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BALANCED SYSTEMS OF EQUATIONS FOR THE ATMOSPHERIC MOTION

A Numerical Experiment, and an Analytical Discussion

By

KAARE PEDERSEN

FREMLAGT I VIDENSKAPS-AKADEMIETS MØTE DEN 26. FEBRUAR 1971 AV HESSTVEDT.

Summary. It is shown that the gradual simplification of the equations governing the motion of the atmosphere as presented by LORENZ (1960) agrees with the simplifications one would make considering the magnitude of the different terms. A 24-hour forecast using the 'Ψ-balanced system' of equations is presented. The boundary condition leading to minimum kinetic energy of the divergent flow is discussed. Finally the sets of equations are linearized, and by using a two level model the speed and growth rate of the waves in the different systems of equations are discussed.

1. The sets of equations. We shall use the following definitions for the horizontal velocity vector

$$\vec{v} = \vec{k} \times \nabla\Psi + \nabla\chi = \vec{v}_\Psi + \vec{v}_\chi \tag{1.1}$$

$\vec{k}$ is the vertical unit vector, Ψ the stream function, and χ the velocity potential.

For an adiabatic and frictionless atmosphere the prognostic equations are:

i) The thermodynamic energy equation

$$g\frac{\partial}{\partial t}\frac{\partial z}{\partial p} + g\vec{v} \cdot \nabla\frac{\partial z}{\partial p} + \sigma\omega = 0 \quad \text{(T.E.)} \tag{1.2}$$

where g is the acceleration of gravity, z the geopotential height, p the pressure, $\omega = \frac{Dp}{dt}$ the vertical motion and

$$\sigma = g\left(\frac{\partial^2 z}{\partial p^2} + \frac{1 - R/Cp}{p}\frac{\partial z}{\partial p}\right).$$

R is specific gas constant and Cp is the specific heat at constant pressure.

ii) The vorticity equation

$$(1.3)\quad \frac{\partial}{\partial t}\nabla^2\Psi+\vec{v}\cdot\nabla(\nabla^2\Psi+f)+\omega\frac{\partial}{\partial p}\nabla^2\Psi+(\nabla^2\Psi+f)\nabla\cdot\vec{v}+\vec{k}\cdot\nabla\omega\times\frac{\partial\vec{v}}{\partial p}=0\quad \text{(V.E.)}$$

where f is the coriolis parameter.

iii) The divergence equation

$$(1.4)\quad \frac{\partial}{\partial t}\nabla^2\chi+\nabla\cdot(v\cdot\nabla v)+\nabla\cdot\left(\omega\frac{\partial\vec{v}}{\partial p}\right)-\mathcal{J}(f,\chi)-\nabla\cdot f\nabla\Psi+g\nabla^2 z=0\quad \text{(D.E.)}$$

where $\mathcal{J}$ denotes the Jacobian.

The continuity equation is

$$(1.5)\quad \nabla^2\chi+\frac{\partial\omega}{\partial p}=0$$

We will rewrite the equations separating the terms (LORENZ 1960). The vorticity equation takes the form

$$\begin{aligned}(1.6)\quad 0=&\frac{\partial}{\partial t}\nabla^2\Psi+\vec{v}_\Psi\cdot\nabla f+\vec{v}_\Psi\cdot\nabla\nabla^2\Psi+f\nabla^2\chi+d\{\nabla f\cdot\nabla\chi\}\\ &+b\left\{\vec{v}_\chi\cdot\nabla\nabla^2\Psi+\omega\frac{\partial}{\partial p}\nabla^2\Psi+\nabla^2\Psi\nabla^2\chi+\vec{k}\cdot\nabla\omega\times\frac{\partial\vec{v}_\Psi}{\partial p}\right\}\\ &+a\left\{\vec{k}\cdot\nabla\omega\times\frac{\partial\vec{v}_\chi}{\partial p}\right\}.\end{aligned}$$

The divergence equation may be written

$$\begin{aligned}(1.7)\quad -\frac{\partial}{\partial t}\nabla^2\chi=&-\nabla\cdot f\nabla\Psi+\nabla^2\phi+b\nabla\cdot(\vec{v}_\Psi\cdot\nabla\vec{v}_\Psi)\\ &+a'\left\{\nabla\cdot(\vec{v}_\Psi\cdot\nabla\vec{v}_\chi)+\nabla\cdot(\vec{v}_\chi\cdot\nabla\vec{v}_\Psi)+\nabla\cdot\left(\omega\frac{\partial\vec{v}_\Psi}{\partial p}\right)\right\}\\ &+a\left\{\mathcal{J}(\chi,f)+\nabla\cdot(\vec{v}_\chi\cdot\nabla\vec{v}_\chi)+\nabla\cdot\left(\omega\frac{\partial v_\chi}{\partial p}\right)\right\}\end{aligned}$$

The thermodynamic energy equation becomes

$$(1.8)\quad 0=\frac{\partial}{\partial t}\frac{\partial\Phi}{\partial p}+\vec{v}_\Psi\cdot\nabla\frac{\partial\Phi}{\partial p}+\sigma\omega+d\left\{\vec{v}_\chi\cdot\nabla\frac{\partial\Phi}{\partial p}\right\}$$

If $\frac{\partial}{\partial t}\nabla^2\chi=0$ and $d=b=a=a'=1$ we have the 'χ-balanced system'. If $\frac{\partial}{\partial t}\nabla^2\chi=0$ and

$a=a'=0$ and $d=b=1$ we have the 'ψ-balanced system'. If $\frac{\partial}{\partial t}\nabla^2\chi=0$, $a'=a=b=0$ and $d=1$ we have the 'geostrophic balanced system'. And finally if $\frac{\partial}{\partial t}\nabla^2\chi=0$, $a'=a=b=d=0$ we have the 'simplified geostrophic balanced system' from which one may derive the 'ω-equation'.

$$(1.9)\qquad \nabla^2(\sigma\omega)+f^2\frac{\partial^2\omega}{\partial p^2}=f\frac{\partial}{\partial p}(\vec{v}_\Psi\cdot\nabla(\nabla^2\Psi+f))-g\nabla^2\left\{\vec{v}_\Psi\cdot\frac{\partial z}{\partial p}\right\}.$$

One finds that this gradual simplification done by LORENZ (1960) also corresponds to simplifications one would make considering the magnitude of the different terms. Tests on actual cases or characteristic scale studies show that the terms in the V.E. with an a in front have an order of magnitude of $10^{-12}s^{-2}$, while those with a b in front are of an order of magnitude of $10^{-11}s^{-2}$ and the rest of the terms are about $10^{-10}s^{-2}$. In the D.E. the terms preceded by a are of an order of magnitude of $10^{-12}s^{-2}$, those preceded by a' of an order of magnitude of $10^{-11}s^{-2}$, the term with b in front is of the order of magnitude $10^{-10}s^{-2}$ and the rest of the order of magnitude of $10^{-9}s^{-2}$.

In a paper by PEDERSEN & GRØNSKEI (1969) solutions of the three systems: the 'χ-balanced system', the 'Ψ-balanced system', and the 'simplified geostrophic balanced system' were studied. It was there found that while the 'simplified geostrophic balanced system' and the 'Ψ-balanced system' gave little differences in the amplitude of ω, the 'χ-balanced system' gave an increased amplitude of ω of about 30%. The aim of the present paper is to study the different sets of equations, using a linearized two-level model. But before entering on this, the author would like to show that the boundary conditions for the velocity potential applied in the paper by PEDERSEN & GRØNSKEI are those giving the divergent fields with a minimum kinetic energy.

2. Boundary conditions. A 24-hour forecast. We consider a region G with a boundary curve S. We may consider the divergent field as made up of two parts, one, χ_1, determined by

$$(2.1)\qquad \nabla^2\chi_1=\nabla^2\chi \text{ in } G \text{ and } \chi_1=0 \text{ on } S.$$

The other part is given by

$$(2.2)\qquad \nabla^2\chi_2=0 \text{ in } G \text{ and } \chi_2=\chi_2(s) \text{ on } S.$$

so that

$$(2.3)\qquad \nabla^2\chi_1+\nabla^2\chi_2=\nabla^2\chi \text{ in } G$$

and the unspecified boundary value of χ is χ_2. The kinetic energy of the divergent flow is

$$(2.4)\qquad \tfrac{1}{2}\int_G(\nabla(\chi_1+\chi^2))^2\delta\sigma=\tfrac{1}{2}\int_G(\nabla\chi_1)^2\delta\sigma+\tfrac{1}{2}\int_G(\nabla\chi_2)^2\delta\sigma+\int_G\nabla\chi_1\cdot\nabla\chi_2\delta\sigma$$

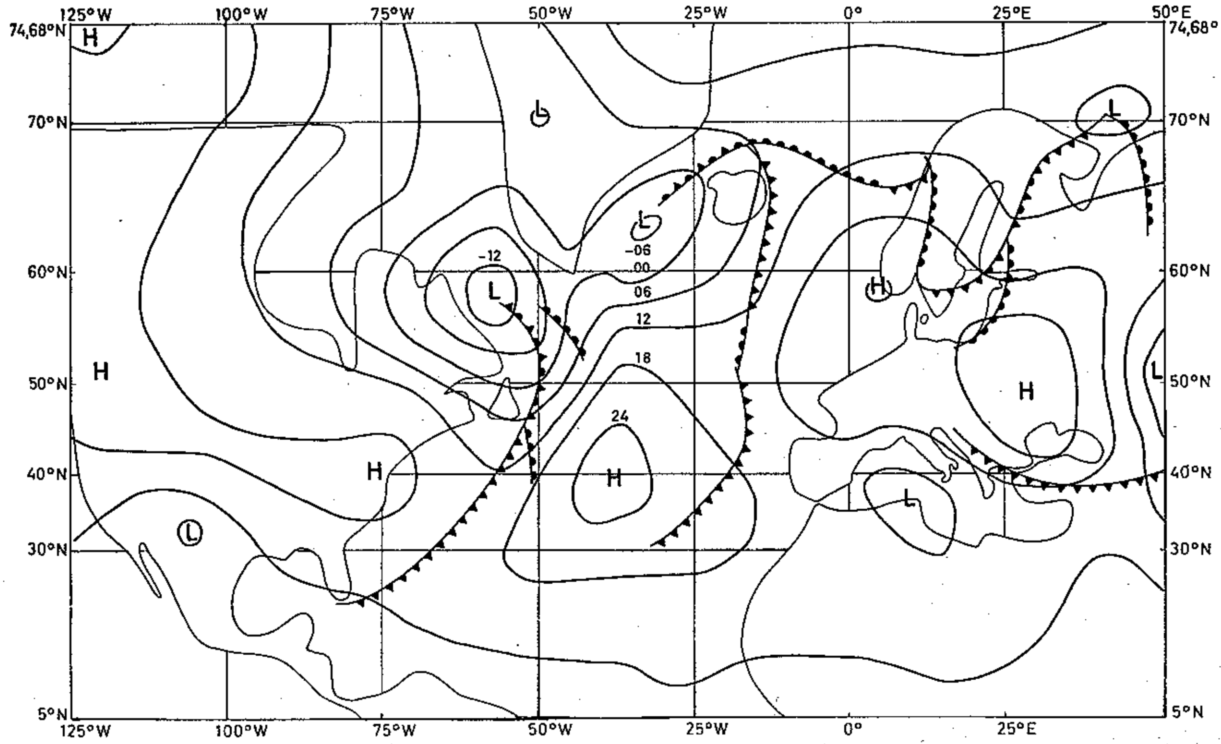

Fig. 1. Z 1000 mb. Unit: 10 m.

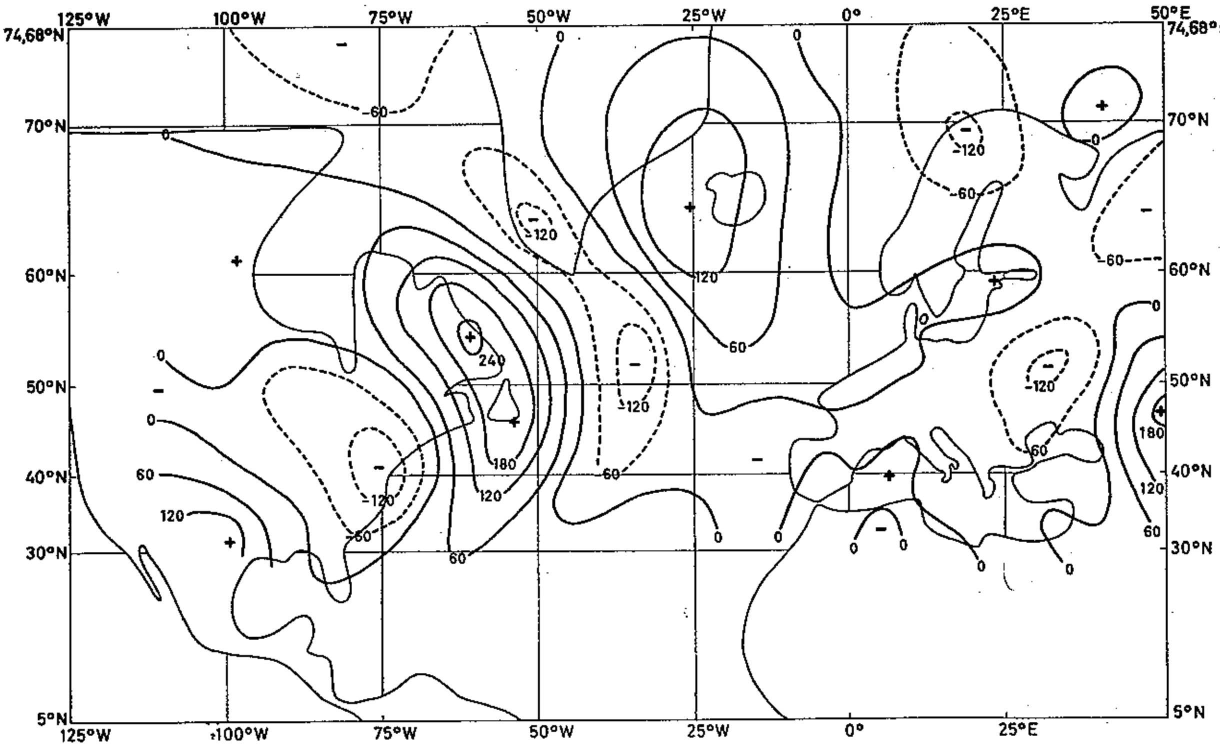

Fig. 2. Observed 24 hour height changes of the 1000 mb level. Unit m.

But the third integral on the right hand side of (3.4) may be written

$$\int_G \nabla \cdot (\chi_1 \nabla \chi_2)\delta\sigma - \int_G \chi_1 \nabla^2 \chi_2 \delta\sigma \tag{2.5}$$

or

$$\int_S \chi_1 \frac{\partial \chi_2}{\delta n}\delta s - \int_G \chi_1 \nabla^2 \chi_2 \delta\sigma \tag{2.6}$$

which with the conditions (2.1) and (2.2) are seen to be zero. The kinetic energy of the divergent flow is therefore a minimum when $\int_G (\nabla\chi_2)^2 \delta\sigma$ is a minimum, that is when $\chi_2(s) = 0$.

As described in section 5 in the paper by PEDERSEN & GRØNSKEI (1969), the equation (1.4) may be used prognostically. With the initial data used in that paper a 24-hour forecast was made using the 'Ψ-balanced system' of equations. The geopotential heights of the 1000 mb level are shown in Fig. 1. Figs. 2 and 3 show the observed and computed 24-hour height changes. The latitude 51.3° N is the central gridpoint line in our area. The observed and computed height changes along this gridline are shown in Fig. 4, together with their difference. The mean value of this difference is found to be 36 m. It is seen that the error seems to have a wavelength of about half the latitudinal width of our area, that is latitudinal wavenumber 4 corresponding to a wavelength at this latitude of about 6000 km.

3. The linearized equations. The basic flow is a constant vertical shear flow, the east-west wind being

$$U = U(p) = -\frac{1}{f}\frac{\partial \Phi}{\partial y}, \quad \frac{dU}{dp} = \text{const.}$$

We will apply the β-plane approximation

$$f = f_0 + \beta_0 y .$$

The static stability σ will be assumed constant. Linearization of the equations (1.6)—(1.8) gives

$$\frac{\partial}{\partial t}\nabla^2\Psi + U\frac{\partial}{\partial x}\nabla^2\Psi + \beta v_\Psi - f\frac{\partial \omega}{\partial p} + d\{\beta v_\chi\} - b\left\{\frac{dU}{dp}\frac{\partial \omega}{\partial y}\right\} = 0 \tag{3.1}$$

$$\frac{\partial}{\partial t}\nabla^2\chi = -\nabla^2\Phi + \nabla \cdot f\nabla\Psi - a'\left\{U\frac{\partial}{\partial x}\nabla^2\chi + \frac{dU}{dp}\frac{\partial \omega}{\partial x}\right\} - a\left\{J(\chi, f)\right\} \tag{3.2}$$

$$\frac{\partial}{\partial t}\frac{\partial \Phi}{\partial p} + U\frac{\partial}{\partial x}\frac{\partial \Phi}{\partial p} - f\frac{dU}{dp}\frac{\partial \Psi}{\partial x} + \sigma\omega - d\left\{f\frac{dU}{dp}\frac{\partial \chi}{\partial y}\right\} = 0 \tag{3.3}$$

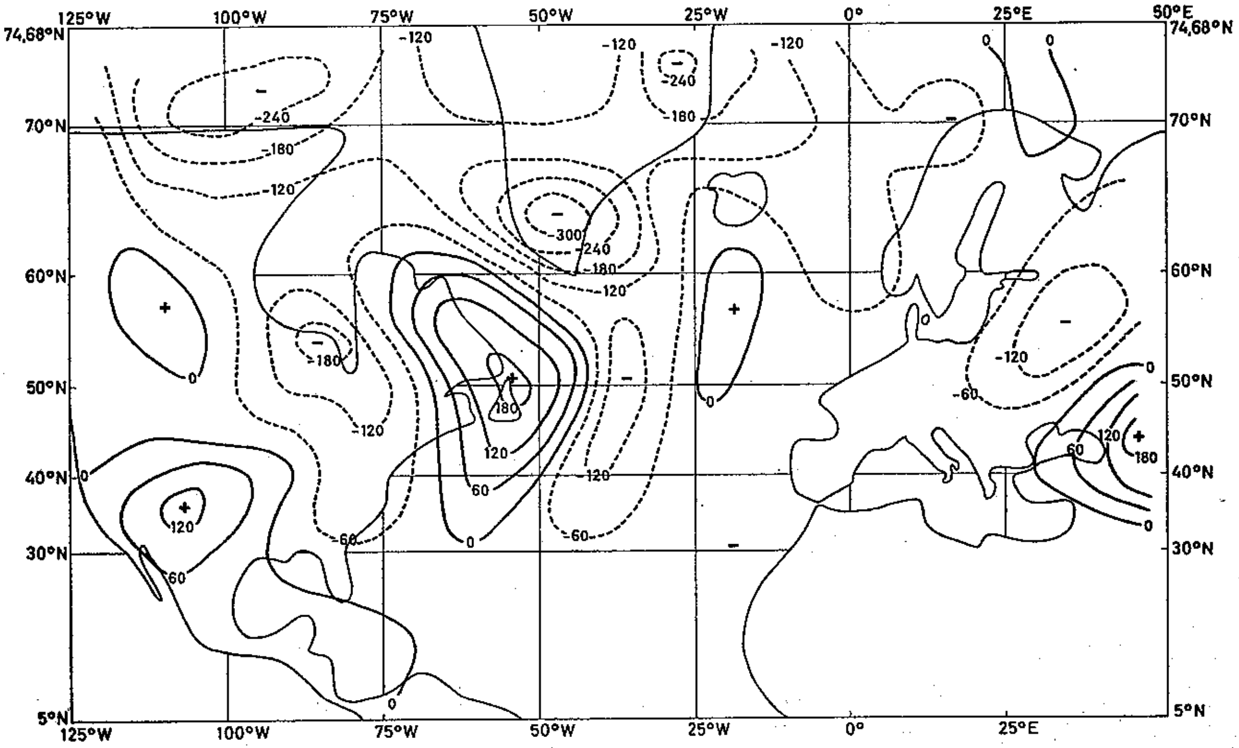

Fig. 3. Computed 24 hour height changes of the 1000 mb level. Unit m.

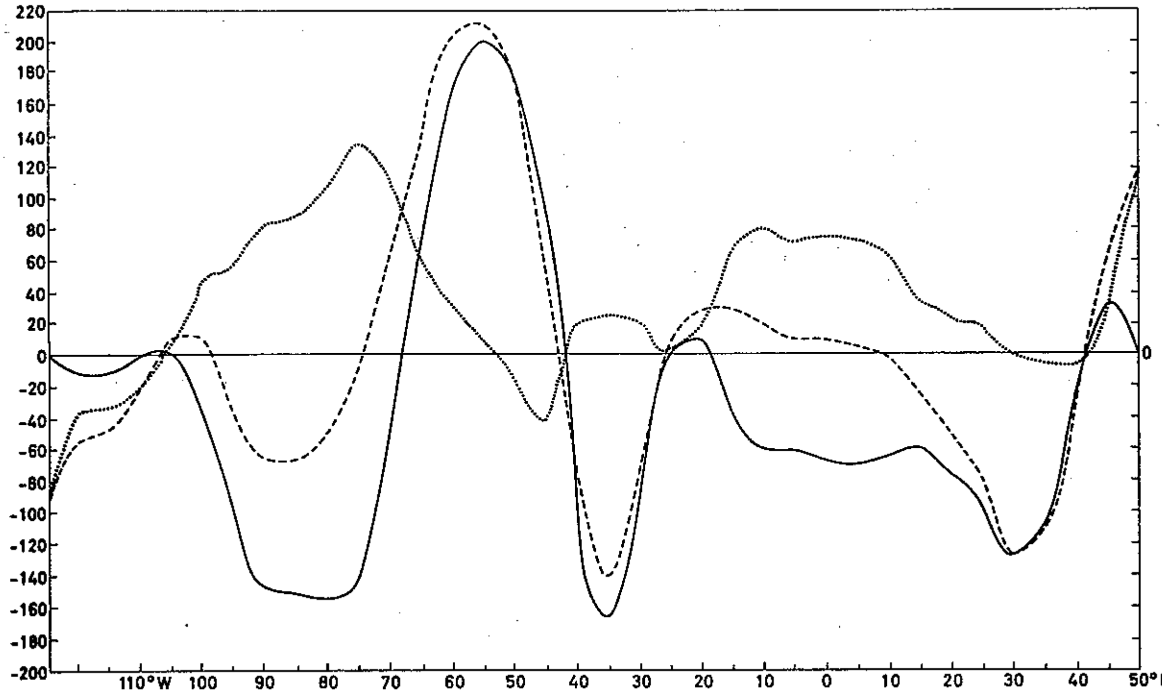

Fig. 4. The observed (- - -) and computed (——) 24 hour height changes along latitude 51.3°N. Dotted curve gives their difference.

The geostrophic balanced system has been studied by many authors, see GARCIA & NORSCINI (1970). In order to treat the two other systems we will simplify to the two level model (see PHILLIPS 1951). The notations we will apply are shown in Fig. 5. We will assume:

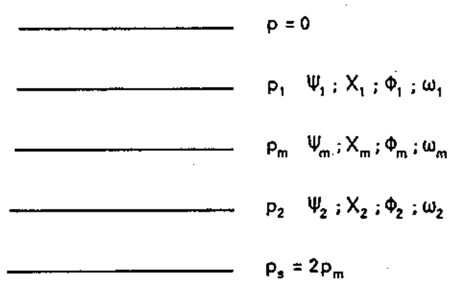

Fig. 5. Notations used in the text.

$$(3.4) \qquad \omega_1 = \omega_2 = \tfrac{1}{2}\omega_m\,, \quad \frac{\partial \omega_1}{\partial p} = \frac{\omega_m}{p_m}\,, \quad \frac{\partial \omega_2}{\partial p} = -\frac{\omega_m}{p_m}\,,$$

For the other variables the following notations will be used

$$\frac{\alpha_1 + \alpha_2}{2} = \alpha_m \text{ and } \frac{\alpha_1 - \alpha_2}{2} = \alpha'$$

We will require that the mean divergence is zero: $\chi_m = 0$. For the sake of simplicity we will put $U_m = 0$ and denote $\frac{1}{2}(U_1 - U_2) = U$. The equations now become:

$$(3.5) \qquad \frac{\partial}{\partial t}\nabla^2\Psi_m + U\frac{\partial}{\partial x}\nabla^2\Psi' + \beta\frac{\partial \Psi_m}{\partial x} + bU\frac{\partial}{\partial y}\left(\frac{\omega_m}{p_m}\right) = 0$$

$$(3.6) \qquad \frac{\partial}{\partial t}\nabla^2\Psi' + U\frac{\partial}{\partial x}\nabla^2\Psi_m - f\left(\frac{\omega_m}{p_m}\right) + \beta\frac{\partial \Psi'}{\partial x} + d\beta\frac{\partial \chi'}{\partial y} = 0$$

$$(3.7) \qquad -f\nabla^2\Psi_m + \nabla^2\Phi_m - \beta\frac{\partial \Psi_m}{\partial y} - 2a'U\frac{\partial}{\delta x}\left(\frac{\omega_m}{p_m}\right) = 0$$

$$(3.8) \qquad -f\nabla^2\Psi' + \nabla^2\Phi' - \beta\frac{\partial \Psi'}{\partial y} + a\beta\frac{\partial \chi'}{\partial x} = 0 = -\frac{\partial}{\partial t}\nabla^2\chi'$$

$$(3.9) \qquad \frac{\partial}{\partial t}\Phi' - fU\frac{\partial \Psi_m}{\partial x} - \gamma^2\frac{\omega_m}{p_m} = 0$$

$$(3.10) \qquad \nabla^2\chi' + \frac{\omega_m}{p_m} = 0$$

here

$$\gamma^2 = p_m\alpha_m\frac{\Theta_1 - \Theta_2}{2\Theta_m}$$

We assume harmonic perturbations

$$\alpha = \hat{\alpha}e^{ik(x-ct)}e^{ily}$$

and we denote $k^2 + l^2 = n^2$.

From equations (3.7) and (3.9) one obtains

$$(3.11) \qquad \frac{\hat{\omega}_m}{p_m} = \frac{-ik(U\hat{\Phi}_m + (1 - iB)c\hat{\Phi}')}{\gamma^2(1 - 2aA - iB)}$$

where

$$A=\frac{k^2}{n^2}\frac{U^2}{\gamma^2} \text{ and } B=\frac{\beta l}{fn^2}$$

By inserting into equations (3.5) and (3.6) from the other equations one may obtain two equations in $\hat{\Phi}_m$ and $\hat{\Phi}'$

$$\left\{(1-iB)c+(1+aA-iB)\frac{U_0}{\kappa^2}-ib(1-iB)\frac{U^2}{\gamma^2}c_l\right\}\hat{\Phi}_m$$
$$-\left\{-2aA(a-iB)\frac{c^2}{U}+ib(1-iB)\frac{U^2}{\gamma^2}\frac{c_l}{U}c-3aA(1-iB)\frac{U_0}{U}\frac{1}{\kappa^2}c\right.$$
$$\left.+(1-2aA-iB)U\right\}\hat{\Phi}'=0 \tag{3.12}$$

and

$$\left\{-(1-B)+(1-B)(1-dB)\frac{1}{\kappa^2}-aA\frac{U^2_0}{U^2}\frac{1}{\kappa^4}-aA\frac{U_0}{U}\frac{1}{\kappa^2}c\right\}U\hat{\Phi}_m$$
$$+\left\{(1-4aA-B+2iaAB)c+(1-iB)^2(1-idB)\frac{c}{\kappa^2}-aA(1-iB)\frac{U_0^2}{U^2}\frac{1}{\kappa^4}c\right.$$
$$\left.-aA(1-iB)\frac{U_0}{U}\frac{1}{\kappa^2}c+(1-2aA-iB)\frac{U_0}{\kappa^2}\right\}\hat{\Phi}=0 \tag{3.13}$$

here

$$U_0=\frac{\beta}{n_0^2},\quad n_0=\frac{f}{\gamma},\quad \kappa=\frac{n}{n_0} \text{ and } c_l=\frac{fl}{n^2}$$

From (3.12) and (3.13) we get the following equation for $\frac{c}{U_0}$

$$-aA\frac{U_0^2}{U^2}\left(\frac{c}{U_0}\right)^3+\left\{1+\kappa^2-(iB+idB+dB^2)-2aA\frac{U_0^2}{U^2}\frac{1}{\kappa^2}\right\}\left(\frac{c}{U_0}\right)^2$$
$$+\left\{2\kappa^2+1-(iB+idB+dB^2)-ib(2-iB)\frac{U^2}{\gamma^2}\cdot\frac{c_l}{U_0}\kappa^4+aA\kappa^2\right.$$
$$\left.-aA\frac{U_0^2}{U^2}\frac{1}{\kappa^2}\right\}\frac{1}{\kappa^2}\frac{c}{U_0}$$
$$+\left\{(1-\kappa^2)\left(\frac{U}{U_0}\right)^2+\frac{1}{\kappa^2}-idB\left(\frac{U}{U_0}\right)^2-ib\frac{U^2}{\gamma^2}\frac{c_l}{U_0}\right\}=0 \tag{3.14}$$

4. The effect of the a term. If we consider one-dimensional perturbations, that is $l=0$, equation (3.14) reduces to

$$-a\frac{U_0^2}{\gamma^2}\left(\frac{c}{U_0}\right)^3+\left\{1+\kappa^2-2a\frac{U_0^2}{\gamma^2}\frac{1}{\kappa^2}\right\}\left(\frac{C}{U_0}\right)^2 \tag{4.1}$$

$$+\left\{2\kappa^2+1+a\frac{U^2}{\gamma^2}\kappa^2-a\frac{U_0^2}{\gamma^2}\right\}\frac{1}{\kappa^2}\frac{1}{k^2}\frac{C}{U_0}$$

$$+\left\{(1-\kappa^2)\left(\frac{U}{U_0}\right)^2+\frac{1}{\kappa^2}\right\}=0$$

In this case it is seen that the equation becomes the same for the 'simplified geostrophic', the 'geostrophic', and the 'Ψ-balanced systems', namely

$$\{1+\kappa^2\}\left(\frac{C}{U_0}\right)^2+\frac{2\kappa^2+1}{\kappa^2}\frac{C}{U_0}+(1-\kappa^2)\left(\frac{U}{U_0}\right)^2+\frac{1}{\kappa^2}=0 \tag{4.2}$$

This equation is thoroughly discussed by WIIN-NIELSEN (1963). If we assume the following values for the parameters:

$$\gamma=50\text{ m/s},\quad f=10^{-4}\text{s}^{-1},\quad \beta=1.5\times10^{-11}\text{s}^{-1}\text{m}^{-1}$$

we get $n_0\approx2\times10^{-6}\text{m}^{-1}$, corresponding to a wavelength $L_0\approx3\times10^6\text{m}$, and $U_0\approx4$ m/s. A realistic value of U is about $2U_0$. For this value of U equation (4.2) gives complex roots for $0.52<\kappa<0.99$ (see Fig. 6). In this range of κ the terms in (4.1) containing

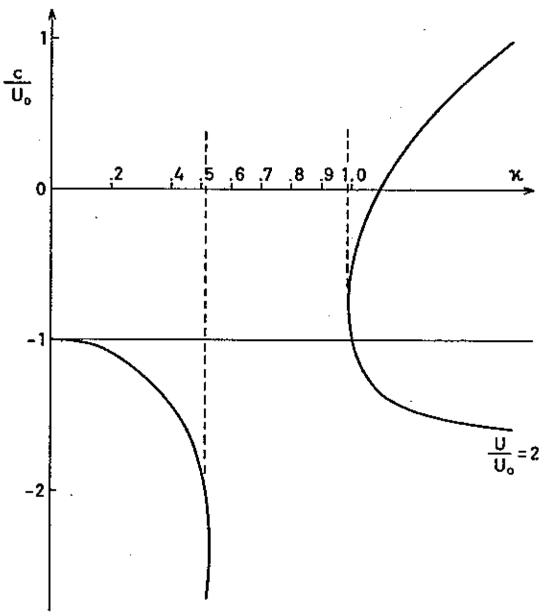

Fig. 6. The full drawn curves give the values of c/U_0 when they are real.

'a' are found to be small compared to unity, thus giving rise to only minor changes in the two roots corresponding to the roots of equation (4.2). Denoting the terms in equation (4.2) by

$$A_0 x_0^2 + B_0 x_0 + C_0 = 0 \tag{4.3}$$

and correspondingly the terms in equation (4.1) by

$$\Delta D(x_0 + \Delta x)^3 + (A_0 + \Delta A)(x_0 + \Delta x)^2 + (B_0 + \Delta B)(x_0 + \Delta x) + C_0 = 0 \tag{4.4}$$

we find a first estimate of Δx by

$$\Delta x = -\frac{\Delta D x_0^3 + \Delta A x_0^2 + \Delta B x_0}{2A_0 x_0 + B_0} \tag{4.5}$$

In our case

$$x_0 = -\frac{2\kappa^2 + 1}{2\kappa^2(1+\kappa^2)} \pm \frac{\sqrt{1 - 4\kappa^4(1-\kappa^4)\left(\frac{U}{U_0}\right)^2}}{2\kappa^2(1+\kappa^2)}$$

$$\Delta A = -2a\frac{U_0^2}{\gamma^2}\frac{1}{\kappa^2}, \quad \Delta B = \left(a\frac{U^2}{\gamma^2}\kappa^2 - a\frac{U_0^2}{\gamma^2}\frac{1}{\kappa}\right)\frac{1}{\kappa^2} \text{ and } \Delta D = -a\frac{U_0^2}{\gamma^2}.$$

For $\kappa^4 = 0.5$ or $\kappa = 0.84$ $4\kappa^4(1-\kappa^4)$ has its maximum value. For this value we find

$$x_0 = \frac{C}{U_0} = -1.00 \pm 0.72i$$

and

$$x_0 + \Delta x = -1.01 \pm 0.71i$$

as we see rather small changes. Also, for the amplitude of $\hat{\omega}_m$ as may be estimated from equation (3.11) one would expect small changes, in contradiction to what we found from the nonlinear equations. The result, however, is a small decrease in the growth rate for κ values larger than about 0.7 (ΔB equal to zero for $\kappa = 0.7$) and a slight increase in the westward propagation speed. For larger κ values one would expect a relatively larger change of the roots; we find for $\kappa = 0.9$:

$$x_0 = \frac{C}{U_0} = -0.89 \pm 0.55i$$

and

$$x_0 + \Delta x = -0.90 \pm 0.54i\,.$$

We may estimate the third root of equation (4.1) from the expression:

$$-a\frac{U_0^2}{\gamma^2}C^3 C_{01} C_{02} = -\left\{(1-\kappa^2)\left(\frac{U}{U_0}\right)^2 + \frac{1}{\kappa^2}\right\} \tag{4.6}$$

We find

$$C^2 = \frac{\gamma^2}{U_0^2}(\kappa^2 + 1)$$

which is an unphysical velocity of about 1000 m/s.

When $\kappa \to \infty$ we find that equation (4.1) has the same two roots as equation (4.2) namely $\frac{C}{U_0} = \pm\frac{U}{U_0}$. When $\kappa \to 0$ we find that equation (4.1) has the root $\frac{C}{U_0} = 0$, while equation (4.2) has the root $\frac{C}{U_0} = -1$.

5. The effect of the b terms. It is seen that both B and c_l have maximum values for $l = k$. We will therefore assume perturbations of that form. With $a = 0$ equation (3.14) becomes

$$\{(1+\kappa^2) - (iB + idB + dB^2)\}\left(\frac{C}{U_0}\right)^2 + \left\{2\kappa^2 + 1 - (iB + idB + dB^2) - ib(2 - iB)\frac{U^2}{\gamma^2}\frac{c_l}{U_0}\kappa^4\right\}\frac{1}{\kappa^2}\frac{C}{U_0} + \left\{(1-\kappa^2)\left(\frac{U}{U_0}\right)^2 + \frac{1}{\kappa^2} - idB\left(\frac{U}{U_0}\right)^2 - ib\frac{U^2}{\gamma^2}\frac{c_l}{U_0}\right\} = 0 \tag{5.1}$$

When $\kappa \to 0$ we find that B and $c_l \to \infty$ as κ; and when $\kappa \to \infty$ we find that B and $c_l \to 0$ as κ. The roots of equation (5.1) as $\kappa \to \infty$ are therefore found to be $\pm\frac{U}{U_0}$ and as $\kappa \to 0$ we find the root zero, as for equation (4.1). For κ values of the order unity we find that $B \approx 0.05$ while $\frac{U^2}{\gamma^2}\frac{c_l}{U_0} \approx 0.2$. We may therefore expect to find an approximate value of Δx_0 by putting

$$\Delta B = -2ib\frac{U^2}{\gamma^2}\frac{c_l}{U_0}\kappa^2 \text{ and } \Delta C = -idB\left(\frac{U}{U_0}\right)^2 - ib\frac{U^2}{\gamma^2}\frac{c_l}{U_0}$$

For $\kappa^4 = 0.50$ we find $B = 0.06$ and $\frac{U^2 c_l}{\gamma^2 U_0} = 0.26$, which gives

$$x_0 + \Delta x = -0.94 \pm 0.82i \tag{5.2}$$

The effect of the 'b' terms only would give

$$x_0 + \Delta x = -1.03 \pm 0.82i \tag{5.3}$$

We find that the increase of the complex value arises from the 'b' term alone. The effect on the growth of the perturbation is opposite to what we found for the 'a' term, and it is an order of magnitude larger.

6. Discussion. That the effect of the 'a' term in a linearized model is so small might have been expected since the magnitude of the 'a' term in equation (1.7) is three orders of magnitude smaller than the largest terms in the equation. In a non-linear model the effect of the 'a'' terms will enter. The effect of these terms on the Ψ-field is not large in magnitude; in fact it was found by Pedersen and Grønskei that the difference

$$\left|\vec{v}_\Psi(\Psi\text{-balanced}) - \vec{v}_g\right| \approx 2\left|\vec{v}_\Psi(\chi\text{-balanced}) - \vec{v}_\Psi(\Psi\text{-balanced})\right|.$$

However, the difference $\vec{v}_\Psi(\chi\text{-balanced}) - \vec{v}_\Psi(\Psi\text{-balanced})$ was perpendicular to the thickness pattern and thus the effect on the temperature advection was large, resulting in a marked difference in the ω-values of the two systems.

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