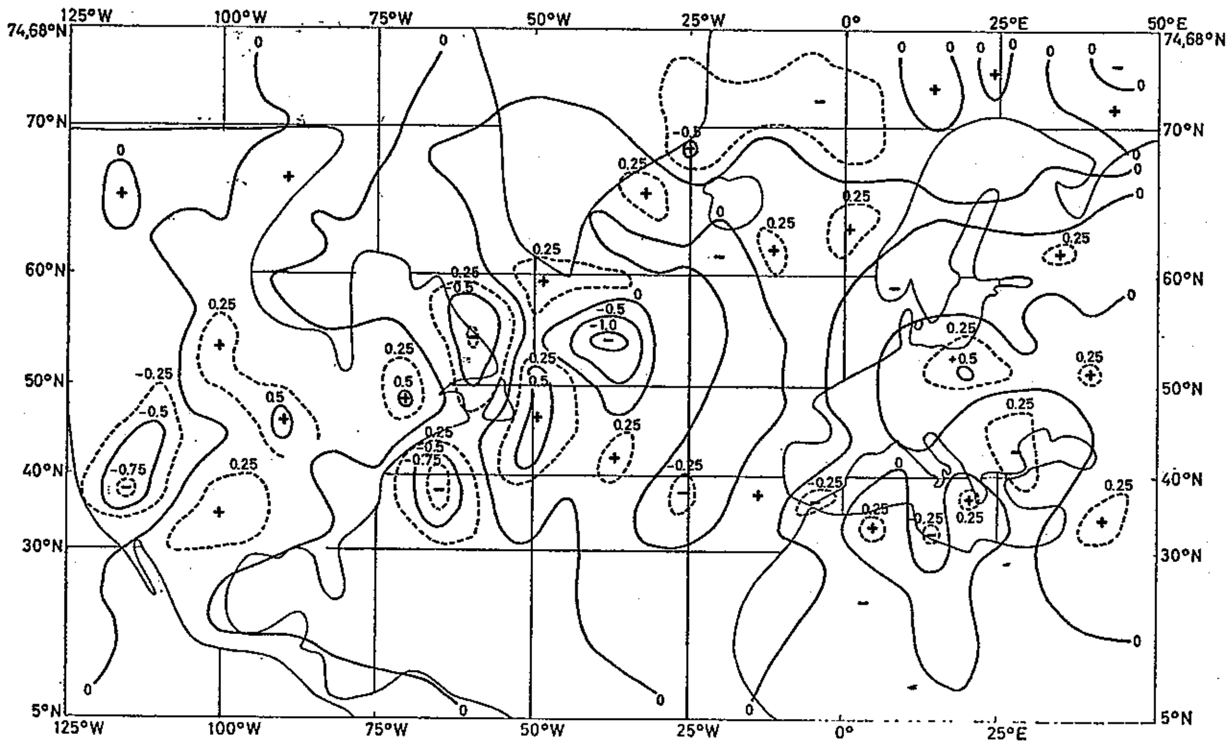
Figur 9. $\omega^{(1)}$ 400 mb. Full lines are drawn for each 1.0 barye/s.



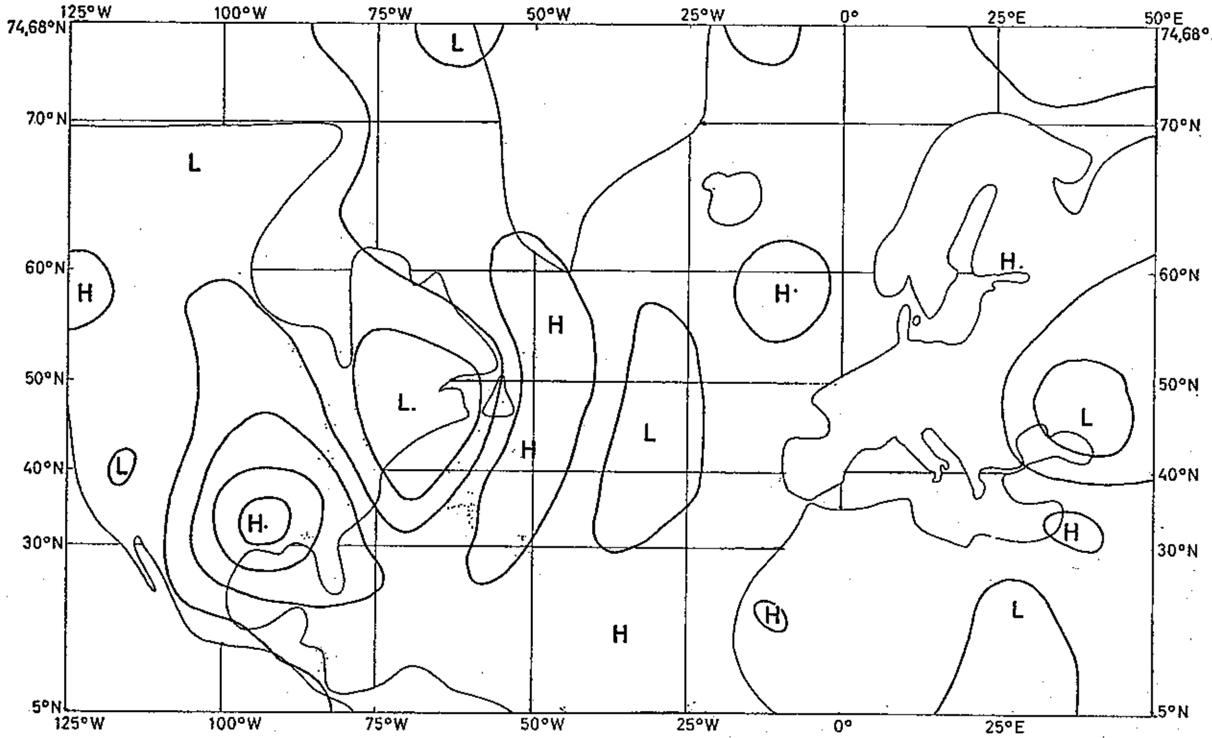
Figur 10. $\omega^{(5)} - \omega^{(1)}$ 925 mb. Full lines are drawn for each 0.5 barye/s.



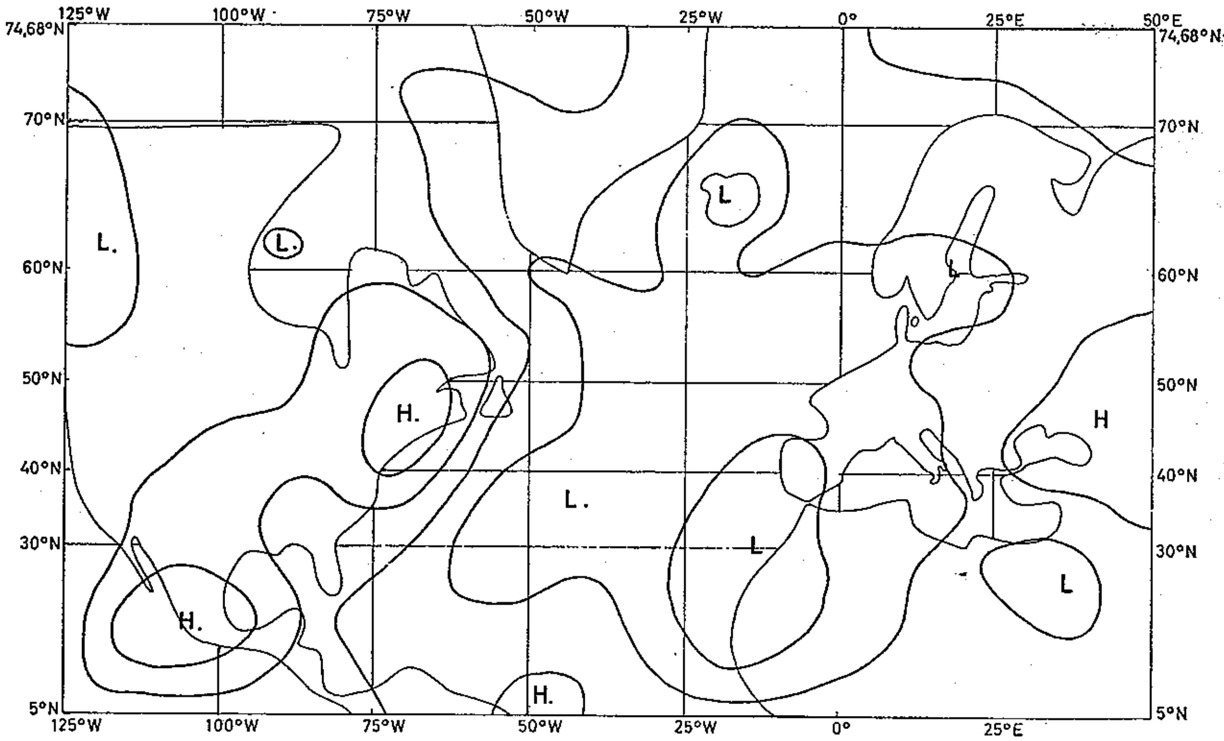
Figur 11. $\omega^{(5)} - \omega^{(1)}$ 600 mb. Full lines are drawn for each 0.5 barye/s.



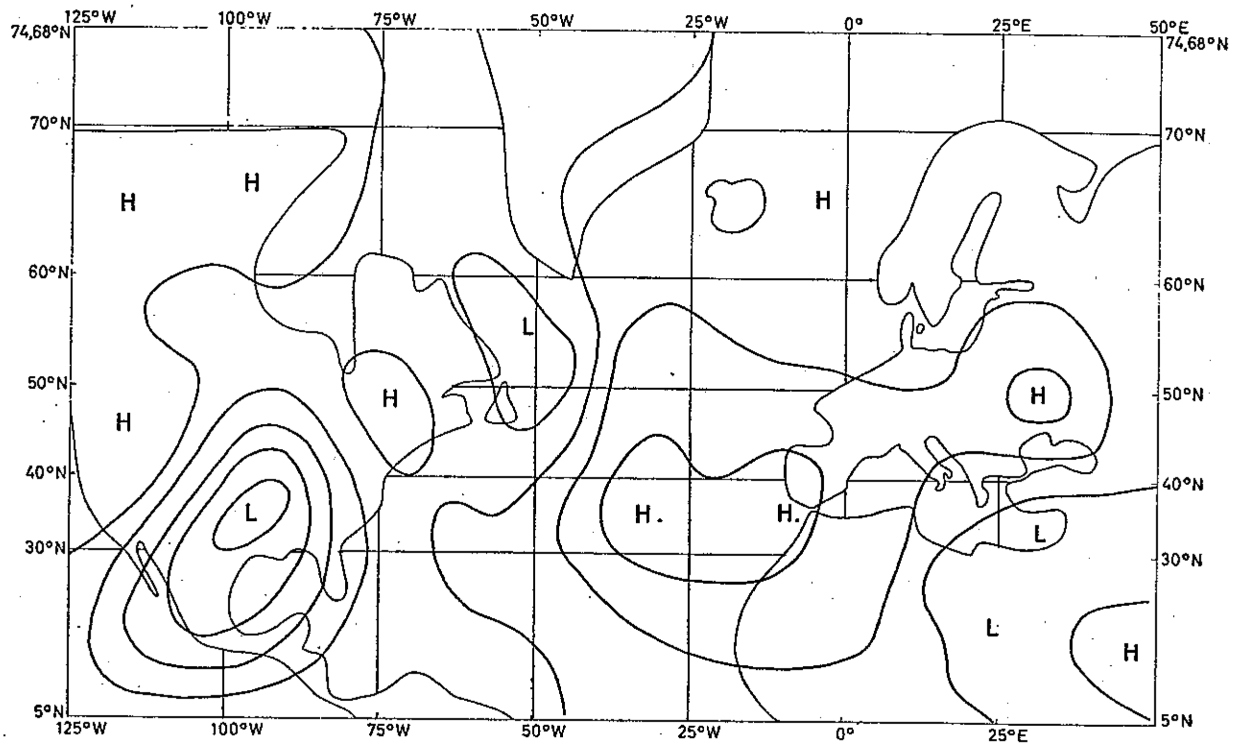
Figur 12. $\omega^{(5)} - \omega^{(1)}$ 400 mb. Full lines are drawn for each 0.5 barye/s.



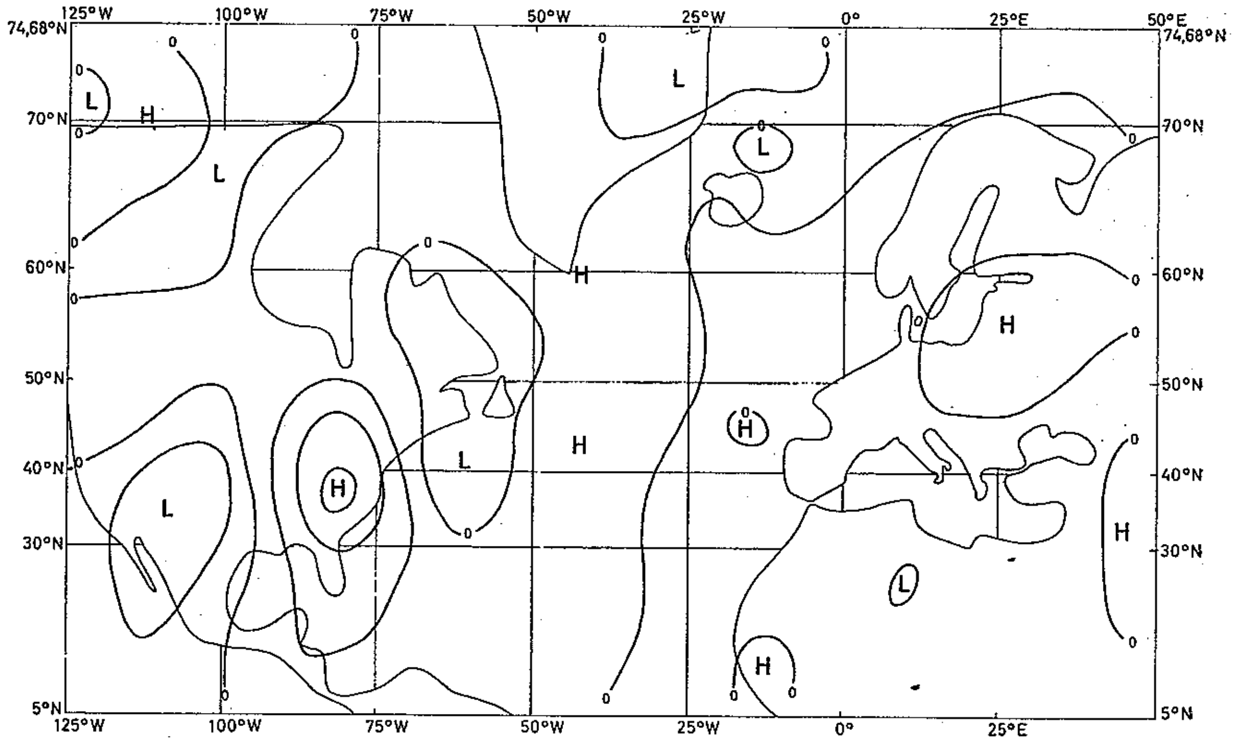
Figur 13. $\chi^{(5)}$ 1000 mb. Full lines are drawn for each $1.0 \cdot 10^{10}$ sm²/s.



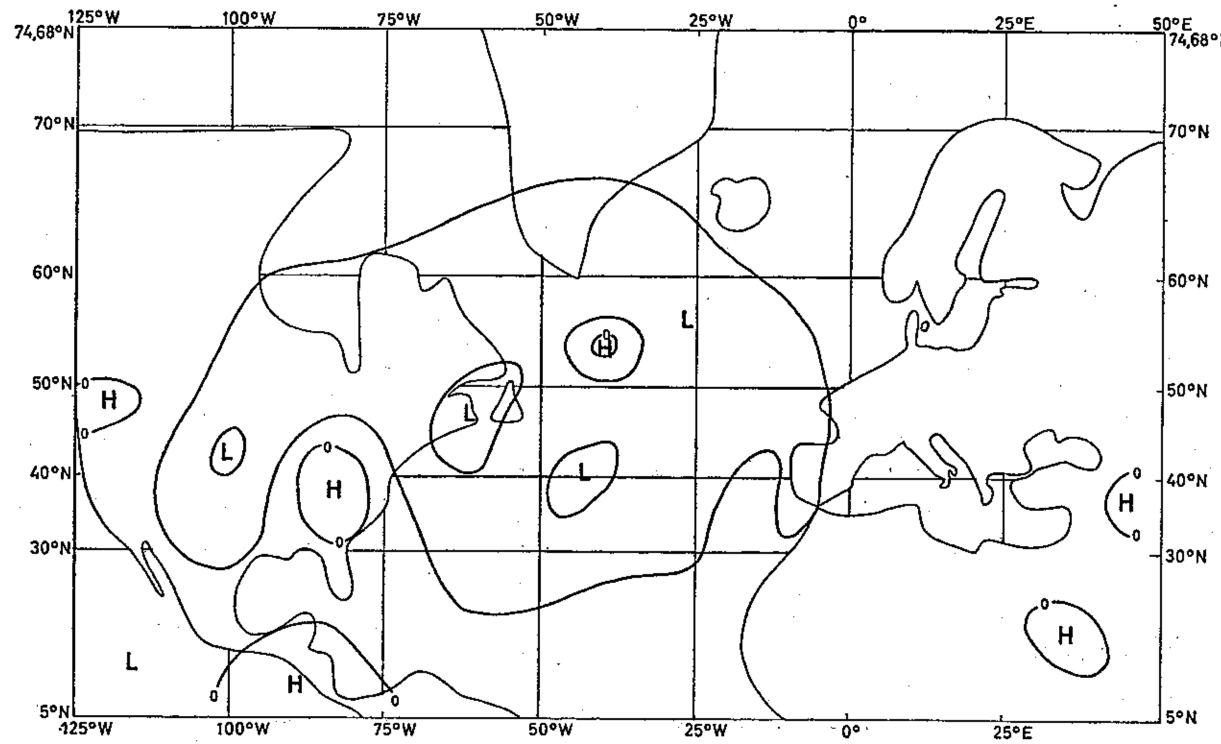
Figur 14. $\chi^{(5)}$ 500 mb. Full lines are drawn for each $5.0 \cdot 10^{9}$ cm²/s.



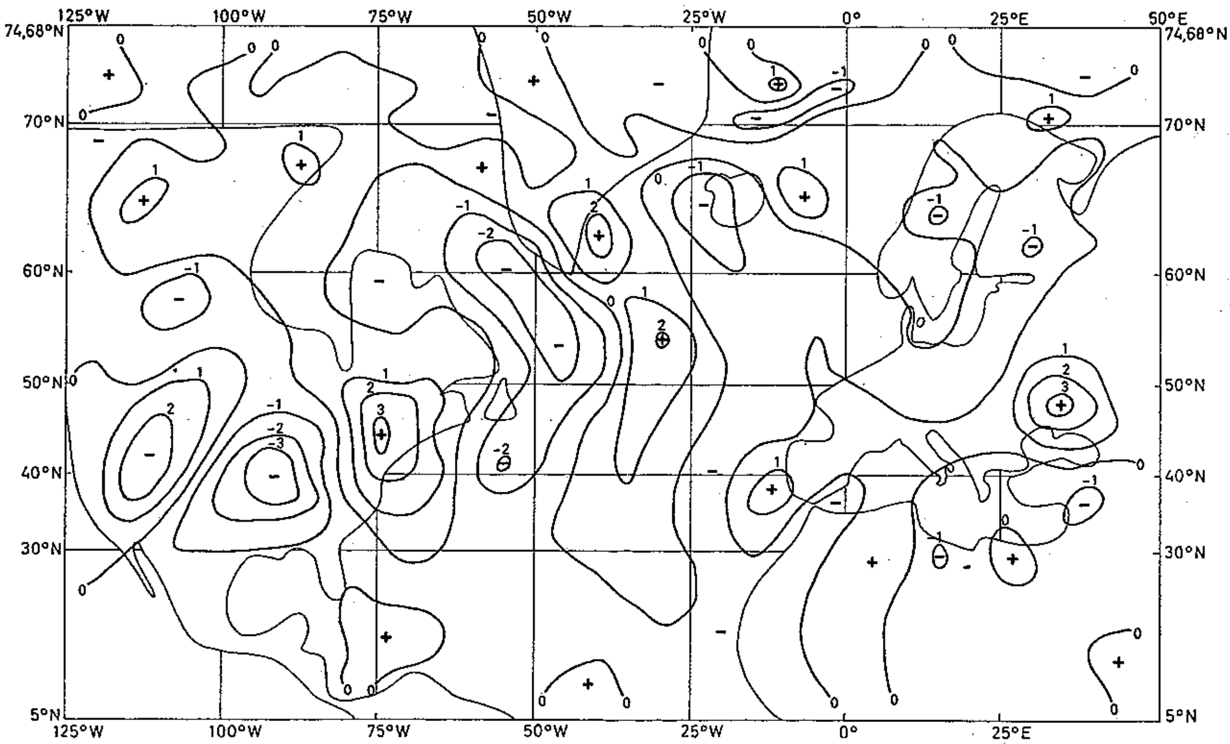
Figur 15. $\chi^{(5)}$ 300 mb. Full lines are drawn for each $1.0 \cdot 10^{10}$ cm²/s.



Figur 16. $\psi^{(1)} - \psi^{(0)}$ 500 mb. Full lines are drawn for each 1.0×10^{10} cm²/s.



Figur 17. $\psi^{(1)} - \psi^{(0)}$ 300 mb. Full lines are drawn for each $1.0 \cdot 10^{10}$ cm²/s.



Figur 18. $\omega^{(9,1)}$ 600 mb. Full lines are drawn for each 1.0 barye/s.

Papers published in *Geofysiske Publikasjoner*, may be obtained from: Universitetsforlaget, Blindern, Oslo 3, Norway.

Vol. XXIII.

No. 1. Bernt Mæhlum: The sporadic E auroral zone. 1962.
» 2. Bernt Mæhlum: Small scale structure and drift in the sporadic E layer as observed in the auroral zone. 1962.
» 3. L. Harang and K. Malmjord: Determination of drift movements of the ionosphere at high latitudes from radio star scintillations. 1962.
» 4. Eyvind Riis: The stability of Couette-flow in non-stratified and stratified viscous fluids. 1962.
» 5. E. Frogner: Temperature changes on a large scale in the arctic winter stratosphere and their probable effects on the tropospheric circulation. 1962.
» 6. Odd H. Sælen: Studies in the Norwegian Atlantic Current. Part II: Investigations during the years 1954—59 in an area west of Stad. 1963.

Vol. XXIV.

In memory of Vilhelm Bjerknes on the 100th anniversary of his birth. 1962.

Vol. XXV.

No. 1. Kaare Pedersen: On the quantitative precipitation forecasting with a quasi-geostrophic model. 1963.
» 2. Peter Thrane: Perturbations in a baroclinic model atmosphere. 1963.
» 3. Eigil Hesstvedt: On the water vapor content in the high atmosphere. 1964.
» 4. Torbjørn Ellingsen: On periodic motions of an ideal fluid with an elastic boundary. 1964.
» 5. Jonas Ekman Fjeldstad: Internal waves of tidal origin. 1964.
» 6. A. Eftestøl and A. Omholt: Studies on the excitation of N_2 and N_2^+ bands in aurora. 1965.

Vol. XXVI.

No. 1. Eigil Hesstvedt: Some characteristics of the oxygen-hydrogen atmosphere. 1965.
» 2. William Blumen: A random model of momentum flux by mountain waves. 1965.
» 3. K. M. Storetvedt: Remanent magnetization of some dolerite intrusions in the Egersund Area, Southern Norway. 1966.
» 4. Martin Mork: The generation of surface waves by wind and their propagation from a storm area. 1966.
» 5. Jack Nordø: The vertical structure of the atmosphere. 1965.
» 6. Alv Egeland and Anders Omholt: Carl Størmer's height measurements of aurora. 1966.
» 7. Gunnvald Bøyum: The energy exchange between sea and atmosphere at ocean weather stations M, I and A. 1966.
» 8. Torbjørn Ellingsen and Enok Palm: The energy transfer from submarine seismic waves to the ocean. 1966.
» 9. Torkild Carstens: Experiments with supercooling and ice formation in flowing water. 1966.
» 10. Jørgen Holmboe: On the instability of stratified shear flow. 1966.
» 11. Lawrence H. Larsen: Flow over obstacles of finite amplitude. 1966.

Vol. XXVII.

No. 1. Arne Grammeltvedt: On the nonlinear computational instability of the equations of one-dimensional flow. 1967.
» 2. Jørgen Holmboe: Instability of three-layer models in the atmosphere. 1968.
» 3. Einar Høiland and Eyvind Riis: On the stability of shear flow of a stratified fluid. 1968.
» 4. Eigil Hesstvedt: On the effect of vertical eddy transport on atmospheric composition in the mesosphere and lower thermosphere. 1968.
» 5. Eigil Hesstvedt: On the photochemistry of ozone in the ozone layer. 1968.
» 6. Arnt Eliassen: On meso-scale mountain waves on the rotating earth. 1968.

A.s John Griegs Boktrykkeri